The 5-Minute Review Session

1. How do \textit{SCCharts} and \textit{SyncCharts} differ?
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1. How do SCCharts and SyncCharts differ?
2. What does the initialize-update-read protocol refer to?
3. What is the SCG?
4. What are basic blocks? What are scheduling blocks?
5. When compiling from the SCG, what types of low-level synthesis do we distinguish? How do they compare?
Safety-Critical Embedded Systems

- Embedded systems often safety-critical
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- Safety-critical systems must react deterministically
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- Computations often exploit concurrency
Safety-Critical Embedded Systems

- Embedded systems often safety-critical
- Safety-critical systems must react deterministically
- Computations often exploit concurrency
- Key challenge: Concurrency must be deterministic!

Thanks to Michael Mendler (U Bamberg) for support with these slides
Implementing (Deterministic) Concurrency

- C, Java, etc.:
Implementing (Deterministic) Concurrency

- C, Java, etc.:
  - Familiar

C, Java, etc.: Familiar
Implementing (Deterministic) Concurrency

- C, Java, etc.:
  - Familiar
  - Expressive sequential paradigm
Implementing (Deterministic) Concurrency

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Implementing (Deterministic) Concurrency

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**Aim:** Deterministic concurrency with synchronous foundations, but without synchronous restrictions.
Comparing Both Worlds

**Sequential Languages**
- C, Java, ...

**Synchronous Languages**
- Esterel, Lustre, Signal, SCADE, SyncCharts ...
Comparing Both Worlds

**Sequential Languages**
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## Comparing Both Worlds

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Comparing Both Worlds

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Comparing Both Worlds (Cont’d)

**Sequential Languages**
- Asynchronous schedule

**Synchronous Languages**
- Clocked, cyclic schedule
Comparing Both Worlds (Cont’d)

**Sequential Languages**
- Asynchronous schedule
  - 😞 No guarantees of determinism or deadlock freedom

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- Clocked, cyclic schedule
  - 😊 Deterministic concurrency and deadlock freedom
### Comparing Both Worlds (Cont’d)

<table>
<thead>
<tr>
<th><strong>Sequential Languages</strong></th>
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Comparing Both Worlds (Cont’d)

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**Sequentially Constructive Model of Computation (SC MoC)**

- 😊 Deterministic concurrency and deadlock freedom
- 😊 Intuitive programming paradigm
Implementing **Deterministic Concurrency**: SC MoC

- **Concurrent** micro-step control flow
Implementing **Deterministic Concurrency**: SC MoC

- **Concurrent** micro-step control flow:
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Implementing **Deterministic Concurrency**: SC MoC

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Implementing **Deterministic** Concurrency: SC MoC

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Implementing **Deterministic Concurrency**: SC MoC

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A Sequentially Constructive Program

Motivation
Formalizing Sequential Constructiveness (SC)
Wrap-Up

The Control Example
A Constructive Game of Schedulability

A Sequentially Constructive Program
A Sequentially Constructive Program (Cont’d)

- Request: request to a resource, resource is pending
  - req
  - pend
- Dispatch: the resource may be free or not
  - grant
  - free
- Control
- Threads: checkReq

Motivation
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C, Java vs. Synchronous Programming

The Control Example
A Constructive Game of Schedulability
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A Sequentially Constructive Program (Cont’d)

**Control**

```plaintext
Req_entry:
pend = false;
if req then
    pend = true;
checkReq = req;
if pend && grant then
    pend = false;
pause;
goto Req_entry;
```

```plaintext
Dis_entry:
grant = false;
if checkReq && free then
    grant = true;
pause;
goto Dis_entry;
```

**Variables:**
- req
- pend
- grant
- free
A Sequentially Constructive Program (Cont’d)

**Imperative** program order (sequential access to shared variables)
A Sequentially Constructive Program (Cont’d)

Imperative program order (sequential access to shared variables)

▶ “write-after-write” can change value sequentially
A Sequentially Constructive Program (Cont’d)

**Imperative** program order (sequential access to shared variables)

- “write-after-write” can change value sequentially
- Prescribed by programmer
**A Sequentially Constructive Program (Cont’d)**

**Imperative** program order (sequential access to shared variables)

- “write-after-write” can change value sequentially
- Prescribed by programmer
  - 😊 Accepted in SC MoC
  - ☹️ Not permitted in standard synchronous MoC
A Sequentially Constructive Program (Cont’d)

Concurrency scheduling constraints (access to shared variables):

```
Req_entry:
pend = false;
if req then
  pend = true;
checkReq = req;
if pend && grant then
  pend = false;
pause;
goto Req_entry;
```

```
Dis_entry:
grant = false;
if checkReq && free then
  grant = true;
pause;
goto Dis_entry;
```
A Sequentially Constructive Program (Cont’d)

\[
\begin{align*}
\text{Req\_entry:} & \\
& \text{pend} = \text{false} ; \\
& \text{if req then} \\
& \quad \text{pend} = \text{true} ; \\
& \quad \text{checkReq} = \text{req} ; \\
& \text{if pend \&\& grant then} \\
& \quad \text{pause} ; \\
& \quad \text{goto Req\_entry} ; \\
\end{align*}
\]

\[
\begin{align*}
\text{Dis\_entry:} & \\
& \text{grant} = \text{false} ; \\
& \text{if checkReq \&\& free then} \\
& \quad \text{grant} = \text{true} ; \\
& \quad \text{pause} ; \\
& \quad \text{goto Dis\_entry} ; \\
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\]

**Concurrency** scheduling constraints (access to shared variables):
- “write-before-read” for concurrent write/reads
Concurrency scheduling constraints (access to shared variables):

- “write-before-read” for concurrent write/reads
- “write-before-write” (i.e., conflicts!) for concurrent & non-confluent writes
A Sequentially Constructive Program (Cont’d)

```
 Req_entry:
 pend = false;
 if req then
   pend = true;
 checkReq = req;
 if pend & grant then
   pend = false;
 pause;
 goto Req_entry;
```

```
 Dis_entry:
 grant = false;
 if checkReq & free then
   grant = true;
 pause;
 goto Dis_entry;
```

**Concurrency** scheduling constraints (access to shared variables):
- “write-before-read” for concurrent write/reads
- “write-before-write” (*i.e.*, conflicts!) for concurrent & non-confluent writes
- Micro-tick thread scheduling prohibits race conditions
Concurrency scheduling constraints (access to shared variables):

- "write-before-read" for concurrent write/reads
- "write-before-write" (*i.e.*, conflicts!) for concurrent & non-confluent writes
- Micro-tick thread scheduling prohibits race conditions
- Implemented by the SC compiler
A Constructive Game of Schedulability

Programmer
- Defines the rules
- Prescribes sequential execution order
- Leaves concurrency to compiler and run-time
- “Free Schedules”
A Constructive Game of Schedulability

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Compiler = Player
- Determines winning strategy
- Restricts concurrency to ensure determinacy and deadlock freedom
- “Admissible Schedules”
A Constructive Game of Schedulability

logically reactive program

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Run-time = Opponent

▶ Tries to choose a *spoiling execution* from admissible schedules
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Sequential Admissibility – Basic Idea
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- *Sequentially ordered* variable accesses
Sequential Admissibility – Basic Idea

- **Sequentially ordered** variable accesses
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The following applies to **concurrent** variable accesses only ...
Organizing Concurrent Variable Accesses

SC Concurrent Memory Access Protocol (per macro tick)

concurrent, multi-writer, multi-reader variables
Organizing Concurrent Variable Accesses

SC Concurrent Memory Access Protocol (per macro tick)

- `\text{tick}`
- `\text{initialise}`
- Concurrent, multi-writer, multi-reader variables
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

- **tick**
- **initialise**
- **modify**
- concurrent, *multi-writer*, multi-reader variables
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

- **Tick**
  - **Initialise**
  - **Modify**
  - **Read**

Concurrent, multi-writer, multi-reader variables
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

confluent
absolute
writes

concurrent, multi-writer, multi-reader variables
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

- Confluent absolute writes
- Before confluent relative writes

**Tick**

- Initialise
- Confluent, multi-writer, multi-reader variables

**Tick**
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

- **Concurrent absolute writes**
- **Concurrent relative writes**
- **Reads**
- **Tick**

**Concurrent, multi-writer, multi-reader variables**
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

- **confluent absolute writes**
- **confluent relative writes**

**concurrent, multi-writer, multi-reader variables**

**Confluent Statements (per macro tick)**
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

confluent absolute writes

confluent relative writes

concurrent, multi-writer, multi-reader variables

**Confluent Statements (per macro tick)**

For all memories Mem, reachable in macro tick:

\[
\begin{align*}
\text{stmt}_1 & \quad \text{Mem}_1 \\
\text{stmt}_2 & \quad \text{Mem}_2
\end{align*}
\]

\[
\text{stmt}_1, \text{stmt}_2 \quad \text{concurrent}
\]
Organizing Concurrent Variable Accesses

**SC Concurrent Memory Access Protocol (per macro tick)**

Confluent absolute writes

Confluent relative writes

Concurrent, multi-writer, multi-reader variables

**Confluent Statements (per macro tick)**

For all memories Mem, reachable in macro tick:

\[ Mem \rightarrow_{stmt_1} Mem_1, Mem_2 \rightarrow_{stmt_2} Mem' \]

\[ stmt_1, stmt_2 \]

Concurrent
Goals and Challenges
The idea behind SC is simple
Goals and Challenges

The idea behind SC is simple – but getting it “right” not so!
Goals and Challenges

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What we are up to:
Goals and Challenges

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1. Want to be conservative wrt “Berry constructiveness”
   - An Esterel program should also be SC
Goals and Challenges

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   ▶ An SC program must be determinate
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   ▶ But what exactly is sequentiality?
Goals and Challenges

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   - A determinate program should also be SC
   - An SC program must be determinate
3. Want to exploit sequentiality as much as possible
   - But what exactly is sequentiality?
4. Want to define not only the exact concept of SC, but also a practical strategy to implement it
   - In practice, this requires conservative approximations
   - Compiler must not accept Non-SC programs
   - Compiler may reject SC programs
References

Most of the material here draws from this reference [TECS]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.

Unless otherwise noted, the numberings of definitions, sections etc. refer to this.

