Synchronous Languages—Lecture 14

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Sequentially Constructive Concurrency in Practice

The 5-Minute Review Session

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5. What is a *thread tree*? How can it be used to define static concurrency?

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4. What is an *SC-schedule*? When is it *valid*?
5. What are conservative, practical approximations of sequential constructiveness?

References

Most of the material here draws from this reference [TECS]:

R. von Hanxleden, M. Mendler, J. Aguado, B. Duderstadt, I. Fuhrmann, C. Motika, S. Mercer, O. O'Brien, and P. Roop.
Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.
ACM Transactions on Embedded Computing Systems, Special Issue on Applications of Concurrency to System Design, July 2014, 13(4s).
https://rtsys.informatik.uni-kiel.de/~biblio/downloads/papers/tecs14.pdf

Unless otherwise noted, the numberings of definitions, sections etc. refer to this.

There is also an extended version [TR]:

R. von Hanxleden, M. Mendler, J. Aguado, B. Duderstadt, I. Fuhrmann, C. Motika, S. Mercer, O. O'Brien, and P. Roop.
Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.
Christian-Albrechts-Universität zu Kiel, Department of Computer Science, Technical Report 1308, ISSN 2192-6247, Aug. 2013, 13(4s).
https://rtsys.informatik.uni-kiel.de/~biblio/downloads/papers/report-1308.pdf

Overview

Conservative Static Approximation

In practice, a compiler must be conservative:

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- May not recognize confluence
- May not recognize that writes are relative

SC-Schedules [Def. 5.1, Lemma 5.3]

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- SC-schedule Σ is subset of G's instantaneous edges: $\Sigma \subseteq E_{ins}$
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Lemma: E_{ins} is valid

Schedule order [Def. 5.2]

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However: valid schedule may still contain conflicting orderings that cannot be satisfied or where it depends on the capabilities of the compiler or the run-time system whether it can be implemented

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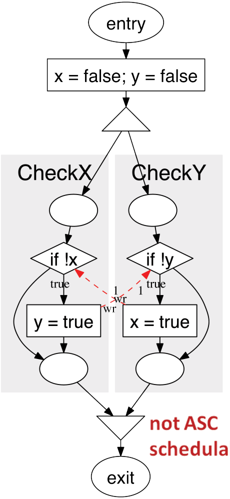
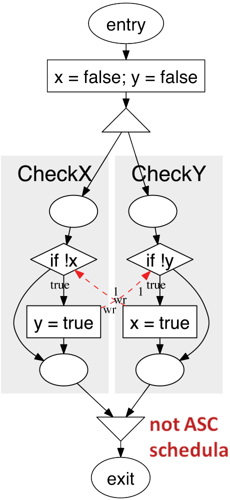
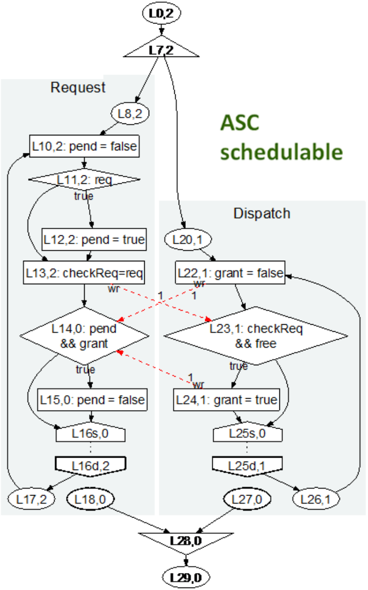
May also relax the sequential order to only order non-confluent statements $\rightsquigarrow$ data-flow acyclic programs



