Synchronous Languages—Lecture 06

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13 Nov. 2018

Last compiled: November 13, 2018, 10:37 hrs

Esterel IV—The Constructive Semantics

The 5-Minute Review Session

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5. When is an Esterel program *logically reactive*?

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5. When is an Esterel program *logically reactive*? ... *correct*?

Overview

The Constructive Semantics

- External Justification vs. Self-Justification
- The Constructive Behavioral Semantics
- The Constructive Operational Semantics

External Justification vs. Self-Justification

- Programming in Esterel:
  - Analyze input events to generate appropriate output signals
  - Use concurrent statements and intermediate local signals to create modular, well-structured programs

External Justification vs. Self-Justification

- Programming in Esterel:
  - Analyze input events to generate appropriate output signals
  - Use concurrent statements and intermediate local signals to create modular, well-structured programs
- Natural way of thinking:
  - Information propagation by *cause* and *effect*

```
present I then
  emit O
end
```

External Justification vs. Self-Justification

```
module P1:
input I;
output O;
signal S1, S2 in
  present I then emit S1 end
||
  present S1 else emit S2 end
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  present S2 then emit O end
end signal
end module
```

- Is this logically correct?

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module P9:
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module P9:
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- Is this logically correct?
  - Yes!
- Is this well-behaved wrt information propagation?
  - No!
- Accepting P9 as correct is
  - Logically possible
  - But against (imperative) intention of the language

External Justification vs. Self-Justification

- "`present` S `then` p `end`":
  - *First* test the status of S, *then* execute p if S is present
  - Status of S should not depend on what p *might* do

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- Synchrony hypothesis:
  - Ordering implicit in the `then` word is not that of time, but that of sequential causality
- Want actual computation:
  - "*Since* S is present, we take the then branch"
- Don't want speculative computation:
  - "*If* we assume S present, then we take the then branch"

External Justification vs. Self-Justification

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module P10:
present O then
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module P10:
present O then
  nothing;
end;
emit O
```

- This is logically correct
- But still want to reject it:
  - In the logical semantics, the information that O is present flows backwards across the sequencing operator
  - Contradicts basic intuition about sequential execution

The Constructive Semantics

- Constructive semantics:
  - Does not check assumptions about signal statuses
  - Instead, propagates facts about control flow and signal statuses

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  - Instead, propagates facts about control flow and signal statuses
- Three equivalent presentations:
  1. Constructive behavioral semantics
  2. Constructive operational semantics
  3. Circuit semantics

The Constructive Semantics

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   - Derived from the logical behavioral semantics
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2. Constructive operational semantics:
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   - Is the simplest way of defining an efficient interpreter
3. Circuit semantics:
   - Translation of programs into constructive circuits
   - Is the core of the Esterel v5 compiler

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- Define disjoint predicates to express
  - "A statement must terminate, must pause, must exit a trap T, or must emit a signal S"
  - "A statement cannot terminate, cannot pause, cannot exit a trap T, or cannot emit a signal S"

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- Define disjoint predicates to express
  - "A statement must terminate, must pause, must exit a trap T, or must emit a signal S"
  - "A statement cannot terminate, cannot pause, cannot exit a trap T, or cannot emit a signal S"
- The *Must* (*Cannot*) predicate determines
  - Which signals are present (absent)
  - Which statements are (cannot be) executed

The Constructive Behavioral Semantics

Recall: Logical Coherence Law

> *A signal S is present in an instant iff an "`emit` S" statement is executed in this instant.*

The Constructive Behavioral Semantics

Recall: Logical Coherence Law

A signal S is present in an instant iff an "`emit` S" statement is executed in this instant.

Replace with disjoint Constructive Coherence Laws:

A signal S is present iff an "`emit` S" statement must be executed.
A signal S is absent iff an "`emit` S" statement cannot be executed

The Constructive Behavioral Semantics

- Define *Must* and *Cannot* predicates by structural induction on statements
- A signal can have three statuses:
  - "$+$": known to be present
  - "$-$": known to be absent
  - "$\perp$": yet unknown

The Constructive Behavioral Semantics

- Define *Must* and *Cannot* predicates by structural induction on statements
- A signal can have three statuses:
  - "$+$": known to be present
  - "$-$": known to be absent
  - "$\perp$": yet unknown
- Is technically easier to define the *Cannot* predicate as the negation of a *Can* predicate
  - No constructiveness problem here as we only deal with finite sets

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- Must (resp. can) execute q if p must (resp. can) terminate

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  - Test behaves as p

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- If S is known to be absent:
  - Test behaves as q
- If S is yet unknown:
  - Test can do whatever p or q can do
  - **There is nothing the test must do**. In particular, it does not even have to do what both p and q have to do—this is the essence of disallowing speculative execution.

The Constructive Behavioral Semantics

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- Consider local signal here; output signal is similar

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Can predicate:

- Recursively analyze p with status $\perp$ for S

The Constructive Behavioral Semantics

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Must predicate:

- Assume we already know that we must execute `signal` S `in` p `end` in some signal context E
- Must compute final status of S to determine signal context of p

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- If S must be emitted:
  - Propagate this information by **reanalyzing** p in E with S present
  - This may generate more information about the other signals

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- Must compute final status of S to determine signal context of p
- First analyze p in E augmented by setting the unknown status $\perp$ for S
- If S must be emitted:
  - Propagate this information by **reanalyzing** p in E with S present
  - This may generate more information about the other signals
- Similarly, if we find that S cannot be emitted:
  - Reanalyze p in E with S absent

Accepting Programs

In the constructive behavioral semantics, a program is accepted as constructive iff fact propagation using the *Must* and *Can* (or *Cannot*) predicates suffices in establishing presence or absence of all output signals (and we can also compute a derivative—see later)