There is also an extended version [TR]:

Sequentially Constructive Concurrency – A Conservative Extension of the Synchronous Model of Computation.
Overview

Motivation

Formalizing Sequential Constructiveness (SC)
The SC Language (SCL) and the SC Graph (SCG) [Sec. 2]
Free Scheduling of SCGs [Sec. 3]
The SC Model of Computation [Sec. 4]

Wrap-Up
The Sequentially Constructive Language (SCL) [Sec. 2.1]

- Foundation for the SC MoC
The Sequentially Constructive Language (SCL) [Sec. 2.1]

- Foundation for the SC MoC
- Minimal Language
The Sequentially Constructive Language (SCL) [Sec. 2.1]

- Foundation for the SC MoC
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- Adopted from C/Java and Esterel
The Sequentially Constructive Language (SCL) [Sec. 2.1]

- Foundation for the SC MoC
- Minimal Language
- Adopted from C/Java and Esterel

\[
s ::= x = e | s ; s | \text{if} (e) s \text{ else } s | l : s | \text{goto} \ l | \text{fork} \ s \ 	ext{par} \ s \ 	ext{join} | \text{pause}
\]

- \(s\) Statement
- \(x\) Variable
- \(e\) Expression
- \(/\) Program label
The SC Graph (SCG) [Sec. 2.3]

The concurrent and sequential control flow of an SCL program is given by an SC Graph (SCG).
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Internal representation for:
- Semantic foundation
- Analysis
- Code generation

The SC Graph (SCG) [Sec. 2.3]

SC Graph: Labeled graph \( G = (N, E) \)
- Nodes \( N \) correspond to statements of sequential program
- Edges \( E \) reflect sequential execution control flow
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The concurrent and sequential control flow of an SCL program is given by an SC Graph (SCG).

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SC Graph:

Labeled graph \( G = (N, E) \)

- Nodes \( N \) correspond to statements of sequential program
- Edges \( E \) reflect sequential execution control flow
Node Types in the SCG

Node $n \in N$ has statement type $n.st$
Node Types in the SCG

Node $n \in N$ has statement type $n.st$

- $n.st \in \{\text{entry, exit, goto, } x = ex, \text{ if } (ex), \text{ fork, join, surf, depth}\}$
- $x$: variable, $ex$: expression.
Edge Types in the SCG [Def. 2.1]

Define edge types:
Edge Types in the SCG [Def. 2.1]

Define edge types:

- **iur-edges** $\alpha_{iur} = \{ww, iu, ur, ir\}$
Define edge types:

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- **instantaneous edges** $\alpha_{ins} = \{ \text{seq} \} \cup \alpha_{iur}$
Define edge types:

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- **flow edges** \( \alpha_{flow} = \{seq, tick\} \)
Edge Types in the SCG [Def. 2.1]

Edge \( e \in E \) has edge type \( e.type \in \alpha_a \)

- Specifies the nature of the particular ordering constraint expressed by \( e \)
- For \( e.type = \alpha \), write \( e.src \xrightarrow{\alpha} e.tgt \), pronounced "\( e.src \alpha\)-precedes \( e.tgt \)"
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- For $e.type = \alpha$, write $e.src \rightarrow_{\alpha} e.tgt$, pronounced “$e.src \alpha$-precedes $e.tgt$”
- $n_1 \rightarrow_{\text{seq}} n_2$: sequential successors
- $n_1 \rightarrow_{\text{tick}} n_2$: tick successors
- $n_1 \rightarrow_{\text{seq}} n_2$, $n_1 \rightarrow_{\text{tick}} n_2$: flow successors, induced directly from source program

Note: $n_1 \rightarrow_{\text{seq}} n_2$ does not imply fixed run-time ordering between $n_1$ and $n_2$ (consider loops)
Edge Types in the SCG [Def. 2.1]

Edge $e \in E$ has edge type $e.type \in \alpha_a$

- Specifies the nature of the particular ordering constraint expressed by $e$
- For $e.type = \alpha$, write $e.src \rightarrow_{\alpha} e.tgt$, pronounced “$e.src$ $\alpha$-precedes $e.tgt$”
- $n_1 \rightarrow_{seq} n_2$: sequential successors
- $n_1 \rightarrow_{tick} n_2$: tick successors
- $n_1 \rightarrow_{seq} n_2, n_1 \rightarrow_{tick} n_2$: flow successors, induced directly from source program
- $\rightarrow_{seq}$: reflexive and transitive closure of $\rightarrow_{seq}$
Edge Types in the SCG [Def. 2.1]

Edge $e \in E$ has edge type $e\.type \in \alpha_a$

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- For $e\.type = \alpha$, write $e\.src \rightarrow_\alpha e\.tgt$, pronounced “$e\.src \alpha$-precedes $e\.tgt$”

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- $\rightarrow_{seq}$: reflexive and transitive closure of $\rightarrow_{seq}$
- **Note**: $n_1 \rightarrow_{seq} n_2$ does not imply fixed run-time ordering between $n_1$ and $n_2$
Edge Types in the SCG [Def. 2.1]

Edge \( e \in E \) has edge type \( e.type \in \alpha_a \)

- Specifies the nature of the particular ordering constraint expressed by \( e \)
- For \( e.type = \alpha \), write \( e.src \to_{\alpha} e.tgt \), pronounced “\( e.src \) \( \alpha \)-precedes \( e.tgt \)”
- \( n_1 \to_{seq} n_2 \): sequential successors
- \( n_1 \to_{tick} n_2 \): tick successors
- \( n_1 \to_{seq} n_2, n_1 \to_{tick} n_2 \): flow successors, induced directly from source program
- \( \to_{seq} \): reflexive and transitive closure of \( \to_{seq} \)
- Note: \( n_1 \to_{seq} n_2 \) does not imply fixed run-time ordering between \( n_1 \) and \( n_2 \) (consider loops)
Mapping SCL & SCG

<table>
<thead>
<tr>
<th>Thread (Region)</th>
<th>Concurrency (Superstate)</th>
<th>Conditional (Trigger)</th>
<th>Assignment (Effect)</th>
<th>Delay (State)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SCG</strong></td>
<td><img src="image1" alt="SCG Diagram" /></td>
<td><img src="image2" alt="SCG Diagram" /></td>
<td><img src="image3" alt="SCG Diagram" /></td>
<td><img src="image4" alt="SCG Diagram" /></td>
</tr>
<tr>
<td><strong>SCL</strong></td>
<td>$t$</td>
<td>fork $t_1$ par $t_2$ join</td>
<td>if $(c)$ $s_1$ else $s_2$</td>
<td>$x = e$</td>
</tr>
</tbody>
</table>

Plus ";" (Sequence) and "goto" to specify sequential successors (solid edges)
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;

    fork {
        // Thread Request
        Request entry:
        pend = false;
        if (req)
            pend = true;
        checkReq = req;
        if ( pend && grant)
            pend = false;
        pause;
        goto Request entry;
    }

    par {
        // Thread Dispatch
        Dispatch entry:
        grant = false;
        if (checkReq && free)
            grant = true;
        pause;
        goto Dispatch entry;
    }

    join;
}
Sequentiaity vs. Concurrency
Static vs. Dynamic Threads

Recall: We want to distinguish between *sequential* and *concurrent* control flow.
Sequentiability vs. Concurrency
Static vs. Dynamic Threads

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Sequentiality vs. Concurrency
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- **Static** threads: Structure of a program (based on SCG)
Recall: We want to distinguish between *sequential* and *concurrent* control flow.
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To get started, distinguish

- **Static** threads: Structure of a program (based on SCG)
- **Dynamic** thread instance
Sequentiality vs. Concurrency
Static vs. Dynamic Threads

Recall: We want to distinguish between *sequential* and *concurrent* control flow.
But what do “sequential” / “concurrent” mean?
This distinction is not as easy to formalize as it may seem . . .

To get started, distinguish

- **Static** threads: Structure of a program (based on SCG)
- **Dynamic** thread instance: thread in execution
Static Threads [Sec. 2.4]

- Given: SCG $G = (N, E)$
- Let $T$ denote the set of threads of $G$
Static Threads [Sec. 2.4]

- Given: SCG $G = (N, E)$
- Let $T$ denote the set of threads of $G$
- $T$ includes a top-level Root thread
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- Given: SCG $G = (N, E)$
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- With each thread $t \in T$, associate unique
  - entry node $t_{en} \in N$
  - exit node $t_{ex} \in N$
- Each $n \in N$ belongs to a thread $th(n)$ defined as
Static Threads [Sec. 2.4]

- **Given**: SCG $G = (N, E)$
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- $T$ includes a top-level Root thread
- With each thread $t \in T$, associate unique
  - entry node $t_{en} \in N$
  - exit node $t_{ex} \in N$
- Each $n \in N$ belongs to a thread $th(n)$ defined as
  - Immediately enclosing thread $t \in T$
  - such that there is a flow path to $n$ that originates in $t_{en}$,
Static Threads [Sec. 2.4]

- **Given:** SCG $G = (N, E)$
- **Let** $T$ denote the set of threads of $G$
- **$T$ includes a top-level Root thread**
- **With each thread** $t \in T$, associate unique
  - entry node $t_{en} \in N$
  - exit node $t_{ex} \in N$
- **Each** $n \in N$ **belongs to a thread** $th(n)$ **defined as**
  - Immediately enclosing thread $t \in T$
  - such that there is a flow path to $n$ that originates in $t_{en}$, does not traverse $t_{ex}$,\(^1\)

\(^1\)Added to definition in paper!
Static Threads [Sec. 2.4]

- **Given:** SCG $G = (N, E)$
- Let $T$ denote the set of threads of $G$
- $T$ includes a top-level **Root** thread
- With each thread $t \in T$, associate unique
  - entry node $t_{en} \in N$
  - exit node $t_{ex} \in N$
- Each $n \in N$ belongs to a thread $th(n)$ defined as
  - Immediately enclosing thread $t \in T$
  - such that there is a flow path to $n$ that originates in $t_{en}$, *does not traverse* $t_{ex}$,\(^1\) and does not traverse any other entry node $t'_{en}$

\(^1\)Added to definition in paper!
Static Threads [Sec. 2.4]

- **Given:** SCG \( G = (N, E) \)
- Let \( T \) denote the **set of threads** of \( G \)
- \( T \) includes a top-level **Root** thread
- With each thread \( t \in T \), associate unique
  - entry node \( t_{en} \in N \)
  - exit node \( t_{ex} \in N \)
- Each \( n \in N \) belongs to a thread \( th(n) \) defined as
  - Immediately enclosing thread \( t \in T \)
  - such that there is a flow path to \( n \) that originates in \( t_{en} \), does not traverse \( t_{ex} \), and does not traverse any other entry node \( t'_{en} \), unless that flow path subsequently traverses \( t'_{ex} \) also

---

\(^1\)Added to definition in paper!
Static Threads [Sec. 2.4]

- **Given:** SCG \( G = (N, E) \)
- **Let** \( T \) denote the set of threads of \( G \)
- **\( T \)** includes a top-level **Root** thread
- **With each thread** \( t \in T \), associate unique
  - entry node \( t_{en} \in N \)
  - exit node \( t_{ex} \in N \)
- **Each** \( n \in N \) belongs to a thread \( th(n) \) defined as
  - Immediately enclosing thread \( t \in T \)
  - such that there is a flow path to \( n \) that originates in \( t_{en} \), does not traverse \( t_{ex} \),\(^1\) and does not traverse any other entry node \( t'_{en} \), unless that flow path subsequently traverses \( t'_{ex} \) also
- **For each thread** \( t \), define \( sts(t) \) as the set of statement nodes \( n \in N \) such that \( th(n) = t \)

