Synchronous Languages—Lecture 14

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Sequentially Constructive
Concurrency in Practice

The 5-Minute Review Session

1. What are goals and challenges in defining the SC MoC?
2. What is confluence in the SC MoC?
3. What is thread reincarnation?
4. In the SC MoC, when are threads considered statically concurrent?
5. What is a thread tree? How can it be used to define static concurrency?

References

Most of the material here draws from this reference [TECS]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.

Unless otherwise noted, the numberings of definitions, sections etc. refer to this.

There is also an extended version [TR]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.
Overview

Conservative Static Approximation of SC

SC-Schedules
Schedule Order
Schedule / Program Classes

Determining SC-Schedules with Priorities

Summary

SC-Schedules [Def. 5.1, Lemma 5.3]

- **Given:** SCG $G = (N, E)$
- SC-schedule $\Sigma$ is subset of $G$’s instantaneous edges: $\Sigma \subseteq E_{ins}$
- $E_{ins}$ is structural SC-schedule; derived solely by analysis of the program structure
- An SC-schedule $\Sigma$ is valid if
  - for every macro tick $R$ of $G$ which can be reached and executed under the SC-admissibility rules,
  - if $(n_1, i_1) \xrightarrow{\alpha} R (n_2, i_2)$ for some node instances $(n_{1,2}, i_{1,2})$ in $R$
  - and some $\alpha \in \alpha_{ins}$,
  - then $(n_1 \rightarrow_{\alpha} n_2) \in \Sigma$.

Validity guarantees:
- If $G$ is executed in an SC-admissible fashion,
- then static node relations $\rightarrow_{\alpha}$ of $\Sigma$ are conservative over-approximation of dynamic relations $\rightarrow_{\alpha}^R$ on node instances

Lemma: $E_{ins}$ is valid

Conservative Static Approximation

In practice, a compiler must be conservative:
- Use a relation $n_1/n_2$ to over-approximate $m_1/m_2$, i.e., what statements are concurrently invoked in the same tick,
  - by considering only static control flow, or
  - ignoring dependency on initial conditions, or
  - by falsely considering nodes to be in the same tick.
- May not recognize confluence
- May not recognize that writes are relative
Schedule order [Def. 5.2]

- Given: Valid SC-schedule \( \Sigma \)
- Schedule order: \( n_1 \rightarrow_{\text{ins}} n_2 \) iff
  1. \( n_1 \parallel n_2 \) and
  2. \( \Sigma \) contains a path from \( n_1 \) to \( n_2 \) that includes an iur-edge

To enforce the iur protocol among concurrent threads, it suffices to always execute \( \rightarrow_{\text{ins}} \)-minimal nodes.

However: valid schedule may still contain conflicting orderings that cannot be satisfied or where it depends on the capabilities of the compiler or the run-time system whether it can be implemented.

Note that (2) is conservative in that it may also impose a scheduling order between nodes if they are not run-time concurrent. We choose this conservative definition to be compatible with the priority-based scheduling scheme introduced in Sec. ??.

A less conservative, thread-instance aware definition of schedule order would for example not consider paths that include \( \text{lcafork}(n_1, n_2) \), since at run time, executing \( \text{lcafork}(n_1, n_2) \) would preclude that the node instances corresponding to \( n_1, 2 \) could be run-time concurrent.

Schedule / Program Classes [Def. 5.4]

Schedule properties

- **acyclic**: does not contain any cycle
- **iur-acyclic**: does not contain any cycle that contains edges induced by \( \rightarrow_{\text{iur}} \)

Program (SCG) properties

- **Acyclic SC (ASC)**: \( \exists \) valid acyclic SC-schedule
- **Iur-acyclic SC (IASC)**: \( \exists \) valid iur-acyclic SC-schedule
- **Structurally acyclic SC (SASC)**: \( E_{\text{ins}} \) is acyclic
- **Structurally iur-acyclic SC (SIASC)**: \( E_{\text{ins}} \) is iur-acyclic

Implications (see also Theorem 5.5):

- \( \text{SASC} \implies \text{SIASC} \implies \text{IASC} \implies \text{SC} \)
- \( \text{SASC} \implies \text{ASC} \implies \text{IASC} \implies \text{SC} \)

May also relax the sequential order to only order non-confluent statements \( \leadsto \) data-flow acyclic programs.
Overview