```
module P1:
input I;
output O;
signal S1, S2 in
  present I then emit S1 end
||
  present S1 else emit S2 end
||
  present S2 then emit O end
end signal
end module
```

Accepting Programs

```
module P2:
signal S in
  emit S;
  present O then
    present S then
      pause
    end;
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  - No need for speculative computation based on *assumptions*
  - Our analysis still "looks ahead" to see what must/cannot be done, but always builds on facts established so far, not on speculations

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- Can analyze this with just propagating *facts*
  - No need for speculative computation based on *assumptions*
  - Our analysis still "looks ahead" to see what must/cannot be done, but always builds on facts established so far, not on speculations
- However, analysis involves recomputations
  - **Avoiding this is goal of operational and circuit semantics!**

Rejecting Programs

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module P3:
output O;
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end module
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module P4:
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Rejecting Programs

- Constructiveness $\Longrightarrow$ logical correctness
- **But not vice versa!**

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module P9:
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module P9:
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- Both O1 and O2 can be emitted

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- No signal must be emitted

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- No signal must be emitted
- No progress—reject P9!

Rejecting Programs

Consider variant of P2:

```
module P11:
signal S
  present O then
    emit S;
    present S then
      pause
    end;
    emit O
  end
end signal
```

Rejecting Programs

Consider variant of P2:

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module P11:
signal S
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- Are not allowed to speculatively execute branches

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Consider variant of P2:

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module P11:
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  present O then
    emit S;
    present S then
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  end
end signal
```

- Are not allowed to speculatively execute branches
- Again no progress—reject P11!

Rejecting Programs

```
module P12:
present O then
  emit O;
else
  emit O
end
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Rejecting Programs

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module P12:
present O then
  emit O;
else
  emit O
end
```

- Must reject P12 as well!

Rejecting Programs