\(^1\)Added to definition in paper!
module Control
input bool free, req;
output bool grant, pend;
{
  bool checkReq;
  fork {
    // Thread Request
    Request entry:
    pend = false;
    if (req)
      pend = true;
    checkReq = req;
    if (pend && grant)
      pend = false;
    pause;
    goto Request entry;
  }
  par {
    // Thread Dispatch
    Dispatch entry:
    grant = false;
    if (checkReq && free)
      grant = true;
    pause;
    goto Dispatch entry;
  }
  join;
}
Threads in Control Example

```plaintext
1 module Control
2 input bool free, req;
3 output bool grant, pend;
4 {
5   bool checkReq;
6   fork {
7     // Thread Request
8     Request entry:
9     pend = false;
10    if (req)
11       pend = true;
12       checkReq = req;
13       if (pend && grant)
14         pend = false;
15       pause;
16       goto Request entry;
17     }
18   }
19   par {
20     // Thread Dispatch
21     Dispatch entry:
22     grant = false;
23     if (checkReq && free)
24       grant = true;
25       pause;
26       goto Dispatch entry;
27     }
28   join;
29 }
```

 Threads \( T = \)
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;
    fork {
        // Thread Request
        Request entry:
        pend = false;
        if (req)
            pend = true;
        checkReq = req;
        if (pend && grant)
            pend = false;
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        goto Request entry;
    }
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        // Thread Dispatch
        Dispatch entry:
        grant = false;
        if (checkReq && free)
            grant = true;
        pause;
        goto Dispatch entry;
    }
    join;
}

 Threads $T = \{ \text{Root}, \text{Request}, \text{Dispatch} \}$
Threads in Control Example

```plaintext
module Control
 input bool free, req;
 output bool grant, pend;
 {
   bool checkReq;
   fork {
     // Thread Request
     Request entry:
     pend = false;
     if (req)
       pend = true;
     checkReq = req;
     if (pend && grant)
       pend = false;
     pause;
     goto Request entry;
   }
   par {
     // Thread Dispatch
     Dispatch entry:
     grant = false;
     if (checkReq && free)
       grant = true;
     pause;
     goto Dispatch entry;
   }
   join;
 }
```

- Threads $T = \{ \text{Root, Request, Dispatch} \}$
- Root thread consists of the statement nodes
Threads in Control Example

```
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;
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        pend = false;
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        // Thread Dispatch
        Dispatch entry:
        grant = false;
        if (checkReq && free)
            grant = true;
        pause;
        goto Dispatch entry;
    }
    join;
}
```

- Threads $T = \{ \text{Root, Request, Dispatch} \}$
- Root thread consists of the statement nodes $\text{sts}(\text{Root}) = \{ L0, L7, L28, L29 \}$
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;

    fork {
        // Thread Request
        Request entry:
        pend = false;
        if (req)
            pend = true;
        checkReq = req;
        if (pend && grant)
            pend = false;
        pause;
        goto Request entry;
    }

    par {
        // Thread Dispatch
        Dispatch entry:
        grant = false;
        if (checkReq && free)
            grant = true;
        pause;
        goto Dispatch entry;
    }

    join;
}

 Threads \( T = \{ \text{Root}, \text{Request}, \text{Dispatch} \} \)

 Root thread consists of the statement nodes
\( \text{sts}(\text{Root}) = \{ L0, L7, L28, L29 \} \)

 The remaining statement nodes of \( N \) are partitioned into
\( \text{sts}(\text{Dispatch}) \) and \( \text{sts}(\text{Request}) \)
Static Thread Concurrency and Subordination [Def. 2.2]

Let $t$, $t_1$, $t_2$ be threads in $T$

$\blacktriangleright \; \text{fork}(t) =_{def} \text{fork node immediately preceding } t_{en}$
Static Thread Concurrency and Subordination [Def. 2.2]

Let $t, t_1, t_2$ be threads in $T$

- $fork(t) \overset{def}{=} \text{fork node immediately preceding } t_{en}$
- For every thread $t \neq \text{Root}$:
  
  $p(t) \overset{def}{=} \text{th}(fork(t))$, the parent thread
Let \( t, t_1, t_2 \) be threads in \( T \)

- \( \text{fork}(t) =_{\text{def}} \) fork node immediately preceding \( t_{en} \)
- For every thread \( t \neq \text{Root} \):
  \[ p(t) =_{\text{def}} \text{th}(\text{fork}(t)) \], the parent thread

- \( p^*(t) =_{\text{def}} \{ t, p(t), p(p(t)), \ldots, \text{Root} \} \), the recursively defined set of ancestor threads of \( t \)
Static Thread Concurrency and Subordination [Def. 2.2]

Let \( t, t_1, t_2 \) be threads in \( T \)

- \( \text{fork}(t) =_{def} \) fork node immediately preceding \( t_{en} \)
- For every thread \( t \neq \text{Root} \):
  \[ p(t) =_{def} \text{th}(\text{fork}(t)), \] the parent thread
- \( p^*(t) =_{def} \{ t, p(t), p(p(t)), \ldots, \text{Root} \} \), the recursively defined set of ancestor threads of \( t \)
- \( t_1 \) is subordinate to \( t_2 \), written \( t_1 \prec t_2 \), if \( t_1 \neq t_2 \land t_1 \in p^*(t_2) \)
Static Thread Concurrency and Subordination [Def. 2.2]

Let $t, t_1, t_2$ be threads in $T$

- $fork(t) = def$ fork node immediately preceding $t_{en}$
- For every thread $t \neq \text{Root}$:
  $p(t) = def th(fork(t))$, the parent thread

- $p^*(t) = def \{ t, p(t), p(p(t)), \ldots, \text{Root} \}$, the recursively defined set of ancestor threads of $t$
- $t_1$ is subordinate to $t_2$, written $t_1 \prec t_2$, if $t_1 \neq t_2 \land t_1 \in p^*(t_2)$
- $t_1$ and $t_2$ are (statically) concurrent, denoted $t_1 \parallel t_2$, iff $t_1$ and $t_2$ are descendants of distinct threads sharing a common fork node, i.e.: 

Denote this common fork node as $lcafork(t_1, t_2)$, the least common ancestor fork

Lift (static) concurrency notion to nodes: $n_1 \parallel n_2 \iff th(n_1) \parallel th(n_2) \iff lcafork(n_1, n_2) = lcafork(th(n_1), th(n_2))$
Static Thread Concurrency and Subordination [Def. 2.2]

Let $t$, $t_1$, $t_2$ be threads in $T$

- $fork(t) =_{def} \text{fork node immediately preceding } t_{en}$
- For every thread $t \neq \text{Root}$:
  $p(t) =_{def} th(fork(t))$, the parent thread

- $p^*(t) =_{def} \{t, p(t), p(p(t)), \ldots, \text{Root}\}$, the recursively defined set of ancestor threads of $t$

- $t_1$ is subordinate to $t_2$, written $t_1 \prec t_2$, if $t_1 \neq t_2 \land t_1 \in p^*(t_2)$

- $t_1$ and $t_2$ are (statically) concurrent, denoted $t_1 \parallel t_2$, iff $t_1$ and $t_2$ are descendants of distinct threads sharing a common fork node, i.e.:
  $\exists t'_1 \in p^*(t_1), t'_2 \in p^*(t_2) : t'_1 \neq t'_2 \land \text{fork}(t'_1) = \text{fork}(t'_2)$
Static Thread Concurrency and Subordination [Def. 2.2]

Let \( t, t_1, t_2 \) be threads in \( T \)

\[ \text{fork}(t) = \text{def} \text{ fork node immediately preceding } t_{en} \]

\[ \text{For every thread } t \neq \text{Root}: \]
\[ p(t) = \text{def} \text{ th(fork}(t)) \), the parent thread \]

\[ p^*(t) = \text{def} \{ t, p(t), p(p(t)), \ldots, \text{Root} \}, \text{ the recursively defined set of ancestor threads of } t \]

\[ t_1 \text{ is subordinate to } t_2, \text{ written } t_1 \prec t_2, \text{ if } t_1 \neq t_2 \land t_1 \in p^*(t_2) \]

\[ t_1 \text{ and } t_2 \text{ are (statically) concurrent, denoted } t_1 \parallel t_2, \text{ iff } \]
\[ t_1 \text{ and } t_2 \text{ are descendants of distinct threads sharing a common fork node, i.e.:} \]
\[ \exists t'_1 \in p^*(t_1), t'_2 \in p^*(t_2) : t'_1 \neq t'_2 \land \text{fork}(t'_1) = \text{fork}(t'_2) \]

\[ \text{Denote this common fork node as } \text{lcafork}(t_1, t_2), \text{ the least common ancestor fork} \]
Static Thread Concurrency and Subordination [Def. 2.2]

Let \( t, t_1, t_2 \) be threads in \( T \)

- \( fork(t) =_{def} \) fork node immediately preceding \( t_{en} \)
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- \( p^*(t) =_{def} \{ t, p(t), p(p(t)), \ldots, \text{Root} \} \), the recursively defined set of ancestor threads of \( t \)

- \( t_1 \) is subordinate to \( t_2 \), written \( t_1 \prec t_2 \), if \( t_1 \neq t_2 \land t_1 \in p^*(t_2) \)

- \( t_1 \) and \( t_2 \) are (statically) concurrent, denoted \( t_1 \parallel t_2 \), iff \( t_1 \) and \( t_2 \) are descendants of distinct threads sharing a common fork node, \( i.e.:\n\exists t'_1 \in p^*(t_1), t'_2 \in p^*(t_2) : t'_1 \neq t'_2 \land fork(t'_1) = fork(t'_2) \)
  - Denote this common fork node as \( lcafork(t_1, t_2) \), the least common ancestor fork

- Lift (static) concurrency notion to nodes: \( n_1 \parallel n_2 \iff th(n_1) \parallel th(n_2) \iff lcafork(n_1, n_2) = lcafork(th(n_1), th(n_2)) \)
module Control
input bool free, req;
output bool grant, pend;
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    if (pend && grant)
      pend = false;
    pause;
    goto Request entry;
  }

  par {
    // Thread Dispatch
    Dispatch entry:
    grant = false;
    if (checkReq && free)
      grant = true;
    pause;
    goto Dispatch entry;
  }

  join;
}
Concurrency and Subordination in Control-Program

```
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;
    fork {
        // Thread Request
        Request entry:
        pend = false;
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        // Thread Dispatch
        Dispatch entry:
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        if (checkReq && free)
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        pause;
        goto Dispatch entry;
    }
    join;
}
```

- **Root ♦ Request** and **Root ♦ Dispatch**
```plaintext
module Control
input bool free, req;
output bool grant, pend;
{
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  fork {
    // Thread Request
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    pend = false;
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  par {
    // Thread Dispatch
    Dispatch entry:
    grant = false;
    if (checkReq && free)
      grant = true;
    pause;
    goto Dispatch entry;
  }

  join;
}
```

- **Root ≺ Request and Root ≺ Dispatch**
- **Request || Dispatch**
Concurrence and Subordination in Control-Program

module Control
input bool free, req;
output bool grant, pend;
{
  bool checkReq;
  fork {
    // Thread Request
    Request entry:
    pend = false;
    if (req)
      pend = true;
    checkReq = req;
    if (pend && grant)
      pend = false;
    pause;
    goto Request entry;
  }
  par {
    // Thread Dispatch
    Dispatch entry:
    grant = false;
    if (checkReq && free)
      grant = true;
    pause;
    goto Dispatch entry;
  }
  join;
}

▶ Root ≺ Request and Root ≺ Dispatch
▶ Request || Dispatch, Root is not concurrent with any thread
module Control
input bool free, req;
output bool grant, pend;
{
bool checkReq;

d {  // Thread Request
    Request entry:
pend = false;
    if (req)
pend = true;
    checkReq = req;
    if (pend && grant)
pend = false;
    pause;
goto Request entry;
}

par {
    // Thread Dispatch
    Dispatch entry:
grant = false;
    if (checkReq && free)
grant = true;
    pause;
goto Dispatch entry;
}

join;

Note: Concurrency on threads, in contrast to concurrency on node instances, is purely static and can be checked with a simple, syntactic analysis of the program structure.

