Synchronous Languages—Lecture 05

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Esterel III—The Logical Semantic

The 5-Minute Review Session

1. How do concurrent threads in Esterel communicate?
2. What is the difference between weak and strong abortion?
3. What is the difference between aborts and traps?
4. What is syntactic sugar, and what is it good for?
5. What is the multiform notion of time?

Overview

Logical Correctness

Causality issues
The logical coherence law
Logical reactivity and determinism
Instantaneous Feedback

The Logical Behavioral Semantics

Causality Problems

- It’s easy to write contradictory programs
- Unfortunate side-effect of instantaneous communication coupled with the single valued signal rule
- These sorts of programs are erroneous and flagged by the Esterel compiler as incorrect
- Note: the first and third example are considered valid in SCEst, see later . . .
Causality Problems

[abort
  emit A
  when immediate B
] !
[
  present A
  then emit B
end;
]

Can be very complicated because of instantaneous communication

Causality Example

emit A;
  present B then emit C end;
  present A else emit B end;

Red statements reachable

Analysis done by original compiler:
- After emit A runs, there's a static path to emit B
- Therefore, the value of B cannot be decided yet
- Execution procedure deadlocks: Program is bad

Causality

- Definition has evolved since first version of the language
- Original compiler had concept of “potentials”
  - Static concept: at a particular program point, which signals could be emitted along any path from that point
- Current definition based on “constructive causality”
  - Dynamic concept: whether there's a “guess-free proof” that concludes a signal is absent

Causality Example

emit A;
  present B then emit C end;
  present A else emit B end;

Red statements reachable

Analysis done by later compilers:
- After emit A runs, it is clear that B cannot be emitted because A's presence runs the "then" branch of the second present
- B declared absent, both present statements run
- Program is OK
Logical Correctness

- The intuitive semantics:
  - Specifies what should happen when executing a program
- However, also want to guarantee that
  - Execution actually exists (at least one possible execution)
  - Execution is unique (at most one possible execution)
- Need extra criteria for this!
- The apparently simplest possible criterion: logical correctness

Recall:
- Signal $S$ is absent by default
- Signal $S$ is present if an $\text{emit } S$ statement is executed

The Logical Coherence Law:

A signal $S$ is present in a tick if and only if an $\text{emit } S$ statement is executed in this tick.

Logical Correctness requires:
- There exists exactly one status for each signal that respects the coherence law

Given:
- Program $P$ and input event $I$
  - $P$ is logically reactive w. r. t. $I$:
    - There is at least one logically coherent global status
  - $P$ is logically deterministic w. r. t. $I$:
    - There is at most one logically coherent global status
  - $P$ is logically correct w. r. t. $I$:
    - $P$ is both logically reactive and deterministic
  - $P$ is logically correct:
    - $P$ is logically correct w. r. t. all possible input events

Is logical correctness decidable?
- Yes!

Pure Esterel programs can be analyzed for logical correctness by performing exhaustive case analysis.
- Given the status of each input signal, one can make all possible assumptions about the global status and check them individually.
- Therefore, logical correctness is decidable.
- We here generally consider just a single reaction. However, in general one also has to consider all possible sequences of reactions and all possible program states. As there is a finite number of program states, this is still decidable.
Logical Correctness

module P1:
input I;
output O;
signal S1, S2 in

| | present I then emit S1 end
| | present S1 else emit S2 end
| | present S2 then emit O end
end signal
end module

Is P1 logically correct?
▶ Yes!

Is P2 logically correct?
▶ Yes!

module P2:
signal S in
emit S;
present O then
| | present S then
| | pause
| | present S else emit S end
| | emit O end
end signal

module P3:
present O else emit O end
end module

Is P3 logically correct?
▶ No!
▶ This is non-reactive

module P4:
present O emit O end
end module

Is P4 logically correct?
▶ No!
▶ This is nondeterministic

module P5:
present O1 then emit O2 end
| | present O2 else emit O1 end

Is P5 logically correct?
▶ No!
▶ This is non-reactive

To make examples shorter, we omit input-output declarations from now on
▶ Inputs will be written I, I1, etc., and outputs will be written O, O1, etc.