```
module P12:
present O then
  emit O;
else
  emit O
end
```

- Must reject P12 as well!
- Does an equivalent HW-circuit always stabilize?
  (*Will come back to this later ...*)

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- *Must* function determines what must be done in a reaction $P \xrightarrow[I]{O} P'$

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- Has the form $Must(p, E) = \langle S, K \rangle$
  - E: partial event, associating status in $B_\perp = \{+, -, \perp\}$ with each signal
  - S: set of signals that p must emit
  - K: set of completion codes that p must return
    - Is either empty or a singleton

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  - S: set of signals that p must emit
  - K: set of completion codes that p must return
    - Is either empty or a singleton
- Use subscripts to access elements of result pair:
  - $Must(p, E) = \langle Must_s(p, E), Must_k(p, E) \rangle$

The *Must*, *Cannot*, and *Can* Functions

- $Cannot^m$ function prunes out false paths
- $Cannot^m(p, E) = \langle Cannot^m_s(p, E), Cannot^m_k(p, E)\rangle = \langle S, K\rangle$
- Extra argument $m \in \{+, \perp\}$ indicates whether it is known that p must be executed in event E
- $Can^m(p, E)$ is component-wise complement

Definitions of *Must* and *Can*

- Completion and signal emission:

$$Must(k, E) = Can^m(k, E) = \langle \emptyset, \{k\} \rangle$$

$$Must(!s, E) = Can^m(!s, E) = \langle \{s\}, \{0\} \rangle$$

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$$Must(!s, E) = Can^m(!s, E) = \langle \{s\}, \{0\} \rangle$$

- Suspension:

$$Must(s \supset p, E) = Must(p, E)$$
$$Can^m(s \supset p, E) = Can^m(p, E)$$

Definitions of *Must* and *Can*

- Signal test:

Definitions of *Must* and *Can*

- Signal test:

$$Must((s?p,q),E) = \begin{cases} Must(p,E) & \text{if } s^{+} \in E \\ Must(q,E) & \text{if } s^{-} \in E \\ \langle \emptyset, \emptyset \rangle & \text{if } s^{\perp} \in E \end{cases}$$

$$Can^{m}((s?p,q),E) = \begin{cases} Can^{m}(p,E) & \text{if } s^{+} \in E \\ Can^{m}(q,E) & \text{if } s^{-} \in E \\ Can^{\perp}(p,E) \cup Can^{\perp}(q,E) & \text{if } s^{\perp} \in E \end{cases}$$

Definitions of *Must* and *Can*

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$$Must(p; q, E) = \begin{cases} Must(p, E) \\ \qquad\qquad \text{if } 0 \notin Must_k(p, E) \\ \langle Must_S(p, E) \cup Must_S(q, E), Must_k(q, E)\rangle \\ \qquad\qquad \text{if } 0 \in Must_k(p, E) \end{cases}$$

$$Can^m(p; q, E) = \begin{cases} Can^m(p, E) \\ \qquad\qquad \text{if } 0 \notin Can^m_k(p, E) \\ \langle Can^m_S(p, E) \cup Can^{m'}_S(q, E), Can^m_k(p, E) \setminus 0 \cup Can^{m'}_k(q, E)\rangle \\ \qquad\qquad \text{if } 0 \in Can^m_k(p, E) \\ \qquad \text{with } m' = \begin{cases} + & \text{if } m = + \wedge 0 \in Must_k(p, E) \\ \perp & \text{otherwise} \end{cases} \end{cases}$$

Definitions of *Must* and *Can*

- Local signal declaration:

Definitions of *Must* and *Can*

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$$Must(p \setminus s, E) = \begin{cases} Must(p, E * s^{+}) \setminus s & \text{if } s \in Must_S(p, E * s^{\perp}) \\ Must(p, E * s^{-}) \setminus s & \text{if } s \notin Can_S^{+}(p, E * s^{\perp}) \\ Must(p, E * s^{\perp}) \setminus s & \text{otherwise} \end{cases}$$

$$Can^m(p \setminus s, E) = \begin{cases} Can^{+}(p, E * s^{+}) \setminus s \\ \qquad \text{if } m = + \text{ and } s \in Must_S(p, E * s^{\perp}) \\ Can^m(p, E * s^{-}) \setminus s \\ \qquad \text{if } s \notin Can_S^{+}(p, E * s^{\perp}) \\ Can^m(p, E * s^{\perp}) \setminus s \\ \qquad \text{otherwise} \end{cases}$$

Definitions of *Must* and *Can*

- Note the *Can*/*Must* asymmetry: in the *Can*-predicate of the local signal declaration, check for $m = +$ before calling *Must* to avoid speculative computation
- Otherwise, would accept program