- Root ≺ Request and Root ≺ Dispatch
- Request || Dispatch, Root is not concurrent with any thread
Thread Trees [TR, Sec. 3.7]

A **Thread Tree** illustrates the static thread relationships.

- Contains subset of SCG nodes:
  1. Entry nodes, labeled with names of their threads
  2. Fork nodes, attached to the entry nodes of their threads

- Similar to the AND/OR tree of Statecharts
Thread Trees [TR, Sec. 3.7]

A **Thread Tree** illustrates the static thread relationships.

- Contains subset of SCG nodes:
  1. Entry nodes, labeled with names of their threads
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Thread tree for Control example:
Thread Trees [TR, Sec. 3.7]

A Thread Tree illustrates the static thread relationships.
- Contains subset of SCG nodes:
  1. Entry nodes, labeled with names of their threads
  2. Fork nodes, attached to the entry nodes of their threads
- Similar to the AND/OR tree of Statecharts

Thread tree for Control example:
Thread Trees – The Reinc2 Example

```c
module Reinc2
output int x, y;
{
  loop:
    fork { // Thread T1
      x = 1;
    }
    par { // Thread T2
      fork { // Thread T21
        y = 1;
      }
      par { // Thread T22
        pause;
        y = 2;
      }
      join;
      fork { // Thread T23
        y = 3;
      }
      par { // Thread T24
        x = 2;
      }
      join
    }
    join;
    goto loop;
}
```

Alternative definition for static thread concurrency:

- Threads are concurrent iff

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Thread Trees – The Reinc2 Example

```plaintext
module Reinc2
output int x, y;
{
  loop:
    fork { // Thread T1
      x = 1;
    }
    par { // Thread T2
      fork { // Thread T21
        y = 1;
      }
      par { // Thread T22
        pause;
        y = 2;
      }
      join;
      fork { // Thread T23
        y = 3;
      }
      par { // Thread T24
        x = 2;
      }
      join
    }
  join;
  goto loop;
}
```

Alternative definition for static thread concurrency:

- Threads are concurrent iff their least common ancestor (lca) in thread tree is
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      }
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    }
    join;
  goto loop;
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```

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Thread Reincarnation – The Reinc Example

Are interested in run-time concurrency, i.e., whether ordering is up to discretion of a scheduler.

module Reinc
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Motivation

Formalizing Sequential Constructiveness (SC)

Wrap-Up

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Free Scheduling of SCGs [Sec. 3]

The SC Model of Computation [Sec. 4]

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- T2 exhibits thread reincarnation
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        pause;
        x = 2;
    }
    join;
    goto loop;
}
```
Statement Reincarnation I

```haskell
module InstLoop
output int x = 0, y = 0;
{
    loop:
        fork {
            // Thread T1
            x += 1;
        }
        par {
            // Thread T2
            y = x;
        }
    join;
    if (y < 2)
        goto loop;
}
```
Statement Reincarnation I

- Accesses to $x$ in $L7$ and $L11$ executed twice within tick
- Denote this as statement reincarnation

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module InstLoop
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Data dependencies $\Rightarrow$ Must schedule $L7$ before $L11$
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Not enough to impose an order on the program statements
$\Rightarrow$ Need to distinguish statement instances
module InstLoop
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Statement Reincarnation II

Traditional synchronous languages: Reject

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        }
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}
```

Traditional synchronous languages: Reject

Instantaneous loops traditionally forbidden, SC: Determinate \[
\Rightarrow
\]
Accept

One might still want to ensure that a program always terminates

But this issue is orthogonal to determinacy and having a well-defined semantics.
Traditional synchronous languages: Reject

- **Instantaneous loops** traditionally forbidden

\[ x = 0; y = 0 \]
\[ \text{exit} \]
\[ L14: y < 2 \]
\[ L7: x += 1 \]
\[ L11: y = x \]

\[
\begin{align*}
\text{module InstLoop} \\
\text{output int x = 0, y = 0;} \\
\{ \\
\text{loop:} \\
\text{fork} \{ \\
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x += 1; \\
\} \\
\text{par} \{ \\
\text{// Thread T2} \\
y = x; \\
\} \\
\text{join}; \\
\text{if } (y < 2) \\
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 ☺ SC: Determinate ⇒ Accept

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Macroticks [Def. 2.3 + 2.4]

- Given: SCG $G = (N, E)$
- (Macro) tick $R$, of length $\text{len}(R) \in \mathbb{N}_{\geq 1}$: mapping from micro tick indices $1 \leq j \leq \text{len}(R)$, to nodes $R(j) \in N$
Motivation
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Wrap-Up

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A macro tick is also: Linearly ordered set of node instances

- Node instance: $ni = (n, i)$,
  - with statement node $n \in N$,
  - micro tick count $i \in \mathbb{N}$
- Can identify macro tick $R$ with set
  $$\{(n, i) \mid 1 \leq i \leq \text{len}(R), n = R(i)\}$$
Run-Time Concurrency [Def. 2.5 + 2.6]

Given: macro tick $R$, index $1 \leq i \leq \text{len}(R)$, node $n \in N$

Def.: $\text{last}(n, i) = \max\{j \mid j \leq i, R(j) = n\}$,
retrieves last occurrence of $n$ in $R$ at or before index $i$. If it does not exist, $\text{last}_R(n, i) = 0.$
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3. their threads have been instantiated by the same instance of the associated least common ancestor fork, i.e., $\text{last}(n, i_1) = \text{last}(n, i_2)$ where $n = \text{lcafork}(n_1, n_2)$
Overview

Motivation

Formalizing Sequential Constructiveness (SC)
   The SC Language (SCL) and the SC Graph (SCG) [Sec. 2]
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Wrap-Up
Continuations & Thread Execution States [Def. 3.1]

A continuation $c$ consists of

1. Node $c$\_node $\in \mathbb{N}$, denoting the current state of each thread, i.e., the node (statement) that should be executed next, similar to a program counter
2. Status $c$\_status $\in \{active, waiting, pausing\}$

In a trace (see later slide), round/square/no parentheses around $n = c$\_node denote $c$\_status, for enabled continuations $c$. 
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Continuation pool: finite set $C$ of continuations
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A configuration is called valid if $C$ is valid
module Control
input bool free, req;
output bool grant, pend;
{
    bool checkReq;

    fork {
        // Thread Request
        Request entry:
        pend = false;
        if (req)
            pend = true;
        checkReq = req;
        if (pend && grant)
            pend = false;
        pause;
        goto Request entry;
    }  
}
par {
    // Thread Dispatch
    Dispatch entry:
    grant = false;
    if (checkReq && free)
        grant = true;
    pause;
    goto Dispatch entry;
    join;
    
    L24,1: grant = true
    
    L25s,0
    true
    
    L26,1
    
    L27,0 
    
    L28,0
    
    L29,0
    
    L22,1: grant = false
    
    L23,1: checkReq & free
    
    L20,1
    
    Request
    L7,2

    L8,2
    
    L10,2: pend = false
    
    L11,2: req
    
    L12,2: pend = true
    
    L13,2: checkReq = req
    
    L14,0: pend & grant
    
    L15,0: pend = false
    
    L16s,0
    
    L16d,2
    
    L17,2
    
    L18,0
    
    L20,1 
    
    L22,1
    
    L23,1
    
    L24,1
    
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    L28,0
    
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Free Scheduling [Sec. 3.2]

Now define free scheduling, to set the stage for later defining “initialize-update-read” protocol (→ SC-admissible scheduling)
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Only restrictions:

1. Execute only $\prec$-maximal threads
   - If there is at least one continuation in $C_{cur}$, then there also is a $\prec$-maximal one, because of the finiteness of the continuation pool
Free Scheduling [Sec. 3.2]

Now define **free scheduling**, to set the stage for later defining "initialize-update-read" protocol

(→ SC-admissible scheduling)

Only restrictions:

1. Execute only \(<\)-maximal threads
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2. Do so in an interleaving fashion
Micro Steps I

Micro step: transition \((C_{\text{cur}}, M_{\text{cur}}) \xrightarrow{c}_{\mu s} (C_{\text{nxt}}, M_{\text{nxt}})\) between two micro ticks

- \((C_{\text{cur}}, M_{\text{cur}})\): current configuration
- \(c\): continuation selected for execution
- \((C_{\text{nxt}}, M_{\text{nxt}})\): next configuration
Micro Steps I

**Micro step:** transition \((C_{\text{cur}}, M_{\text{cur}}) \xrightarrow{c} \mu s (C_{\text{nxt}}, M_{\text{nxt}})\) between two micro ticks

- \((C_{\text{cur}}, M_{\text{cur}})\): current configuration
- \(c\): continuation selected for execution
- \((C_{\text{nxt}}, M_{\text{nxt}})\): next configuration

The **free schedule** is permitted to pick any one of the \(\prec\)-maximal continuations \(c \in C_{\text{cur}}\) with \(c.status = \text{active}\) and execute it in the current memory \(M_{\text{cur}}\).
(Recall:) Micro step: transition \((C_{cur}, M_{cur}) \xrightarrow{c}_{\mu s} (C_{nxt}, M_{nxt})\)
Micro Steps II

(Recall:) Micro step: transition \((C_{cur}, M_{cur}) \xrightarrow{c}_{\mu_{s}} (C_{nxt}, M_{nxt})\)

- Executing \(c\) yields a new memory \(M_{nxt} = \mu M(c, M_{cur})\) and a (possibly empty) set of new continuations \(\mu C(c, M_{cur})\) by which \(c\) is replaced, i.e., \(C_{nxt} = C_{cur} \setminus \{c\} \cup \mu C(c, M_{cur})\)
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- If \(\mu C(c, M_{cur}) = \emptyset\): status flags set to active for all \(c' \in C_{nxt}\) that become \(\prec\)-maximal by eliminating \(c\) from \(C\)
Micro Steps II

