

# Synchronous Languages—Lecture 14

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*Sequentially Constructive  
Concurrency in Practice*

## The 5-Minute Review Session

1. What are goals and challenges in defining the SC MoC?
2. What is *confluence* in the SC MoC?
3. What is *thread reincarnation*?
4. In the SC MoC, when are threads considered *statically concurrent*?
5. What is a *thread tree*? How can it be used to define static concurrency?

## The 5-Minute Review Session

1. How is *run-time concurrency* defined? How does it relate to static concurrency?
2. What is *SC-admissibility*?
3. When is a program *sequentially constructive*?
4. What is an *SC-schedule*? When is it *valid*?
5. What are conservative, practical approximations of sequential constructiveness?

## References

Most of the material here draws from this reference [TECS]:



R. von Hanxleden, M. Mendler, J. Aguado, B. Duderstadt, I. Fuhrmann, C. Motika, S. Mercer, O. O'Brien, and P. Roop.

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.

ACM Transactions on Embedded Computing Systems, Special Issue on Applications of Concurrency to System Design, July 2014, 13(4s).

<https://rtsys.informatik.uni-kiel.de/~biblio/downloads/papers/tecs14.pdf>

Unless otherwise noted, the numberings of definitions, sections etc. refer to this.

There is also an extended version [TR]:



R. von Hanxleden, M. Mendler, J. Aguado, B. Duderstadt, I. Fuhrmann, C. Motika, S. Mercer, O. O'Brien, and P. Roop.

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.

Christian-Albrechts-Universität zu Kiel, Department of Computer Science, Technical Report 1308, ISSN 2192-6247, Aug. 2013, 13(4s).

<https://rtsys.informatik.uni-kiel.de/~biblio/downloads/papers/report-1308.pdf>

# Overview

## Conservative Static Approximation of SC

- SC-Schedules

- Schedule Order

- Schedule / Program Classes

## Determining SC-Schedules with Priorities

## Summary

## Conservative Static Approximation

In practice, a compiler must be conservative:

- ▶ Use a relation  $n_1|n_2$  to over-approximate  $n_1|_R n_2$ , *i. e.*, what statements are **concurrently** invoked in the same tick,
  - ▶ by considering only static control flow, or
  - ▶ ignoring dependency on initial conditions, or
  - ▶ by falsely considering nodes to be in the same tick.
- ▶ May not recognize confluence
- ▶ May not recognize that writes are relative

## SC-Schedules [Def. 5.1, Lemma 5.3]

- ▶ Given: SCG  $G = (N, E)$
- ▶ SC-schedule  $\Sigma$  is subset of  $G$ 's instantaneous edges:  $\Sigma \subseteq E_{ins}$
- ▶  $E_{ins}$  is structural SC-schedule; derived solely by analysis of the program structure
- ▶ An SC-schedule  $\Sigma$  is valid if
  - ▶ for every macro tick  $R$  of  $G$  which can be reached and executed under the SC-admissibility rules,
  - ▶ if  $(n_1, i_1) \rightarrow_{\alpha}^R (n_2, i_2)$  for some node instances  $(n_{1,2}, i_{1,2})$  in  $R$  and some  $\alpha \in \alpha_{ins}$ ,
  - ▶ then  $(n_1 \rightarrow_{\alpha} n_2) \in \Sigma$ .

Validity guarantees:

- ▶ If  $G$  is executed in an SC-admissible fashion,
- ▶ then static node relations  $\rightarrow_{\alpha}$  of  $\Sigma$  are conservative over-approximation of dynamic relations  $\rightarrow_{\alpha}^R$  on node instances

Lemma:  $E_{ins}$  is valid

## Schedule order [Def. 5.2]

- ▶ Given: Valid SC-schedule  $\Sigma$
- ▶ **Schedule order:**  $n_1 \rightarrow_{ins}^{\Sigma} n_2$  iff
  1.  $n_1 \parallel n_2$  and
  2.  $\Sigma$  contains a path from  $n_1$  to  $n_2$  that includes an iur-edge