Conservative Static Approximation of SC

Determining SC-Schedules with Priorities
Priority-Based Scheduling [Sec. 5.2]
Computing Priorities [Sec. 5.3]

Summary

Priorities [Def. 5.6, Lemma 5.7]

- **Given:** valid SC-schedule $\Sigma$
- **Priority** $n.pr$ of statement $n \in N$: maximal number of $\rightarrow_{ir}$ edges traversed by any path in $\Sigma$ that originates in $n$

**Lemma:** Priorities implement the schedule order

**Given:**
- Priority assignment according to some SC-schedule $\Sigma$
- Run-time (and hence also statically) concurrent statements $n_1, n_2 \in N$

**Then:** $n_1 \rightarrow_{ins} n_2$ implies $n_1.pr > n_2.pr$

**Priority-Based Scheduler [Theorem 5.8]**

*Priority-based scheduler:* always gives control to the thread with highest priority, chosen from the set of threads that are still active in the current tick

- Never allows a statement that is ready for execution to wait on another statement with lower priority
- Implements a valid schedule, as can be verified from the SCG construction
- For example $n_1 \rightarrow_{iu} n_2$ implies $n_1 \rightarrow_{iir} n_2$, which implies, by definition of priorities, $n_1.pr > n_2.pr$, which in turn implies that $n_1$ gets scheduled before $n_2$

**Theorem**

A program is IASC iff there exists a valid SC-schedule such that all statement priorities are finite
Computing Priorities for IASC Programs

- Given a valid SC-schedule Σ, can formulate the calculation of priorities as longest weighted path problem.
- Assign to each edge \( e \in \Sigma \) a weight \( e.w \), with \( e.w = 0 \) iff \( e.src \rightarrow_{seq} e.tgt \), and \( e.w = 1 \) iff \( e.src \rightarrow_{iur} e.tgt \).
- As relations \( \rightarrow_{iur} \) and \( \rightarrow_{seq} \) exclude each other, weight of each edge is uniquely determined.
- \( n.pr \) becomes maximal weight of any path originating in \( n \).
- Difficulty: want to handle (sequential) loops, i.e., cyclic SCGs.
- For arbitrary (i.e., possibly cyclic) weighted graphs, the computation of the longest weighted path is NP-hard.
- However, can exclude all graphs with a positive weight cycle.

Algorithm for Computing Priorities I

1. Detect whether \( \Sigma \) has a positive weight cycle.
   We can do so by computing the Strongly Connected Components (SCCs), e.g., by Tarjan’s algorithm, and checking if any SCC contains a node that is connected to another node within the same SCC by a \( \rightarrow_{iur} \) edge.
2. If a positive weight cycle exists, then \( \Sigma \) is not \( iur \)-acyclic; we then reject the program.
   Otherwise, we accept the program, and continue.
   Now nodes in the same SCC can reach each other, but only through paths with weight 0, and therefore must have the same priority.

Algorithm for Computing Priorities II

3. From the SCCs, construct the directed acyclic graph \( G_{SCC} = (N_{SCC}, E_{SCC}) \), where \( N_{SCC} \subset N \) contains a representative node from each SCC of \( G \) (using e.g., the SCC roots computed by Tarjan’s algorithm), and \( E_{SCC} \) contains an edge from one SCC representative to another iff the corresponding SCCs are connected in \( G \).
   Here we assign an edge in \( E_{SCC} \) the maximum weight of the corresponding edges in \( \Sigma \).
4. Compute for each \( n_{SCC} \in N_{SCC} \) the maximum weighted length (priority) \( n_{SCC}.pr \) of any path originating in \( n_{SCC} \), e.g., with a depth-first recursive traversal of all edges in the acyclic \( G_{SCC} \).
5. Assign each statement \( n \in N \) the priority computed for its SCC.

Complexity: linear in size of SCG.
Summary I

Underlying idea of sequential constructiveness rather simple
- Prescriptive instead of descriptive sequentiality
- Thus circumventing “spurious” causality problems
- Initialize-update-read protocol

However, precise definition of SC MoC not trivial
- Challenging to ensure conservativeness relative to Berry-constructiveness
- Plain initialize-update-read protocol does not accommodate, e.g., signal re-emissions
- Restricting attention to concurrent, non-confluent node instances is key

Summary II

ASC-schedulability
- Is conservative approximation to SC
- Basis for practical implementation

Future work
- Plenty of it (SC+, optimized code gen, improved SCCharts transformations, . . .
- Talk to us if you want to be part of it