(Recall:) Micro step: transition $(C_{cur}, M_{cur}) \xrightarrow{c} \mu_s (C_{nxt}, M_{nxt})$

- Executing $c$ yields a new memory $M_{nxt} = \mu M(c, M_{cur})$ and a (possibly empty) set of new continuations $\mu C(c, M_{cur})$ by which $c$ is replaced, i.e., $C_{nxt} = C_{cur} \setminus \{c\} \cup \mu C(c, M_{cur})$

- If $\mu C(c, M_{cur}) = \emptyset$: status flags set to active for all $c' \in C_{nxt}$ that become $\prec$-maximal by eliminating $c$ from $C$

- Actions $\mu M$ and $\mu C$ (made precise in paper) depend on the statement $c.node.st$ to be executed
Micro Steps II

(Recall:) Micro step: transition \( (C_{cur}, M_{cur}) \xrightarrow{c} \mu_s (C_{nxt}, M_{nxt}) \)

- Executing \( c \) yields a new memory \( M_{nxt} = \mu M(c, M_{cur}) \) and a (possibly empty) set of new continuations \( \mu C(c, M_{cur}) \) by which \( c \) is replaced, i.e., \( C_{nxt} = C_{cur} \setminus \{c\} \cup \mu C(c, M_{cur}) \)
- If \( \mu C(c, M_{cur}) = \emptyset \): status flags set to active for all \( c' \in C_{nxt} \) that become \( \prec \)-maximal by eliminating \( c \) from \( C \)
- Actions \( \mu M \) and \( \mu C \) (made precise in paper) depend on the statement \( c.node.st \) to be executed
- \( (C_{nxt}, M_{nxt}) \) uniquely determined by \( c \), thus may write \( (C_{nxt}, M_{nxt}) = c(C_{cur}, M_{cur}) \)
Clock Steps 1

**Quiescent configuration** \((C, M)\):
- No active \(c \in C\)
Clock Steps I

**Quiescent configuration** \((C, M)\):

- No active \(c \in C\)
- All \(c \in C\) pausing or waiting
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- No active \(c \in C\)
- All \(c \in C\) pausing or waiting

If \(C = \emptyset\):

- Main program terminated

Otherwise:

- Scheduler can perform a global clock step
Clock Steps II

Global clock step: \((C_{\text{cur}}, M_{\text{cur}}) \xrightarrow{\text{tick}} (C_{\text{nxt}}, M_{\text{nxt}})\)

- Transition between last micro tick of the current macro tick to first micro tick of the subsequent macro tick
Clock Steps II

Global clock step: \((C_{cur}, M_{cur}) \rightarrow_{tick} (C_{nxt}, M_{nxt})\)

- Transition between last micro tick of the current macro tick to first micro tick of the subsequent macro tick
- All pausing continuations of \(C\) advance from their surf node to the associated depth node:

\[
C_{nxt} = \{c[\text{active} :: \text{tick}(n)] \mid c[\text{pausing} :: n] \in C_{cur}\} \cup \{c[\text{waiting} :: n] \mid c[\text{waiting} :: n] \in C_{cur}\}
\]
Clock Steps III

Global clock step updates the memory:

- Let $I = \{x_1, x_2, \ldots, x_n\}$ be the designated input variables of the SCG, including input/output variables
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- All other memory locations persist unchanged into the next macro tick.
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- All other memory locations persist unchanged into the next macro tick.

Formally,

$$M_{nxt}(x) = \begin{cases} v_i, & \text{if } x = x_i \in I, \\ M_{cur}(x), & \text{if } x \notin I. \end{cases}$$
Macro Ticks

Scheduler runs through sequence

\[(C_0^a, M_0^a) \xrightarrow{c_1^a} \mu s (C_1^a, M_1^a) \xrightarrow{c_2^a} \mu s \cdots \xrightarrow{c_{k(a)}^a} \mu s (C_{k(a)}^a, M_{k(a)}^a) \quad (1)\]

to reach final quiescent configuration \((C_{k(a)}^a, M_{k(a)}^a)\)
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Sequence (1) is macro tick (synchronous instant) \(a\):

\[(R^a, V_I^a) : (C_0^a, M_0^a) \rightarrow (C_{k(a)}^a, M_{k(a)}^a) \quad (2)\]

- \(V_I^a\): projects the initial input, \(V_I^a(x) = M_0^a(x)\) for \(x \in I\)
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\(R^a\): sequence of statement nodes executed during \(a\)

- \(\text{len}(R^a) = k(a)\) is length of \(a\)
- \(R^a\) is function mapping each micro tick index \(1 \leq j \leq k(a)\) to node \(R^a(j) = c_j^a.\text{node}\) executed at index \(j\)
Runs and Traces

**Run** of $G$: sequence of macro ticks $R^a$ and external inputs $V_i^a$, with
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- all macro tick configurations are connected by clock steps, i.e., $(C_{k(a)}^a, M_{k(a)}^a) \rightarrow \text{tick} (C_{0+1}^a, M_{0+1}^a)$
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**Trace**: externally visible output values at each macro tick $R$ [TR, Sec. 3.9]
Determinacy

Recall:

\[ (C_0^a, M_0^a) \xrightarrow{c_1^a} \mu_s (C_1^a, M_1^a) \xrightarrow{c_2^a} \mu_s \cdots \xrightarrow{c_{k(a)}^a} \mu_s (C_{k(a)}^a, M_{k(a)}^a) \]  

(1)

\[ (R^a, V_l^a) : (C_0^a, M_0^a) \Rightarrow (C_{k(a)}^a, M_{k(a)}^a) \]  

(2)
Determinacy

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▶ **Macro (tick) configuration**: end points of a macro tick (2)
▶ **Micro (tick) configuration**: all other intermediate configurations \((C_i^a, M_i^a), 0 < i < k(a)\) seen in (1)
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**Synchrony hypothesis:**

- only macro configurations are observable externally (in fact, only the memory component of those)
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- only macro configurations are observable externally (in fact, only the memory component of those)
- **Suffices to ensure that**
  - sequence of macro ticks \(\Rightarrow\) is determinate
- Micro tick behavior \(\rightarrow_{\mu_s}\) may well be non-determinate
Active and Pausing Continuations are Concurrent [TR, Prop. 2]

Given:

- \((C, M)\), reachable (micro or macro tick) configuration
- \(c_1, c_2 \in C\), active or pausing continuations with \(c_1 \neq c_2\)
Active and Pausing Continuations are Concurrent [TR, Prop. 2]

Given:
- \((C, M)\), reachable (micro or macro tick) configuration
- \(c_1, c_2 \in C\), active or pausing continuations with \(c_1 \neq c_2\)

Then:
- \(c_1.node \neq c_2.node\)
- \(th(c_1.node) \parallel th(c_2.node)\)
- No instantaneous sequential path from \(c_1.node\) to \(c_2.node\) or vice versa

(Proof: see [TR])
Concurrency vs. Sequentiality Revisited I

Recall: Want to exploit sequentiality as much as possible
  ▶ Thus, consider only run-time concurrent data dependencies
Concurrency vs. Sequentiality Revisited I

**Recall:** Want to exploit sequentiality as much as possible
  - Thus, consider only run-time concurrent data dependencies

**Recall:** Static concurrency $\not\Rightarrow$ run-time concurrency
  - Consider Reinc example
  - Thus, can ignore some statically concurrent data dependencies
Concurrency vs. Sequentiality Revisited II

Question: Does (static) sequentiality preclude run-time concurrency?

Then we could ignore data dependencies between nodes that are sequentially ordered.

But the answer is: no

Counterexample: Reinc3 (SCG shown on right)

Assignments to x run-time concurrent?

Assignments to x sequentially ordered?

Thus, concurrency and (static) sequentiality are not mutually exclusive, but orthogonal!

However, (instantaneous) run-time sequentiality (on node instances) does exclude run-time concurrency
Concurrence vs. Sequentiality Revisited II

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Concurrentcs vs. Sequentiality Revisited II

**Question:** Does (static) sequentiality preclude runtime concurrency?

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Notes on Free Scheduling I

Key to determinacy:
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rule out uncertainties due to unknown scheduling mechanism
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- Like the synchronous MoC, the SC MoC ensures macro-tick determinacy by inducing certain scheduling constraints on variable accesses

- **Unlike** the synchronous MoC, the SC MoC tries to take **maximal advantage of the execution order already expressed by the programmer** through sequential commands

- A scheduler can only affect the order of variable accesses through **concurrent** threads
Notes on Free Scheduling II

Recall:

- If variable accesses (within tick) are already sequentialized by $\rightarrow_{\text{seq}}$, they cannot appear simultaneously in the active continuation pool.
- Hence, no way for thread scheduler to reorder them and thus lead to a non-determinate outcome.

- Similarly, threads are not concurrent with parent thread.
- Because of path ordering $\prec$, a parent thread is always suspended when a child thread is in operation.
- Thus, not up to scheduler to decide between parent and child thread.
- No race conditions between variable accesses performed by parent and child threads; no source of non-determinacy.
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The Aim

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In the following, will define such a restriction:
The Aim

Want to find a suitable restriction on the “free” scheduler which is

1. easy to compute
2. leaves sufficient room for concurrent implementations
3. still (predictably) sequentializes any concurrent variable accesses that may conflict and produce unpredictable responses

In the following, will define such a restriction: the SC-admissible schedules
Guideline for SC-admissibility

- Initialize-Update-Read protocol, for concurrent accesses
- Want to conservatively extend Esterel’s “Write-Read protocol” (must do emit *before* testing)
- But does Esterel *always* follow write-read protocol?
Write After Read Revisited

```
module WriteAfterRead
output x, y, z;
emit x;
[
    present x then
    emit y
end
||
    present y then
    emit z
end;
emit x
]
end
```

Esterel version
Write After Read Revisited

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```

SCL version

```plaintext
module WriteAfterRead
output int x, y, z;
{
    x = 1;
    fork
        y = x;
    par
        z = y;
    x = 1;
    join
}
```
Write After Read Revisited

**Esterel version**

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**SCG**
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Esterel version

Concurrent emit after present test

SCG

SCG
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- Concurrent emit after present test
- But WriteAfterRead is BC

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SCG
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SCL version
Write After Read Revisited

Esterel version

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- **Observation:** second emit is ineffective, i.e., does not change value

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SCG
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SCG
Ineffectiveness – 1st Try [TR, Sec. 5.2]

```java
module InEffective1
{
  int y;
  {
    fork
    if (x == 2) {
      y = 1;
      x = 7
    }
    else
      y = 0
    par
    x = 7
    join
  }
}
```

If L13 is scheduled before L6:
▶ L13 is effective
▶ No out-of-order write
▶ y = 0

If L13 is scheduled after L8 (and L6):
▶ L13 is out-of-order write
▶ However, L13 is ineffective
▶ y = 1 (→ non-determinacy!)