To enforce the iur protocol among concurrent threads, it suffices to always execute  $\rightarrow_{ins}^{\Sigma}$ -minimal nodes

However: valid schedule may still contain conflicting orderings that cannot be satisfied or where it depends on the capabilities of the compiler or the run-time system whether it can be implemented



## Schedule / Program Classes [Def. 5.4]

### Schedule properties

- ▶ **acyclic**: does not contain any cycle
- ▶ **iur-acyclic**: does not contain any cycle that contains edges induced by  $\rightarrow_{iur}$

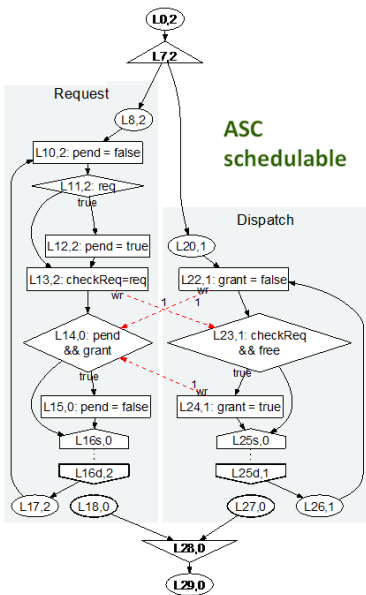
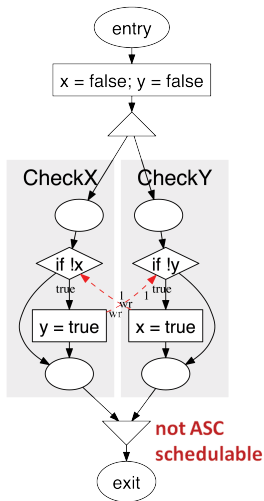
### Program (SCG) properties

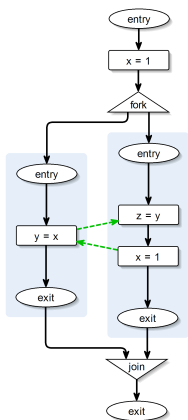
- ▶ **Acyclic SC (ASC)**:  $\exists$  valid acyclic SC-schedule
- ▶ **Iur-acyclic SC (IASC)**:  $\exists$  valid iur-acyclic SC-schedule
- ▶ **Structurally acyclic SC (SASC)**:  $E_{ins}$  is acyclic
- ▶ **Structurally iur-acyclic SC (SIASC)**:  $E_{ins}$  is iur-acyclic

### Implications (see also Theorem 5.5):

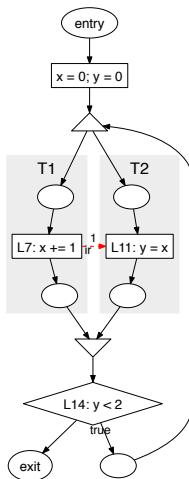
- ▶  $SASC \implies SIASC \implies IASC \implies SC$
- ▶  $SASC \implies ASC \implies IASC \implies SC$

May also relax the sequential order to only order non-confluent statements  $\rightsquigarrow$  **data-flow acyclic** programs





ASC, and hence SC, but not SIASC, hence not SASC



SC, but not IASC, and hence not SIASC/ASC/SASC

## Overview

Conservative Static Approximation of SC

Determining SC-Schedules with Priorities

Priority-Based Scheduling [Sec. 5.2]

Computing Priorities [Sec. 5.3]

Summary

## Priorities [Def. 5.6, Lemma 5.7]

- ▶ **Given:** valid SC-schedule  $\Sigma$
- ▶ **Priority**  $n.pr$  of statement  $n \in N$ : maximal number of  $\rightarrow_{iur}$  edges traversed by any path in  $\Sigma$  that originates in  $n$

**Lemma:** Priorities implement the schedule order

**Given:**

- ▶ Priority assignment according to some SC-schedule  $\Sigma$
- ▶ Run-time (and hence also statically) concurrent statements  $n_{1,2} \in N$

