Synchronous Languages—Lecture 14

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Sequentially Constructive
Concurrency in Practice
The 5-Minute Review Session
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1. What are goals and challenges in defining the SC MoC?
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2. What is *confluence* in the SC MoC?
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2. What is *confluence* in the SC MoC?
3. What is *thread reincarnation*?
4. In the SC MoC, when are threads considered *statically concurrent*?
5. What is a *thread tree*? How can it be used to define static concurrency?
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4. What is an SC-schedule? When is it valid?
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1. How is *run-time concurrency* defined? How does it relate to static concurrency?
2. What is *SC-admissibility*?
3. When is a program *sequentially constructive*?
4. What is an *SC-schedule*? When is it *valid*?
5. What are conservative, practical approximations of sequential constructiveness?
Most of the material here draws from this reference [TECS]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.

Unless otherwise noted, the numberings of definitions, sections etc. refer to this.

There is also an extended version [TR]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.
Overview

Conservative Static Approximation of SC
  SC-Schedules
  Schedule Order
  Schedule / Program Classes

Determining SC-Schedules with Priorities

Summary
Conservative Static Approximation

In practice, a compiler must be conservative:

- Use a relation $n_1 | n_2$ to over-approximate $n_1 |_R n_2$, i.e., what statements are concurrently invoked in the same tick,
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SC-Schedules [Def. 5.1, Lemma 5.3]

- Given: SCG $G = (N, E)$
- SC-schedule $\Sigma$ is subset of $G$'s instantaneous edges: $\Sigma \subseteq E_{ins}$
- $E_{ins}$ is structural SC-schedule; derived solely by analysis of the program structure
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- $E_{ins}$ is structural SC-schedule; derived solely by analysis of the program structure
- An SC-schedule $\Sigma$ is valid if
  - for every macro tick $R$ of $G$ which can be reached and executed under the SC-admissibility rules,
  - if $(n_1, i_1) \xrightarrow{\alpha}^R (n_2, i_2)$ for some node instances $(n_{1,2}, i_{1,2})$ in $R$ and some $\alpha \in \alpha_{ins}$,
  - then $(n_1 \xrightarrow{\alpha} n_2) \in \Sigma$. 

Validity guarantees:

- If $G$ is executed in an SC-admissible fashion,
  - then static node relations $\xrightarrow{\alpha}$ of $\Sigma$ are conservative over-approximation of dynamic relations $\xrightarrow{R\alpha}$ on node instances.
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**Lemma:** \( E_{\text{ins}} \) is valid
Schedule order [Def. 5.2]

- **Given:** Valid SC-schedule $\Sigma$
- **Schedule order:** $n_1 \xrightarrow{\Sigma}_{ins} n_2$ iff
  1. $n_1 \parallel n_2$ and
  2. $\Sigma$ contains a path from $n_1$ to $n_2$ that includes an iur-edge
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However: valid schedule may still contain conflicting orderings that cannot be satisfied or where it depends on the capabilities of the compiler or the run-time system whether it can be implemented
Schedule / Program Classes [Def. 5.4]

Schedule properties

▶ **acyclic**: does not contain any cycle
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Program (SCG) properties

- **Acyclic SC (ASC)**: $\exists$ valid acyclic SC-schedule
- **Iur-acyclic SC (IASC)**: $\exists$ valid iur-acyclic SC-schedule
- **Structurally acyclic SC (SASC)**: $E_{ins}$ is acyclic
- **Structurally iur-acyclic SC (SIASC)**: $E_{ins}$ is iur-acyclic

Implications (see also Theorem 5.5):

- $SASC \implies SIASC \implies IASC \implies SC$
- $SASC \implies ASC \implies IASC \implies SC$

May also relax the sequential order to only order non-confluent statements; data-flow acyclic programs
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May also relax the sequential order to only order non-confluent statements $\rightsquigarrow$ **data-flow acyclic** programs
entry

x = false; y = false

CheckX

if !x

if y

y = true

x = true

not ASC schedulable

exit
ASC, and hence SC, but not SIASC, hence not SASC

```
entry
x = 0; y = 0
exit
L14: y < 2
L7: x += 1
L11: y = x
```

T1 T2
entry
x = 0; y = 0
exit
L14: y < 2
L7: x += 1
L11: y = x

ir
true
ASC, and hence SC, but not SIASC, hence not SASC
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SC, but not IASC, and hence not SIASC/ASC/SASC
Overview

Conservative Static Approximation of SC

Determining SC-Schedules with Priorities
  Priority-Based Scheduling [Sec. 5.2]
  Computing Priorities [Sec. 5.3]

Summary
Priorities [Def. 5.6, Lemma 5.7]

- **Given**: valid SC-schedule $\Sigma$
- **Priority** $n.pr$ of statement $n \in N$: maximal number of $\rightarrow_{iur}$ edges traversed by any path in $\Sigma$ that originates in $n$
Priorities [Def. 5.6, Lemma 5.7]