The problem: L8 hides the potential effectiveness of L13 wrt. L6!

▶ Both schedules would be permitted under a scheduling regime that permits ineffective writes
▶ → Strengthen notion of “ineffective writes”:
▶ Consider writes “ineffective” only if they do not change read!
Ineffectiveness – 1st Try [TR, Sec. 5.2]

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Ineffectiveness – 1st Try [TR, Sec. 5.2]

If L13 is scheduled before L6:
▷ L13 is effective
▷ No out-of-order write
▷ y = 0

If L13 is scheduled after L8 (and L6):
▷ L13 is out-of-order write
▷ However, L13 is ineffective
▷ y = 1 (→ non-determinacy!)

module InEffective1
output int x = 2;
int y;
{
  fork
    if (x == 2) {
      y = 1;
      x = 7
    }
  else
    y = 0
  par
  x = 7
  join
Ineffectiveness – 1st Try [TR, Sec. 5.2]

If L13 is scheduled before L6:
- L13 is effective
- No out-of-order write
- \( y = 0 \)

If L13 is scheduled after L8 (and L6):
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- **The problem:** L8 hides the potential effectiveness of L13 wrt. L6!
Ineffectiveness – 1st Try [TR, Sec. 5.2]

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- Both schedules would be permitted under a scheduling regime that permits ineffective writes
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- \( y = 1 \) \( \rightarrow \) non-determinacy!
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- Both schedules would be permitted under a scheduling regime that permits ineffective writes
- \( \rightarrow \) Strengthen notion of “ineffective writes”: 

```plaintext
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▶ However, L13 is ineffective
▶ $y = 1$ (→ non-determinacy!)
▶ The problem: L8 hides the potential effectiveness of L13 wrt. L6!

▶ Both schedules would be permitted under a scheduling regime that permits ineffective writes
▶ → Strengthen notion of “ineffective writes”:
▶ Consider writes “ineffective” only if they do not change read!
Ineffectiveness – 2nd Try

```c
module InEffective2

output bool x = false;

int y;
{
  fork
    if (!x) {
      y = 1;
      x = x xor true
    } else
      y = 0
  par
  x = x xor true;
  join
}
```

"x = x xor true"

▶ Relative writes
▶ Equivalent to "x = !x"

Sequence L13; L6; L11:
▶ y = 0

Sequence L6; L7; L8; L13:
▶ Q: Is L13 ineffective relative to L6?
▶ A: Yes!
▶ L13 is out-of-order . . .
▶ but writes x = true, which is what L6 read!
▶ y = 1 (→ again non-determinacy!)

Again, both schedules would be permitted under a scheduling regime that permits ineffective writes
→ Replace "ineffectiveness" by "confluence"
module InEffective2
output bool $x = false$
  int $y$
{
  fork
    if (!$x) {
      $y = 1;
      $x = $x xor true
    }
  else
    $y = 0
  par
    $x = $x xor true;
  join
}
Ineffectiveness – 2nd Try

```
module InEffective2
output bool x = false;
  int y;
{
  fork
    if (!x) {
      y = 1;
      x = x xor true
    }
  else
    y = 0
  par
    x = x xor true;
  join
}
```

“\(x = x \text{ xor true}\)”

- Relative writes
- Equivalent to “\(x = !x\)”

Sequence L13; L6; L11:
- \(y = 0\)

Sequence L6; L7; L8; L13:
Ineffectiveness – 2nd Try

```
module InEffective2
output bool x = false;
  int y;
{
  fork
    if (!x) {
      y = 1;
      x = x xor true
    }
  else
    y = 0
  par
    x = x xor true;
  join
```

“\(x = x \text{xor} \text{true}\)”

- Relative writes
- Equivalent to “\(x = !x\)”

Sequence L13; L6; L11:
- \(y = 0\)

Sequence L6; L7; L8; L13:
- \(Q:\) Is L13 ineffective relative to L6?

\(\text{\textit{A: Yes!}}\) 
\(\text{\textit{L13 is out-of-order. . .\)}}\)
\(\text{\textit{but writes x = true, which is what L6 read!}}\) 
\(\text{\textit{y = 1 (→ again non-determinacy!}}\) 
\(\text{\textit{Again, both schedules would be permitted under a scheduling regime that permits ineffective writes \arrow{→} \text{Replace “ineffectiveness” by “confluence”}}}\)
Ineffectiveness – 2nd Try

```
module InEffective2
output bool x = false;
  int y;
{
  fork
    if (!x) {
      y = 1;
      x = x xor true
    }
  else
    y = 0
  par
    x = x xor true;
  join
```

“\(x = x \text{ xor} \true\)”

- Relative writes
- Equivalent to “\(x = \!x\)”

Sequence L13; L6; L11:
- \(y = 0\)

Sequence L6; L7; L8; L13:
- Q: Is L13 ineffective relative to L6?
- A: Yes!

\[\text{Replace “ineffectiveness” by “confluence”}\]
Ineffectiveness – 2nd Try

```
module InEffective2
output bool x = false;
  int y;
{ }
  fork
    if (!x) {
      y = 1;
      x = x xor true
    }
  else
    y = 0
par
  x = x xor true;
join
```

“x = x xor true”

- Relative writes
- Equivalent to “x = !x”

Sequence L13; L6; L11:
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Sequence L6; L7; L8; L13:
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- L13 is out-of-order . . .
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"x = x xor true"

- Relative writes
- Equivalent to “x = !x”

Sequence L13; L6; L11:

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Sequence L6; L7; L8; L13:

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- A: Yes!

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- but writes x = true, which is what L6 read!
- y = 1 (→ again non-determinacy!)

Again, both schedules would be permitted under a scheduling regime that permits ineffective writes
Ineffectiveness – 2nd Try