▶ Notice that P2 is inputless
▶ Inputless programs react on empty input events, i.e., on clock ticks
Logical Correctness

module P6:
present O1 then emit O2 end
||
present O2 then emit O1 end

Is P6 logically correct?
▶ No!
▶ This is nondeterministic

module P7:
present O then pause end;
emit O

Is P7 logically correct?
▶ No!
▶ This is non-reactive

Logical Correctness

module P8:
trap T in

present I else pause end;
emit O
||
present O then exit T end
end trap;
emit O

Is this logically correct?
▶ Yes for I present
▶ Nondeterministic for I absent

Instantaneous Feedback

▶ Want to reject logically incorrect programs at compile time
▶ One option:
  ▶ Forbid static self-dependency of signals
  ▶ Similar to acyclicity requirement for electrical circuits
  ▶ This is what the Esterel v4 compiler did

module P3:
present O else emit O end
end module
≡ O = not O

module P4:
present O emit O end
end module
≡ O = O
Logical Correctness—Assessment

- We now introduced logical correctness
- But do we want to use it as basis for the language?
  - sound
  - sometimes unintuitive (consider P9)
  - computationally complex
- Alternative 1: allow only programs that are acyclic
  - simple
  - too restrictive (consider GoodCycle1/2)
- Alternative 2: accept everything for which the compiler finds a static execution schedule
  - relatively simple for the compiler
  - definition not precise, depends on abilities of compiler (different compilers accept different programs)
- Alternative 3: the constructive semantics
  - analysis not trivial
  - clear semantics

Overview

- Logical Correctness
  - The Logical Behavioral Semantics
    - Notation and Definitions
    - The Basic Broadcasting Calculus
    - Transition Rules
    - Reactivity and Determinism
The Semantics of Esterel

1. **Logical Behavioral Semantics**
   - Rewriting rules defining reactivity, determinism, and logical correctness
   - Signal coherence law embedded in rules for local signals
2. **Constructive Behavioral Semantics**
   - Refines logical behavioral semantics
   - Based on must and cannot analysis
3. **Logical/Constructive State Behavioral Semantics**
   - Replaces rewriting with marking of active delays (v5 debugger)
4. **Constructive State Operational Semantics**
   - Defines reaction as sequence of microsteps (v3 compiler)
5. **Constructive Circuit Semantics**
   - Translates Esterel programs into Boolean digital circuits (v5 compiler)

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**Notation and Definitions**

- **Sort** $S$: A set of signals
- **Signal statuses**: $B = \{+,-\}$
- **Event** $E$:
  - Given sort $S$, defines status $E(s) \in B$ for each $s \in S$
  - Obtain sort of $E$ as $S(E) = S$
- **Two equivalent representations for $E$**:
  - As subset of $S$: $E = \{s \in S \mid E(s) = +\}$
  - As a mapping from $S$ to $B$: $E = \{(s, b) \mid b = E(s)\}$

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- The logical behavioral semantics accepts more programs than we would like (for example, program P9 presented in Lecture 03)
- However, the logical behavioral semantics is important in that all other semantics should be a refinement of it, and it is also a natural starting point
- The constructive semantics are equivalent; the constructive behavioral semantics is the most intuitive, and can be derived fairly directly from the logical behavioral semantics, so we will focus on these two semantics here
- Note that the terminology (and categorization) used in different references (and sometimes within the same reference—e.g., in Berry’s draft book) is a bit in flux; the keywords to look out for to distinguish which is which are “logical” vs. “constructive”, “state” and “behavioral” vs. “operational”
- In this class, will focus on semantics 1, 2, and 5

- Allowing to represent events in alternate ways somewhat simplifies the subsequent presentation of the rewriting rules
Notation and Definitions

- Write $s^+ \in E$ iff $E(s) = +$
- Write $s^- \in E$ iff $E(s) = -$
- Write $E' \subseteq E$ iff $\forall s \in S(E'): s^+ \in E' \implies s^+ \in E$
- Given signal $s$, define singleton event \{\(s^+\)}:
  - $\{s^+\}(s) = +$
  - $\forall s' \neq s: \{s^+\}(s') = -$\n- Given signal set $S$ and signal $s \in S$, write $S \setminus s = S - \{s\}$
- Given $E$ and $s \in S(E)$, write $E \setminus s$ to denote event of sort $S(E) \setminus s$, which coincides with $E$ on all signals but $s$
- Define $E \ast s^b$ as event $E'$ of sort $S(E) \cup \{s\}$ with
  - $E'(s) = b$, $E'(s') = E(s')$ for $s' \neq s$

Note that in the definition of $E \ast s^b$, $s$ may or may not be in $S(E)$; in the former case, the status of $s$ in $E$ is lost in $E \ast s^b$

Notation and Definitions

- Will present formal semantics as Plotkin's Structural Operational Semantics (SOS) inference rules
- Behavioral Semantics formalizes reaction of program $P$ as behavioral transition
  \[
  \begin{array}{c}
  P \\
  \xrightarrow{O} \\
  P'
  \end{array}
  \]