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**Lemma:** Priorities implement the schedule order

**Given:**
- Priority assignment according to some SC-schedule $\Sigma$
- Run-time (and hence also statically) concurrent statements $n_1, n_2 \in N$

**Then:** $n_1 \rightarrow_{ins}^{\Sigma} n_2$ implies
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**Then:** $n_{1} \xrightarrow{\Sigma_{ins}} n_{2}$ implies $n_{1}.pr > n_{2}.pr$
Priority-Based Scheduler [Theorem 5.8]

Priority-based scheduler: always gives control to the thread with
Priority-Based Scheduler [Theorem 5.8]

Priority-based scheduler: always gives control to the thread with highest priority, chosen from the set of threads that are

Never allows a statement that is ready for execution to wait on another statement with lower priority

Implements a valid schedule, as can be verified from the SCG construction

For example

\( \text{n}_1 \rightarrow \text{iu} \text{n}_2 \) implies \( \text{n}_1 \rightarrow \text{iur} \text{n}_2 \), which implies, by definition of priorities, \( \text{n}_1.pr > \text{n}_2.pr \), which in turn implies that \( \text{n}_1 \) gets scheduled before \( \text{n}_2 \)
Priority-Based Scheduler [Theorem 5.8]

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Theorem
A program is IASC iff
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Theorem
A program is IASC iff there exists a valid SC-schedule such that all statement priorities are finite
Computing Priorities for IASC Programs

Given a valid SC-schedule Σ, can formulate the calculation of priorities as longest weighted path problem.

Assign to each edge $e \in \Sigma$ a weight $w_e$, with $e \cdot w_e = 0$ iff $e \cdot \text{src} \to \text{seq} e \cdot \text{tgt}$, and $e \cdot w_e = 1$ iff $e \cdot \text{src} \to \text{iur} e \cdot \text{tgt}$.

As relations $\to \text{iur}$ and $\to \text{seq}$ exclude each other, weight of each edge is uniquely determined.

The difficulty is to handle (sequential) loops, i.e., cyclic SCGs.

For arbitrary (i.e., possibly cyclic) weighted graphs, the computation of the longest weighted path is NP-hard.

However, can exclude all graphs with a positive weight cycle.
Computing Priorities for IASC Programs

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- However, can exclude all graphs with a positive weight cycle.
Algorithm for Computing Priorities I

1. Detect whether Σ has a positive weight cycle. We can do so by computing the Strongly Connected Components (SCCs), e.g., by Tarjan’s algorithm, and checking if any SCC contains a node that is connected to another node within the same SCC by a → iur edge.

2. If a positive weight cycle exists, then Σ is not iur-acyclic; we then reject the program. Otherwise, we accept the program, and continue. Now nodes in the same SCC can reach each other, but only through paths with weight 0, and therefore must have the same priority.
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2. If a positive weight cycle exists, then $\Sigma$ is not $iur$-acyclic; we then reject the program. Otherwise, we accept the program, and continue. Now nodes in the same SCC can reach each other, but only through paths with weight 0, and therefore must have the same priority.
Algorithm for Computing Priorities II

3. From the SCCs, construct the directed acyclic graph $G_{SCC} = (N_{SCC}, E_{SCC})$, where $N_{SCC} \subset N$ contains a representative node from each SCC of $G$ (using e.g. the SCC roots computed by Tarjan’s algorithm), and $E_{SCC}$ contains an edge from one SCC representative to another iff the corresponding SCCs are connected in $G$. Here we assign an edge in $E_{SCC}$ the maximum weight of the corresponding edges in $\Sigma$. 
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4. Compute for each $n_{SCC} \in N_{SCC}$ the maximum weighted length (priority) $n_{SCC}.pr$ of any path originating in $n_{SCC}$, e.g., with a depth-first recursive traversal of all edges in the acyclic $G_{SCC}$. 
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Complexity:
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Complexity: linear in size of SCG
Overview

Conservative Static Approximation of SC

Determining SC-Schedules with Priorities

Summary
Summary I

Underlying idea of sequential constructiveness rather simple
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- Prescriptive instead of descriptive sequentiality
- Thus circumventing “spurious” causality problems
- Initialize-update-read protocol
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Underlying idea of sequential constructiveness rather simple

- Prescriptive instead of descriptive sequentiality
- Thus circumventing “spurious” causality problems
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However, precise definition of SC MoC not trivial

- Challenging to ensure conservativeness relative to Berry-constructiveness
- Plain initialize-update-read protocol does not accomodate, e.g., signal re-emissions
- Restricting attention to concurrent, non-confluent node instances is key
Summary II

ASC-schedulability

- Is conservative approximation to SC
- Basis for practical implementation
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ASC-schedulability
- Is conservative approximation to SC
- Basis for practical implementation

Future work
- Plenty of it (SC+, optimized code gen, improved SCCharts transformations, …)
- Talk to us if you want to be part of it