```
module InEffective2
output bool x = false;
  int y;
{
    fork
     if (!x) {
       y = 1;
       x = x xor true
     }
   else
     y = 0
par
  x = x xor true;
join
```

“\(x = x \oplus true\)”

- Relative writes
- Equivalent to “\(x = !x\)”

Sequence L13; L6; L11:

- \(y = 0\)

Sequence L6; L7; L8; L13:

- **Q**: Is L13 ineffective *relative to L6*?
- **A**: Yes!
- L13 is out-of-order . . .
- but writes \(x = \text{true}\), which is what L6 read!
- \(y = 1\) (→ again non-determinacy!)

- Again, both schedules would be permitted under a scheduling regime that permits ineffective writes
- → Replace “ineffectiveness” by “confluence”
Overview

Motivation

Formalizing Sequential Constructiveness (SC)
The SC Language (SCL) and the SC Graph (SCG) [Sec. 2]
Free Scheduling of SCGs [Sec. 3]
The SC Model of Computation [Sec. 4]

Wrap-Up
Combination Functions [Def. 4.1]

Combination function $f$:
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Combination function $f$:

- $f(f(x, e_1), e_2) = f(f(x, e_2), e_1)$
  for all $x$ and all side-effect free expressions $e_1, e_2$

- **Sufficient condition:**
Combination Functions [Def. 4.1]

Combination function $f$:
- $f(f(x, e_1), e_2) = f(f(x, e_2), e_1)$
  for all $x$ and all side-effect free expressions $e_1, e_2$
- **Sufficient condition:** $f$ is *commutative* and *associative*
- **Examples:**
Combination Functions [Def. 4.1]

Combination function $f$:

- $f(f(x, e_1), e_2) = f(f(x, e_2), e_1)$ for all $x$ and all side-effect free expressions $e_1, e_2$
- **Sufficient condition**: $f$ is *commutative* and *associative*
- **Examples**: $\ast, +, -, \max, \text{and, or}$
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$

- $f$ must be a combination function

Absolute writes ("write" / "initialize"): $x = e$
Relative and Absolute Writes [Def. 4.2]

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- \( f \) must be a combination function
- Evaluation of \( e \) must be free of side effects
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$
- $f$ must be a combination function
- Evaluation of $e$ must be free of side effects
- Thus, schedules
  - $'x = f(x, e_1); x = f(x, e_2)'$ and
  - $'x = f(x, e_2); x = f(x, e_1)'$ yield same result for $x$

Absolute writes ("write" / "initialize"): $x = e$
- Writes that are not relative
- E.g., $x = 0$, $x = 2y + 5$, $x = f(z)$
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$

- $f$ must be a combination function
- Evaluation of $e$ must be free of side effects
- Thus, schedules
  
  '\begin{align*}
  x &= f(x, e_1) \quad ; \quad x = f(x, e_2) \\
  x &= f(x, e_2) \quad ; \quad x = f(x, e_1)
  \end{align*}'
  
  yield same result for $x$

- Thus, writes are
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$

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- Thus, writes are confluent
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Relative writes, of type \( f \) ("increment" / "modify"): \( x = f(x, e) \)

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- Thus, schedules
  \[ x = f(x, e_1); x = f(x, e_2) \]
  \[ x = f(x, e_2); x = f(x, e_1) \]
  yield same result for \( x \)
- Thus, writes are confluent
- E.g., \( x++ \)
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Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$

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- Evaluation of $e$ must be free of side effects
- Thus, schedules
  - $'x = f(x, e_1); x = f(x, e_2)'$ and $'x = f(x, e_2); x = f(x, e_1)'$ yield same result for $x$
- Thus, writes are confluent
- E.g., $x++$, $x = 5*x$
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ (“increment” / “modify”): $x = f(x, e)$

- $f$ must be a combination function
- Evaluation of $e$ must be free of side effects
- Thus, schedules
  
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  '$x = f(x, e_2); x = f(x, e_1)$' yield same result for $x$

- Thus, writes are confluent

- E.g., $x++$, $x = 5 \times x$, $x = x-10$
Relative and Absolute Writes [Def. 4.2]

Relative writes, of type $f$ ("increment" / "modify"): $x = f(x, e)$
- $f$ must be a combination function
- Evaluation of $e$ must be free of side effects
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  $'x = f(x, e_1); x = f(x, e_2)'$ and
  $'x = f(x, e_2); x = f(x, e_1)'$ yield same result for $x$
- Thus, writes are confluent
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  \[ x = f(x, e_1); x = f(x, e_2) \]
  \[ x = f(x, e_2); x = f(x, e_1) \]
  yield same result for \( x \)
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- E.g., \( x++ \), \( x = 5*x \), \( x = x-10 \)

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- Thus, schedules
  \[
  \begin{align*}
  x &= f(x, e_1); x = f(x, e_2) \quad \text{and} \\
  x &= f(x, e_2); x = f(x, e_1)
  \end{align*}
  \]
  yield same result for \( x \)
- Thus, writes are confluent
- E.g., \( x++ \), \( x = 5 \times x \), \( x = x - 10 \)

Absolute writes ("write" / "initialize"): \( x = e \)

- Writes that are not relative
- E.g., \( x = 0 \)
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  - \( 'x = f(x, e_2); x = f(x, e_1)' \) yield same result for \( x \)
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Absolute writes ("write" / "initialize"): \( x = e \)
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- E.g., \( x = 0 \), \( x = 2 \times y + 5 \), \( x = f(z) \)
iur Relations [Def. 4.3]

Given two statically concurrent accesses $n_1 \parallel n_2$ on some variable $x$, we define the iur relations
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Given two statically concurrent accesses $n_1 \parallel n_2$ on some variable $x$, we define the iur relations

- $n_1 \xrightarrow{ww} n_2$ iff $n_1$ and $n_2$ both initialize $x$ or both perform updates of different type. We call this a $ww$ conflict.
- $n_1 \xrightarrow{iu} n_2$ iff $n_1$ initializes $x$ and $n_2$ updates $x$.
- $n_1 \xrightarrow{ur} n_2$ iff $n_1$ updates $x$ and $n_2$ reads $x$.
- $n_1 \xrightarrow{ir} n_2$ iff $n_1$ initializes $x$ and $n_2$ reads $x$. 

Since $n_1 \xrightarrow{ww} n_2$ implies $n_2 \xrightarrow{ww} n_1$, abbreviate the conjunction of $n_1 \xrightarrow{ww} n_2$ and $n_2 \xrightarrow{ww} n_1$ with $n_1 \xleftrightarrow{ww} n_2$. By symmetry, $\xrightarrow{ww}$ implies $\xleftrightarrow{ww}$. 
iur Relations [Def. 4.3]

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Since $n_1 \rightarrow_{ww} n_2$ implies $n_2 \rightarrow_{ww} n_1$:

- abbreviate the conjunction of $n_1 \rightarrow_{ww} n_2$ and $n_2 \rightarrow_{ww} n_1$ with $n_1 \leftrightarrow_{ww} n_2$. 

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Since $n_1 \rightarrow_{ww} n_2$ implies $n_2 \rightarrow_{ww} n_1$:

- abbreviate the conjunction of $n_1 \rightarrow_{ww} n_2$ and $n_2 \rightarrow_{ww} n_1$ with $n_1 \leftrightarrow_{ww} n_2$.
- by symmetry $\rightarrow_{ww}$ implies $\leftrightarrow_{ww}$.
Confluence of Nodes [Def. 4.4]

Given:

- Valid configuration \((C, M)\) of SCG
- Nodes \(n_1, n_2 \in N\)

\(n_1, n_2\) are conflicting in \((C, M)\) iff

\[
\begin{align*}
\text{1. } & n_1, n_2 \text{ active in } C, \text{ i.e. } \exists c_1, c_2 \in C \text{ with } c_i.\text{status} = \text{active} \text{ and } n_i = c_i.\text{node} \\
\text{2. } & c_1(c_2(C, M)) \neq c_2(c_1(C, M)) \\
\end{align*}
\]

\(n_1, n_2\) are confluent with each other in \((C, M)\), written:

\[
\begin{align*}
n_1 \sim (C, M) n_2, \text{ iff } & \not\exists \text{Sequence of micro steps } (C, M) \rightarrow \mu s (C', M') \text{ such that } n_1 \text{ and } n_2 \text{ are conflicting in } (C', M')
\end{align*}
\]
Confluence of Nodes [Def. 4.4]

Given:
- Valid configuration \((C, M)\) of SCG
- Nodes \(n_1, n_2 \in N\)

\(n_1, n_2\) are **conflicting** in \((C, M)\) iff

1. \(n_1, n_2\) active in \(C\)
Confluence of Nodes [Def. 4.4]

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- Valid configuration \((C, M)\) of SCG
- Nodes \(n_1, n_2 \in N\)

\(n_1, n_2\) are **conflicting** in \((C, M)\) iff

1. \(n_1, n_2\) active in \(C\),
   - i.e., \(\exists c_1, c_2 \in C\) with
     - \(c_i.status = active\) and \(n_i = c_i.node\)
Confluence of Nodes [Def. 4.4]

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   \(c_i.status = active\) and \(n_i = c_i.node\)
2. \(c_1(c_2(C, M)) \neq c_2(c_1(C, M))\)
Confluence of Nodes [Def. 4.4]

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\(n_1, n_2\) are **confluent with each other** in \((C, M)\), written: \(n_1 \sim_{(C,M)} n_2\), iff
Confluence of Nodes [Def. 4.4]

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\(n_1, n_2\) are confluent with each other in \((C, M)\), written: \(n_1 \sim_{(C,M)} n_2\), iff

- \(\not\exists\) Sequence of micro steps \((C, M) \xrightarrow{\mu_s} (C', M')\) such that \(n_1\) and \(n_2\) are conflicting in \((C', M')\)
Notes on Confluence

(From definition:) \( n_1 \sim_{(C,M)} n_2 \) iff

- Sequence of micro steps \((C, M) \rightarrow_{\mu s} (C', M')\)
  such that \(n_1\) and \(n_2\) are conflicting in \((C', M')\)

Observations I

- Confluence is taken *relative* to valid configurations \((C, M)\)
  and *indirectly* as the absence of conflicts

- Instead of requiring that confluent nodes commute with each other for *arbitrary* memories, we only consider those configurations \((C', M')\) that are *reachable* from \((C, M)\)

- *E.g.*, if it happens for a given program that in all memories \(M'\) reachable from a configuration \((C, M)\) two expressions \(ex_1\) and \(ex_2\) evaluate to the same value, then the assignments \(x = ex_1\) and \(x = ex_2\) are confluent in \((C, M)\)
Notes on Confluence

(From definition:) \( n_1 \sim_{(C,M)} n_2 \) iff

- \( \forall \) Sequence of micro steps \((C, M) \rightarrow_{\mu} (C', M')\)
  such that \( n_1 \) and \( n_2 \) are conflicting in \((C', M')\)

Observations II

- Similarly, if the two assignments are never jointly active in any reachable continuation pool \( C' \), they are confluent in \((C, M)\), too
- Thus, statements may be confluent for some program relative to some reachable configuration, but not for other configurations or in another program
Notes on Confluence

(From definition:) \( n_1 \sim_{(C,M)} n_2 \) iff

- \( \forall \) Sequence of micro steps \((C, M) \rightarrow_{\mu s} (C', M')\)
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Observations II

- Similarly, if the two assignments are never jointly active in any reachable continuation pool \( C' \), they are confluent in \((C, M)\), too

- Thus, statements may be confluent for some program relative to some reachable configuration, but not for other configurations or in another program

- However, notice that relative writes of the same type are confluent in the absolute sense, \( i.e. \), for all valid configurations \((C, M)\) of all programs
Notes on Confluence

(From definition:) $n_1 \sim_{(C,M)} n_2$ iff

- No sequence of micro steps $(C, M) \xrightarrow{\mu_s} (C', M')$ such that $n_1$ and $n_2$ are conflicting in $(C', M')$

Observations III

- Confluence $n_1 \sim_{(C,M)} n_2$ requires conflict-freeness for all configurations $(C', M')$ reachable from $(C, M)$ by arbitrary micro-sequences under free scheduling
- Will use this notion of confluence to define the restricted set of SC-admissible macro ticks
Notes on Confluence

(From definition:) \( n_1 \sim (C,M) n_2 \) iff

- \( \nexists \) Sequence of micro steps \( (C, M) \rightarrow_{\mu s} (C', M') \)
  such that \( n_1 \) and \( n_2 \) are conflicting in \( (C', M') \)

Observations III

- Confluence \( n_1 \sim (C,M) n_2 \) requires conflict-freeness for all
  configurations \( (C', M') \) reachable from \( (C, M) \) by arbitrary
  micro-sequences under free scheduling

- Will use this notion of confluence to define the restricted set
  of \( SC\)-admissible macro ticks

- Since compiler will ensure \( SC\)-admissibility of the execution
  schedule, one might be tempted to define confluence relative to these
  \( SC\)-admissible schedules;
Notes on Confluence

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- Since compiler will ensure \(SC\)-admissibility of the execution schedule,
  one might be tempted to define confluence relative to these \(SC\)-admissible schedules;
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Observations IV

- This relative view of confluence keeps the scheduling constraints on SC-admissible macro ticks sufficiently weak
Notes on Confluence

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- There exists a sequence of micro steps \((C, M) \rightarrow_{\mu_s} (C', M')\) such that \(n_1\) and \(n_2\) are conflicting in \((C', M')\).

Observations IV

- This relative view of confluence keeps the scheduling constraints on SC-admissible macro ticks sufficiently weak.
- **Note:** two nodes confluent in some configuration are still confluent in every later configuration reached through an arbitrary sequence of micro steps.
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(From definition:) \( n_1 \sim (C,M) n_2 \) iff

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Observations IV

- This relative view of confluence keeps the scheduling constraints on SC-admissible macro ticks sufficiently weak
- Note: two nodes confluent in some configuration are still confluent in every later configuration reached through an arbitrary sequence of micro steps
- However, more nodes may become confluent in later configurations, because some conflicting configurations are no longer reachable
Notes on Confluence

(From definition:) \( n_1 \sim_{(C,M)} n_2 \) iff

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Observations IV

- This relative view of confluence keeps the scheduling constraints on SC-admissible macro ticks sufficiently weak
- **Note**: two nodes confluent in some configuration are still confluent in every later configuration reached through an arbitrary sequence of micro steps
- However, more nodes may become confluent in later configurations, because some conflicting configurations are no longer reachable
- Exploit this in following definition of confluence of node instances by making confluence of node instances within a macro tick relative to the index position at which they occur
Confluence of Node Instances [Def. 4.5]

Given:

- Macro tick $R$
- $(C_i, M_i)$ for $0 \leq i \leq \text{len}(R)$, the configurations of $R$
- Node instances $n_{i_1} = (n_1, i_1)$ and $n_{i_2} = (n_2, i_2)$ in $R$
Confluence of Node Instances [Def. 4.5]

Given:

- Macro tick $R$
- $(C_i, M_i)$ for $0 \leq i \leq \text{len}(R)$, the configurations of $R$
- Node instances $ni_1 = (n_1, i_1)$ and $ni_2 = (n_2, i_2)$ in $R$, i.e., $1 \leq i_1, i_2 \leq \text{len}(R)$, $n_1 = R(i_1)$, $n_2 = R(i_2)$
Confluence of Node Instances [Def. 4.5]

Given:
- Macro tick \( R \)
- \((C_i, M_i)\) for \(0 \leq i \leq \text{len}(R)\), the configurations of \( R \)
- Node instances \( ni_1 = (n_1, i_1) \) and \( ni_2 = (n_2, i_2) \) in \( R \), i.e., \(1 \leq i_1, i_2 \leq \text{len}(R)\), \( n_1 = R(i_1) \), \( n_2 = R(i_2) \)

Call node instances confluent in \( R \), written \( ni_1 \sim_R ni_2 \), iff
Confluence of Node Instances [Def. 4.5]

Given:
- Macro tick $R$
- $(C_i, M_i)$ for $0 \leq i \leq \text{len}(R)$, the configurations of $R$
- Node instances $n_i_1 = (n_1, i_1)$ and $n_i_2 = (n_2, i_2)$ in $R$, i.e., $1 \leq i_1, i_2 \leq \text{len}(R)$, $n_1 = R(i_1)$, $n_2 = R(i_2)$

Call node instances confluent in $R$, written $n_i_1 \sim_R n_i_2$, iff
- for $i = \text{min}(i_1, i_2) - 1$
- $n_1 \sim (C_i, M_i) n_2$
Recall sequence L6; L7; L8; L13:

Q: Is L13 ineffective relative to L6?
A: Yes!

L13 is out-of-order . . . but writes x = false, which is what L6 read!

Q: Are L6 and L13 confluent?
A: No!

L6 and L13 conflict at point of execution of L6 → Def. of SC-admissibility – specifically, the underlying scheduling relations – uses confluence condition
InEffective2 Revisited

Recall sequence L6; L7; L8; L13:

▶ Q: Is L13 ineffective relative to L6?

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Recall sequence L6; L7; L8; L13:

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Recall sequence L6; L7; L8; L13:

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Recall sequence L6; L7; L8; L13:

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- **Q:** Is L13 ineffective *relative to L6*?
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- L13 is out-of-order . . .
- but writes \( x = \) false, which is what L6 read!

- **Q:** Are L6 and L13 confluent?
- **A:** No!
- L6 and L13 conflict at point of execution of L6

→ Def. of SC-admissibility – specifically, the underlying scheduling relations – uses confluence condition
Scheduling Relations [Def 4.6]

Given:
- Macro tick $R$ with
- Node instances $ni_{1,2} = (n_{1,2}, i_{1,2})$, i.e., $1 \leq i_{1,2} \leq \text{len}(R)$ and $n_{1,2} = R(i_{1,2})$
- $ni_{1,2}$ concurrent in $R$, i.e., $ni_1 \mid_R ni_2$
- $ni_{1,2}$ not confluent in $R$, i.e., $ni_1 \not\sim_R ni_2$

Then:
- $ni_1 \rightarrow_R \alpha ni_2$ iff $n_1 \rightarrow_\alpha n_2$ for some $\alpha \in \alpha_{iur}$
- $ni_1 \rightarrow_R ni_2$ iff $i_1 < i_2$; i.e., $ni_1$ happens before $ni_2$ in $R$. 
Sequential Admissibility [Def. 4.7]

A macro tick $R$ is SC-admissible iff

1. for all node instances $ni_{1,2} = (n_{1,2}, i_{1,2})$ in $R$, with $1 \leq i_{1,2} \leq \text{len}(R)$ and $n_{1,2} = R(i_{1,2})$,

2. for all $\alpha \in \alpha_{iur}$

the scheduling condition $SC_\alpha$ holds:
Sequential Admissibility [Def. 4.7]

A macro tick $R$ is **SC-admissible** iff

- for all node instances $ni_{1,2} = (n_{1,2}, i_{1,2})$ in $R$, with $1 \leq i_{1,2} \leq \text{len}(R)$ and $n_{1,2} = R(i_{1,2}),$

- for all $\alpha \in \alpha_{iur}$

the scheduling condition $\text{SC}_\alpha$ holds:
if $ni_1 \rightarrow^R_{\alpha} ni_2$ then $ni_1 \rightarrow^R ni_2.$

A run for an SCG is **SC-admissible** if all macro ticks $R$ in this run are SC-admissible.
SC-admissibility vs. Determinacy
SC-admissibility vs. Determinacy