**Then:**  $n_1 \rightarrow_{ins}^{\Sigma} n_2$  implies  $n_1.pr > n_2.pr$

## Priority-Based Scheduler [Theorem 5.8]

**Priority-based scheduler:** always gives control to the thread with highest priority, chosen from the set of threads that are still active in the current tick

- ▶ Never allows a statement that is ready for execution to wait on another statement with lower priority
- ▶ Implements a valid schedule, as can be verified from the SCG construction
- ▶ For example  $n_1 \rightarrow_{iu} n_2$  implies  $n_1 \rightarrow_{iur} n_2$ , which implies, by definition of priorities,  $n_1.pr > n_2.pr$ , which in turn implies that  $n_1$  gets scheduled before  $n_2$

### Theorem

A program is IASC iff there exists a valid SC-schedule such that all statement priorities are finite

## Computing Priorities for IASC Programs

- ▶ Given a valid SC-schedule  $\Sigma$ , can formulate the calculation of priorities as longest weighted path problem
- ▶ Assign to each edge  $e \in \Sigma$  a weight  $e.w$ , with  $e.w = 0$  iff  $e.src \rightarrow_{seq} e.tgt$ , and  $e.w = 1$  iff  $e.src \rightarrow_{iur} e.tgt$
- ▶ As relations  $\rightarrow_{iur}$  and  $\rightarrow_{seq}$  exclude each other, weight of each edge is uniquely determined
- ▶  $n.pr$  becomes maximal weight of any path originating in  $n$
- ▶ **Difficulty:** want to handle (sequential) loops, *i. e.*, cyclic SCGs
- ▶ For arbitrary (*i. e.*, possibly cyclic) weighted graphs, the computation of the longest weighted path is NP-hard
- ▶ However, can exclude all graphs with a positive weight cycle

# Algorithm for Computing Priorities I

1. Detect whether  $\Sigma$  has a positive weight cycle.  
We can do so by computing the Strongly Connected Components (SCCs), e. g., by Tarjan's algorithm, and checking if any SCC contains a node that is connected to another node within the same SCC by a  $\rightarrow_{iur}$  edge.
2. If a positive weight cycle exists, then  $\Sigma$  is not *iur*-acyclic; we then **reject** the program.  
Otherwise, we **accept** the program, and continue.  
Now nodes in the same SCC can reach each other, but only through paths with weight 0, and therefore must have the same priority.



## Algorithm for Computing Priorities II

3. From the SCCs, construct the directed acyclic graph  $G_{SCC} = (N_{SCC}, E_{SCC})$ , where  $N_{SCC} \subset N$  contains a representative node from each SCC of  $G$  (using e. g. the SCC roots computed by Tarjan's algorithm), and  $E_{SCC}$  contains an edge from one SCC representative to another iff the corresponding SCCs are connected in  $G$ .  
Here we assign an edge in  $E_{SCC}$  the maximum weight of the corresponding edges in  $\Sigma$ .
4. Compute for each  $n_{SCC} \in N_{SCC}$  the maximum weighted length (priority)  $n_{SCC}.pr$  of any path originating in  $n_{SCC}$ , e. g., with a depth-first recursive traversal of all edges in the acyclic  $G_{SCC}$ .
5. Assign each statement  $n \in N$  the priority computed for its SCC.

Complexity: linear in size of SCG

# Overview

Conservative Static Approximation of SC

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Summary

## Summary I

Underlying idea of sequential constructiveness rather simple

- ▶ Prescriptive instead of descriptive sequentiality
- ▶ Thus circumventing “spurious” causality problems
- ▶ Initialize-update-read protocol

However, precise definition of SC MoC not trivial

- ▶ Challenging to ensure conservativeness relative to Berry-constructiveness
- ▶ Plain initialize-update-read protocol does not accomodate, e. g., signal re-emissions
- ▶ Restricting attention to *concurrent, non-confluent* node instances is key

## Summary II

### ASC-schedulability

- ▶ Is conservative approximation to SC
- ▶ Basis for practical implementation

### Future work

- ▶ Plenty of it (SC+, optimized code gen, improved SCCharts transformations, ...)
- ▶ Talk to us if you want to be part of it