- $I$: input event
- $O$: output event
- $P'$: derivative of $P$—the program for the next instance
Notation and Definitions

- Auxiliary statement transition relation:

\[
\begin{align*}
E' & \xrightarrow{E,k} p' \\
p & \xrightarrow{E} p
\end{align*}
\]

- \( p \): program body (of \( P \))
- \( E \): event defining status of all signals declared in scope of \( p \)
- \( E' \): event composed of all signals emitted by \( p \) in the reaction
- \( k \): completion code returned by \( p \) (0 iff \( p \) terminates)
- \( p' \): derivative of \( p \)
- Logical coherence (or broadcasting invariant):

\[
E' \subset E
\]

Here, we consider an Esterel program to consist of an input/output signal interface and an executable body

Note that the event \( E \) is an assumption in the sense of the logical semantics

Notation and Definitions

- Given:
  - Program \( P \) with body \( p \)
  - Input event \( I \)
- Define program transition of \( P \) by statement transition of \( p \):

\[
\begin{align*}
P & \xrightarrow{O,I} P' \text{ iff } p \xrightarrow{O,k} p' \text{ for some } k \\
P & \xrightarrow{O,I} P'
\end{align*}
\]

- These program transitions, yielding an output reaction \( O \) and a derivative \( P' \), determine the logical behavioral semantics of \( P \)

Note how the definition of the program transition reflects the logical coherence
The Basic Broadcasting Calculus

▸ For concise presentation of rules: Replace Esterel syntax with terser process-calculus syntax
▸ Use parenthesis for grouping statements

```
nothing                      0
pause                        1
emit s                       s
present s then p else q end  s ? p, q
p.q                          p, q
loop p end                   p
p || q                       p, q
signal s in p end            p \ s
suspend p when s end         s \ p
trap T in p end              {p}
exit T k with k ≥ 2          ↑p
[no concrete syntax]
```

Example

```
pause;
emit O1;
loop
  pause;
  emit O2;
  [present I1 then emit O2 end present ]
  | present I3 else emit O3 end present 
end loop
≡
1; !O1; (1; ((I1 ? !O2, 0) | (I3 ? 0, !O3)))*
```

Recall: trap T in p end
▸ Defines a lexically scoped exit point T for p
▸ Immediately starts its body p and behaves as p until termination or exit
▸ If p terminates, so does the trap statement
▸ If p exits T, then the trap statement terminates instantaneously
▸ If p exits an enclosing trap U, this exit is propagated by the trap statement
▸ Is part of pure Esterel
Basic Transition Rules

The null process 0:

\[ \begin{array}{c}
0 & \overset{E}{\rightarrow} & 0 \\
\end{array} \]  
(null)

The unit delay process 1:

\[ \begin{array}{c}
1 & \overset{E}{\rightarrow} & 0 \\
\end{array} \]  
(unit delay)

Signal emission:

\[ \begin{array}{c}
!s \{s\}, 0 & \overset{E}{\rightarrow} & 0 \\
\end{array} \]  
(emit)

Deduction Rules

- In addition to simple transition rules, will also use deduction rules

- Hypothesis: If sub-instructions behave like this...

\[\begin{array}{c}
p_1 \overset{E_1,k_1}{\rightarrow} p'_1 \\
p_2 \overset{E_2,k_2}{\rightarrow} p'_2 \\
\text{Other hypotheses}
\end{array}\]

Instruction\( (p_1, p_2) \overset{E(\{E_1,E_2\})}{\rightarrow} K(k_1,k_2) \overset{E}{\rightarrow} \text{Instruction}'(p'_1, p'_2) \)

- Conclusion: ...then the compound instruction behaves like that

- The null process 0 terminates instantaneously and rewrites into itself

- The unit delay process 1 waits in the current reaction and rewrites itself into 0 for the next reaction
Deduction Rules—Sequencing

\[
\begin{align*}
\frac{\epsilon \quad \epsilon' \quad k \neq 0}{\epsilon'} & \quad \text{(seq1)} \\
\frac{\epsilon \rightarrow \epsilon'}{\epsilon \rightarrow \epsilon'} & \quad \text{(loop)}
\end{align*}
\]

\[
\begin{align*}
\frac{\epsilon_1 \rightarrow \epsilon_2 \quad \epsilon_2 \rightarrow \epsilon'}{\epsilon_1 \rightarrow \epsilon'} & \quad \text{(seq2)}
\end{align*}
\]