```plaintext
module NonDet
output bool x = false, y = false;
{
    fork { // Thread CheckX
        if (!x)
            y = true;
    }
    par { // Thread CheckY
        if (!y)
            x = true
    }
    join
}
```

Thus:
SC-admissibility \(\not\Rightarrow\) Determinacy
SC-admissibility vs. Determinacy

```plaintext
module NonDet
output bool x = false, y = false;
{
  fork { // Thread CheckX
    if (!x)
      y = true;
  }
  par { // Thread CheckY
    if (!y)
      x = true
  }
  join
}
```

▶ Admissible runs?

Thus: SC-admissibility \(\not\Rightarrow\) Determinacy
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▶ Admissible runs? Yes, multiple
SC-admissibility vs. Determinacy

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      x = true
  }
  join
}
```

- Admissible runs? Yes, multiple
- Determinate?
SC-admissibility vs. Determinacy

```python
module NonDet
output bool x = false, y = false;
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  fork { // Thread CheckX
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      y = true;
  }
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  }
  join
}
```

- Admissible runs? Yes, multiple
- Determinate? No
SC-admissibility vs. Determinacy

Admissible runs? Yes, multiple
Determinate? No

```c
module NonDet
output bool x = false, y = false;
{
    fork { // Thread CheckX
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            y = true;
    }
    par { // Thread CheckY
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    join
}
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SC-admissibility vs. Determinacy

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module NonDet
output bool x = false, y = false;
{
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    }
    join
}
```

- Admissible runs? Yes, multiple
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Thus: SC-admissibility $\not\Rightarrow$ Determinancy
SC-admissibility vs. Determinacy
SC-admissibility vs. Determinacy

```
module Fail
output bool z = false;
{
  fork {
    if (!z)
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  }
  par {
    if (z)
      z = true
  }
  join
}
```
SC-admissibility vs. Determinacy

```plaintext
module Fail
output bool z = false;
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    fork {
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Thus: Determinacy \(\not\Rightarrow\) SC-admissibility
SC-admissibility vs. Determinacy

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  }
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▶ Admissible runs? No

Thus:

Determinacy \( \not\Rightarrow \) SC-admissibility
SC-admissibility vs. Determinacy

```plaintext
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      z = true
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  join
}
▶ Admissible runs? No
▶ Determinate?
SC-admissibility vs. Determinacy

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```

- Admissible runs? No
- Determinate? Yes
SC-admissibility vs. Determinacy

```plaintext
module Fail
output bool z = false;
{
  fork {
    if (!z)
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  }
  par {
    if (z)
      z = true
  }
  join
}
```

▶ Admissible runs? No
▶ Determinate? Yes

Thus: Determinacy $\nRightarrow$ SC-admissibility
Sequential Constructiveness [Def. 4.8]

Definition: A program $P$ is **sequentially constructive (SC)** iff for each initial configuration and input sequence:

1. There exists an SC-admissible run ($P$ is **reactive**)
2. Every SC-admissible run generates the same determinate sequence of macro responses ($P$ is **determinate**)

Motivation

Formalizing Sequential Constructiveness (SC)

Wrap-Up
Overview

Motivation

Formalizing Sequential Constructiveness (SC)

Wrap-Up
  Synchronous Program Classes
  Summary
Synchronous Program Classes [TR, Sec. 9]

Sequentially Constructive (SC)  
Acyclic Schedulable (ASC)  
Sequences of values  
Static cycles  
Dynamic scheduling
Synchronous Program Classes [TR, Sec. 9]

- **Sequentially Constructive (SC)**
- **Berry Constructive (BC)**
- **Acyclic Schedulable (ASC)**

**Static cycles**

**Dynamic scheduling**

**Sequences of values**
Synchronous Program Classes [TR, Sec. 9]

- Sequentially Constructive (SC)
- Logically Correct (LC)
- Acyclic Schedulable (ASC)
- Speculate on absence or presence
- Sequences of values
- Static cycles
  - Dynamic scheduling
- Out-of-order scheduling
- Berry Constructive (BC)
- Logically Correct (LC)
Synchronous Program Classes [TR, Sec. 9]

Sequentially Constructive (SC)

Logically Correct (LC)

Pnueli-Shalev (PC)

Berry Constructive (BC)

Acyclic Schedulable (ASC)

Static cycles
Dynamic scheduling

Sequences of values

Out-of-order scheduling

Speculate on absence
Speculate on absence or presence

Speculate on absence

Sequences of values

Static cycles
Dynamic scheduling

Out-of-order scheduling
Synchronous Program Classes [TR, Sec. 9]

- Seementially Constructive (SC)
- Logically Correct (LC)
- Pnueli-Shalev (PC)
- Berry Constructive (BC)
- Acyclic Schedulable (ASC)
- Out-of-order scheduling
- Cycle of concurrent dependencies, or concurrent writes
- Sequences of values
- Static cycles
- Dynamic scheduling
- Speculate on absence
- Speculate on absence or presence

All Programs
Synchronous Program Classes [TR, Sec. 9]

- Sequentially Constructive (SC)
- Logically Correct (LC)
- Pnueli-Shalev (PC)
- Berry Constructive (BC)
- Acyclic Schedulable (ASC)
- NonDet
- Out-of-order scheduling
- Cycle of concurrent dependencies
- or concurrent writes
- Speculate on absence
- Speculate on absence or presence
- All Programs

Static cycles
Dynamic scheduling
Sequences of values
P_{AS}
P_{APS}
P_{ALPS}
P_{ABLPS}
P_{BLPS}
P_{LP}
P_{LS}
P_{PS}
P_{P}
P_{LPS}
Synchronous Program Classes

Example $P_{APS} =$
Example $P_{APS} = \text{if } (x) x = 1$
Example $P_{AS} =$
Example $P_{AS} = \text{if } (\neg x) x = 1$
Synchronous Program Classes

Example $P_{ALS} =$
Synchronous Program Classes

Example $P_{ALS} = \text{if} \ (\neg x) \ x = 1 \ \text{else} \ x = 1$
Example $P_{ALPS} =$
Example $P_{ALPS} = \text{if } (!x \&\& y) \{ x = 1; y = 1 \}$
Summary

Underlying idea of sequential constructiveness rather simple
Summary

Underlying idea of sequential constructiveness rather simple

- Prescriptive instead of descriptive sequentiality
- Thus circumventing “spurious” causality problems
- Initialize-update-read protocol
Summary

Underlying idea of sequential constructiveness rather simple
- Prescriptive instead of descriptive sequentiality
- Thus circumventing “spurious” causality problems
- Initialize-update-read protocol

However, precise definition of SC MoC not trivial
- Challenging to ensure conservativeness relative to Berry-constructiveness
- Plain initialize-update-read protocol does not accommodate, e.g., signal re-emissions
- Restricting attention to concurrent, non-confluent node instances is key
Conclusions

▶ Clocked, **synchronous model of execution** for imperative, shared-memory multi-threading

▶ Conservatively extends synchronous programming (Esterel) by **standard sequential control flow** (Java, C)

▶ \(\Rightarrow\) Deterministic concurrency with synchronous foundations, but without synchronous restrictions
  
  ▶ ☺ Expressive and intuitive sequential paradigm
  
  ▶ ☺ Predictable concurrent threads
Future Work

Plenty of extensions/adaptations possible . . .

▶ Alternative notions of sequential constructiveness:
  ▶ A truly “constructive” approach that sharpens SC admissibility to determinate schedules
  ▶ Extension of iur-protocol, e.g., to model ForeC

▶ Improved synthesis & analysis — see also next lecture