```
present O then
  signal S in
    emit S
  ||
    present S else emit O end
  end
end
```

Definitions of *Must* and *Can*

- Loop:

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$$Must(p*, E) = Must(p, E)$$
$$Can^m(p*, E) = Can^m(p, E)$$

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- Parallel:

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- Parallel:

$$\begin{aligned}Must(p|q, E) = \langle & Must_S(p, E) \cup Must_S(q, E),\\ & Max(Must_k(p, E), Must_k(q, E))\rangle\end{aligned}$$

$$\begin{aligned}Can^m(p|q, E) = \langle & Can^m_S(p, E) \cup Can^m_S(q, E),\\ & Max(Can^m_k(p, E), Can^m_k(q, E))\rangle\end{aligned}$$

The *Max*-operator on sets of completion codes is defined as

$$Max(K, L) = \begin{cases} \emptyset & \text{if } K = \emptyset \text{ or } L = \emptyset \\ \{\max(k, l) \mid k \in K, l \in L\} & \text{if } K, L \neq \emptyset \end{cases}$$

Definitions of *Must* and *Can*

- Trap:

Definitions of *Must* and *Can*

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$$Must(\{p\}, E) = \langle Must_S(p, E), {\downarrow} Must_k(p, E)\rangle$$

$$Can^m(\{p\}, E) = \langle Can^m_S(p, E), {\downarrow} Can^m_k(p, E)\rangle$$

- Shift:

Definitions of *Must* and *Can*

- Trap:

$$Must(\{p\}, E) = \langle Must_S(p, E), \downarrow Must_k(p, E)\rangle$$

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- Shift:

$$Must(\uparrow p, E) = \langle Must_S(p, E), \uparrow Must_k(p, E)\rangle$$

$$Can^m(\uparrow p, E) = \langle Can^m_S(p, E), \uparrow Can^m_k(p, E)\rangle$$

Definition of the Constructive Behavioral Semantics

The constructive behavioral semantics of a given program is defined by a two-step procedure, yielding the current reaction and the derivative:

1. Compute output event O using *Must* and *Cannot* predicates
   - This fails if status of some output signal cannot be determined to be $+$ or $-$

Definition of the Constructive Behavioral Semantics

The constructive behavioral semantics of a given program is defined by a two-step procedure, yielding the current reaction and the derivative:

1. Compute output event O using *Must* and *Cannot* predicates
   - This fails if status of some output signal cannot be determined to be $+$ or $-$
2. Compute behavioral transition yielding program derivative
   - This fails if body of some loop is found to terminate instantaneously
   - This also fails if we cannot establish the presence/absence of a local signal

Definition of the Constructive Semantics

Step 1: Compute output event O

Approach:

- Start with undefined O (all output signal statuses $= \perp$)
- Iteratively enrich O using *Must* and *Can* information
- Terminate when this stabilizes (guaranteed by monotonicity)

Formalize this as computation of a least fixed point (see draft book)

Algorithm to Compute Outputs