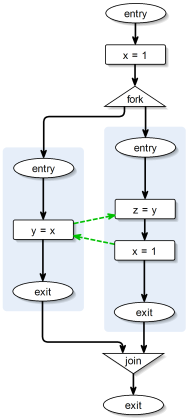
not ASC schedulable

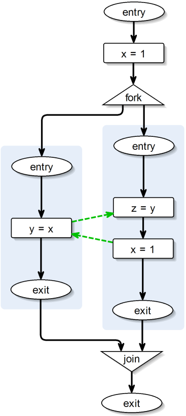


not ASC schedulable



ASC schedulable


entry
x = 1
fork
entry
entry
z = y
y = x
x = 1
exit
exit
join
exit



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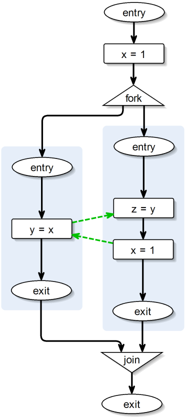

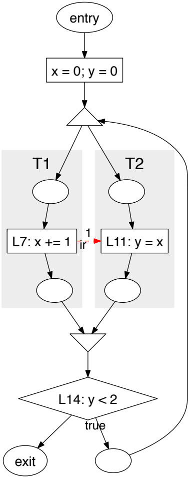

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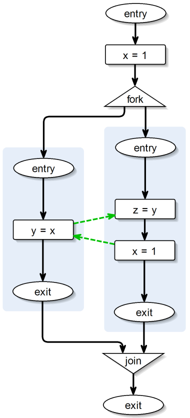

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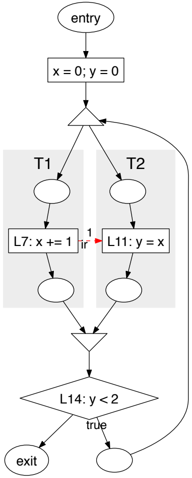

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Overview

Conservative Static Approximation of SC

Determining SC-Schedules with Priorities

- Priority-Based Scheduling [Sec. 5.2]
- Computing Priorities [Sec. 5.3]

Summary

Priorities [Def. 5.6, Lemma 5.7]

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Lemma: Priorities implement the schedule order
Given:

- Priority assignment according to some SC-schedule Σ
- Run-time (and hence also statically) concurrent statements $n_{1,2} \in N$

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- For example $n_1 \rightarrow_{iu} n_2$ implies $n_1 \rightarrow_{iur} n_2$, which implies, by definition of priorities, $n_1.pr > n_2.pr$, which in turn implies that n_1 gets scheduled before n_2

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Theorem

A program is IASC iff there exists a valid SC-schedule such that all statement priorities are finite

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- However, can exclude all graphs with a positive weight cycle

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1. Detect whether Σ has a positive weight cycle.
   We can do so by computing the Strongly Connected Components (SCCs), *e.g.*, by Tarjan's algorithm, and checking if any SCC contains a node that is connected to another node within the same SCC by a $\rightarrow_{iur}$ edge.

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2. If a positive weight cycle exists, then Σ is not *iur*-acyclic; we then **reject** the program.
   Otherwise, we **accept** the program, and continue.
   Now nodes in the same SCC can reach each other, but only through paths with weight 0, and therefore must have the same priority.

Algorithm for Computing Priorities II

3. From the SCCs, construct the directed acyclic graph $G_{SCC} = (N_{SCC}, E_{SCC})$, where $N_{SCC} \subset N$ contains a representative node from each SCC of G (using *e.g.* the SCC roots computed by Tarjan's algorithm), and E_{SCC} contains an edge from one SCC representative to another iff the corresponding SCCs are connected in G.
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Complexity:

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Complexity: linear in size of SCG

Overview

Summary I

Underlying idea of sequential constructiveness rather simple

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However, precise definition of SC MoC not trivial

- Challenging to ensure conservativeness relative to Berry-constructiveness
- Plain initialize-update-read protocol does not accomodate, *e. g.*, signal re-emissions
- Restricting attention to *concurrent*, *non-confluent* node instances is key

Summary II

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Future work

- Plenty of it (SC+, optimized code gen, improved SCCharts transformations, . . .)
- Talk to us if you want to be part of it