```
function computeOut(P, I)
  E = I ∪ {s⊥ | s ∈ Out(P)}
  do
    E' = E
    can = Can+_S(P, E)
    must = Must_S(P, E)
    E = I ∪ {s+ | s ∈ must}
          ∪ {s- | s ∈ Out(P) \ can}
          ∪ {s⊥ | s ∈ can \ must}
  while (E' ≠ E)
  if ∃s : s⊥ ∈ E then error ("not constructive")
  return E
```

Example for *Can* analysis

Consider the program $p = !S; S?!O, 1$ and environment $\{S^{\perp}, O^{\perp}\}$.

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Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$.

$$Can^+(!S, \{S^\perp, O^\perp\}) = \langle\{S\}, \{0\}\rangle$$
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Example for *Can* analysis

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$$Can^+(!S, \{S^\perp, O^\perp\}) = \langle\{S\}, \{0\}\rangle$$
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Example for *Can* analysis

Consider the program $p =!S; S?!O, 1$ and environment $\{S^{\perp}, O^{\perp}\}$.

$$
\begin{aligned}
Can^{+}(!S, \{S^{\perp}, O^{\perp}\}) &= \langle\{S\}, \{0\}\rangle \\
Must_{k}(!S, \{S^{\perp}, O^{\perp}\}) &= \{0\} \\
Can^{\perp}(!O, \{S^{\perp}, O^{\perp}\}) &=
\end{aligned}
$$

Example for *Can* analysis

Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$.

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&\qquad Can^{\perp}(1, \{S^{\perp}, O^{\perp}\}) =
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Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$.

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\end{aligned}
$$

Gives no new information on signal status

Example for *Must* analysis

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Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$

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   $Must(S?!O, 1, \{S^\perp, O^\perp\}) =$

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Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$

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2. Update environment to $\{S^+, O^\perp\}$

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2. Update environment to $\{S^+, O^\perp\}$
3. $Must(!S, \{S^+, O^\perp\}) =$

Example for *Must* analysis

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Example for *Must* analysis

Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$

1. $$Must(!S, \{S^\perp, O^\perp\}) = \langle\{S\}, \{0\}\rangle$$
   $$Must(S?!O, 1, \{S^\perp, O^\perp\}) = \langle\emptyset, \emptyset\rangle$$
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2. Update environment to $\{S^+, O^\perp\}$
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   $$Must(!S; S?!O, 1, \{S^+, O^\perp\}) = \langle\{S, O\}, \{0\}\rangle$$
4. Update environment to $\{S^+, O^+\}$

Example for *Must* analysis

Consider the program $p = !S; S?!O, 1$ and environment $\{S^\perp, O^\perp\}$

1. $$\begin{aligned} Must(!S, \{S^\perp, O^\perp\}) &= \langle\{S\}, \{0\}\rangle \\ Must(S?!O, 1, \{S^\perp, O^\perp\}) &= \langle\emptyset, \emptyset\rangle \\ Must(!S; S?!O, 1, \{S^\perp, O^\perp\}) &= \langle\{S\}, \emptyset\rangle \end{aligned}$$
2. Update environment to $\{S^+, O^\perp\}$
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4. Update environment to $\{S^+, O^+\}$
5. All signals have a defined status $\rightarrow$ done

Definition of the Constructive Semantics

Step 2: Compute transition

Rules are exactly as for logical behavioral semantics—except for changed rules for local signals

$$\boxed{\frac{p \xrightarrow[E * s^+]{E' * s^+, k} p' \quad S(E') = S(E) \setminus s}{p \setminus s \xrightarrow[E]{E', k} p' \setminus s}} \qquad (\text{sig } +)$$

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is replaced with

$$\boxed{\dfrac{s \in Must_s(p, E * s^\perp) \quad p \xrightarrow[E*s^+]{E'*s^+,k} p' \quad S(E') = S(E) \setminus s}{p \setminus s \xrightarrow[E]{E',k} p' \setminus s}} \qquad (\text{csig } +)$$

Definition of the Constructive Semantics

$$\boxed{\frac{p \xrightarrow[E*s^-]{E'*s^-,k} p' \quad S(E') = S(E) \setminus s}{p \setminus s \xrightarrow[E]{E',k} p' \setminus s}} \qquad (\text{sig } -)$$

is replaced with

$$\boxed{\frac{s \in Cannot_s^+(p, E * s^\perp) \quad p \xrightarrow[E*s^-]{E'*s^-,k} p' \quad S(E') = S(E) \setminus s}{p \setminus s \xrightarrow[E]{E',k} p' \setminus s}} \qquad (\text{csig } -)$$

The Constructive Operational Semantics

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  - ☺ Instead of reasoning about what we must do, just do it

The Constructive Operational Semantics

- . . . is defined by a rewriting-based interpretation scheme
  - ☺ Instead of reasoning about what we must do, just do it
  - ☹ Formal definition and technical treatment of the constructive *operational* semantics is much heavier than that of the constructive *behavioral* semantics
- Will still take constructive *behavioral* semantics as the primary semantics

The Constructive Operational Semantics

- Decorate signal declarations with status $+$, $-$, $\perp$
- Initially, all signals except inputs unknown
- Constructive operational semantics is a micro-step semantics
  - Current state indicated by •

The Constructive Operational Semantics

Consider P1 with I present:

```
module P1:
input I+;
output O⊥;
•signal S1⊥, S2⊥ in
  present I then emit S1 end
||
  present S1 else emit S2 end
||
  present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Fork of the parallel statement:

```
module P1:
input I⁺;
output O⊥;
signal S1⊥, S2⊥ in
  •present I then emit S1 end
||
  •present S1 else emit S2 end
||
  •present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Only first thread can continue:

```
module P1:
input I^+;
output O^⊥;
signal S1^⊥, S2^⊥ in
  present I then •emit S1 end
||
  •present S1 else emit S2 end
||
  •present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Now emit S1:

```
module P1:
input I+;
output O⊥;
signal S1+, S2⊥ in
  present I then emit S1 end●
||
  ●present S1 else emit S2 end
||
  ●present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Now the 2nd branch can continue:

```
module P1:
input I^+;
output O^⊥;
signal S1^+, S2^⊥ in
  present I then emit S1 end•
||
  present S1 else emit S2 end•
||
  •present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Cannot emit S2 any more:

```
module P1:
input I+;
output O⊥;
signal S1+, S2− in
  present I then emit S1 end•
||
  present S1 else emit S2 end•
||
  •present S2 then emit O end
end signal
end module
```

The Constructive Operational Semantics

Now 3rd branch can continue:

```
module P1:
input I⁺;
output O⊥;
signal S1⁺, S2⁻ in
  present I then emit S1 end•
||
  present S1 else emit S2 end•
||
  present S2 then emit O end•
end signal
end module
```

The Constructive Operational Semantics

Cannot emit O any more:

```
module P1:
input I+;
output O−;
signal S1+, S2− in
  present I then emit S1 end●
||
  present S1 else emit S2 end●
||
  present S2 then emit O end●
end signal
end module
```

The Constructive Operational Semantics

Synchronize the terminated threads:

```
module P1:
input I+;
output O−;
signal S1+, S2− in
  present I then emit S1 end
||
  present S1 else emit S2 end
||
  present S2 then emit O end
end signal●
end module
```

The Constructive Operational Semantics

Now consider P2:

```
module P2:
output O⊥;
•signal S⊥ in
  emit S;
  present O then
    present S then
      pause
    end;
    emit O
  end
end signal
end module
```

The Constructive Operational Semantics

After 3 microsteps:

```
module P2:
output O⊥;
signal S+ in
  emit S;
  •present O then
    present S then
      pause
    end;
    emit O
  end
end signal
end module
```

The Constructive Operational Semantics

Perform cannot analysis (as in constructive behavioral semantics)—and set O absent:

```
module P2:
output O⁻;
signal S⁺ in
  emit S;
  •present O then
    present S then
      pause
    end;
    emit O
  end
end signal
end module
```

The Constructive Operational Semantics

Take implicit `else` branch of test:

```
module P2:
output O⁻;
signal S⁺ in
  emit S;
  present O then
    present S then
      pause
    end;
    emit O
  end
end signal●
end module
```

The Constructive Operational Semantics

- Statuses evolve monotonically
  - Hence avoid most of the recomputations that take place in the constructive behavioral semantics
- Rejecting programs is similar to constructive behavioral semantics

The Constructive Operational Semantics

- Statuses evolve monotonically
  - Hence avoid most of the recomputations that take place in the constructive behavioral semantics
- Rejecting programs is similar to constructive behavioral semantics

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- No possible initial microstep $\Longrightarrow$ cannot set O^+
- Potential path to `emit O` $\Longrightarrow$ cannot set O^-

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- Signals are shared objects with status $\{+, -, \perp\}$
- Signal status initialization:
  - Input signals are initialized according to the input event
  - Other signals initialized to $\perp$
- Signal status changes:
  - Status of a signal S changes from $\perp$ to $+$ as soon as an "`emit` S" statement is executed
  - Status of a signal S changes from $\perp$ to $-$ as soon as all the "`emit` S" statements have been found unreachable by the cannot false path analysis

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- Sequential threads of control forked by parallel statements
- When a thread reaches a "`present` S" statement:
  - As long as the status of S is $\perp$:
    - Control remains there, frozen,
  - As soon as S has a non-$\perp$ status:
    - Control can resume
- If several threads are enabled, any one of them can be chosen

Summary of Constructive Interpretation

Control:

- Threads are stopped by termination or by executing `pause` or `exit` statements
- Parallel statements synchronize stopped threads, as explained in the intuitive semantics
- Finally, the false path analysis explores all possible instantaneous paths towards emit statements
  - Takes into account all facts established so far
  - No speculative reasoning

Summary of Constructive Interpretation

Program Acceptance:

- Given an input, a program is accepted if the analysis succeeds in setting each signal status to a defined value $+$ or $-$
- Logical correctness is guaranteed for accepted programs

To Go Further

- Albert Benveniste, Paul Caspi, Stephen A. Edwards, Nicolas Halbwachs, Paul Le Guernic, Robert De Simone, The synchronous languages 12 years later, *Proceedings of the IEEE*, Jan. 2003 vol. 91, issue 1, pages 64–83, `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.96.1117`