► If the first component of a sequence waits, the sequence also waits
   ► For reasons that will become clear later, write waiting as \( k \neq 0 \) instead of \( k = 1 \)

► If the first component of a sequence terminates, the second is started (in zero delay), in the same environment \( E \), and the emitted signals are merged
   ► Using same \( E \) for both premises implements forward broadcasting from \( p \) to \( q \), as broadcasting invariant of first premise implies \( E_p \subset E \)
   ► However, with the same reasoning we have backward broadcasting from \( q \) to \( p \), conflicting with our requirement for causality—will rule this out later

Deduction Rules—Looping and Parallel

\[
\begin{align*}
\frac{\epsilon \rightarrow \epsilon'}{\epsilon \rightarrow \epsilon'} & \quad \text{(seq2)}
\end{align*}
\]

► Note how the global broadcasting invariant expresses that signals are broadcast between parallel branches: \( E_p' \cup E_q' \subset E \) holds iff both \( E_p' \subset E \) and \( E_q' \subset E \) hold
   ► Note that parallel constructs where all threads have terminated get cleaned up by the (seq2) rule or (trap1)
Deduction Rules—Conditional

\[
\frac{s^+ \in E \quad p}{s?p, q} \quad \text{(present +)}
\]

\[
\frac{s^- \in E \quad q}{s?p, q} \quad \text{(present -)}
\]

Zero delay: can use decision trees to test for arbitrary Boolean conditions:

- \((s_1 \land s_2)?p, q\) is \(s_1? (s_2?q, p), q\)
- \((s_1 \lor s_2)?p, q\) is \(s_1?p, (s_2?p, q)\)
- \(\neg s?p, q\) is \(s?q, p\)

Example: loop emit S; pause; emit T end.

In the process calculus: \((!S; 1; !T)^*\)

Calculating initial reaction, as a derivative tree (\textit{Ableitungsbaum}):

\[
\begin{array}{c}
\text{seq1} \\
\text{seq2} \\
\text{loop}
\end{array}
\]

- \((!S; 1; !T)^* \xrightarrow{\text{seq1}} 0; !T; (!S; 1; !T)^*\)
- \((!S; 0) \xrightarrow{\text{seq2}} 0; 0; (S; 0) \xrightarrow{\text{seq1}} 0; 0; (S; 0) \xrightarrow{\text{loop}} 0; !T; (!S; 1; !T)^*\)

See next note for an alternative notation.

Similarly, for next reaction (and all following):

\[0; !T; (!S; 1; !T)^* \xrightarrow{S,T} 0; !T; (!S; 1; !T)^*\]

Deduction Rules—Restriction

\[
\frac{p}{E^s?k} \quad p' S(E') = S(E)\setminus s
\]

\[
\frac{p \setminus s}{E^s?k} \quad p' S(E') = S(E)\setminus s
\]

Note: This also properly handles nested restrictions of the same signal.
Traps—Example

The trap exit encoding is

- $k = 2$ if the closest enclosing trap is exited, and
- $k = n + 2$ if $n$ trap declarations have to be traversed

\[
\text{trap } U \text{ in trap } T \text{ in nothing } \equiv \{0 \mid 1 \mid 2 \mid 3 \} 2
\]

The Shift Operator

- ↑ ("shift") shifts exit numbers of $p$ by 1 when placing $p$ in a trap block
- May use ↑ in definitions of derived operators

Two Operators on Completion Codes

- The ↓$k$ operator computes completion code of $\{p\}$ from that of $p$:
  - $\downarrow k = 0$ if $k = 0$ or $k = 2$
  - $\downarrow k = 1$ if $k = 1$
  - $\downarrow k = k - 1$ if $k > 2$

- The ↑$k$ operator computes completion code of ↑$p$ from that of $p$; want $\{\uparrow p\} \equiv p$:
  - $\uparrow k = k$ if $k = 0$ or $k = 1$
  - $\uparrow k = k + 1$ if $k > 1$
Traps—The Rules

\[ \emptyset \xrightarrow{k \in \mathcal{E}} 0 \]  
(exit)

\[ p \xrightarrow{E'} \xrightarrow{k \in \mathcal{E}} p' \quad k = 0 \text{ or } k = 2 \]  
(trap1)

\[ s \supset p \xrightarrow{E'} \xrightarrow{k \in \mathcal{E}} 0 \]  
(suspend1)

\[ p \xrightarrow{E'} \xrightarrow{k \in \mathcal{E}} p' \quad k = 1 \text{ or } k > 2 \]  
(trap2)

\[ s \supset p \xrightarrow{E'} \xrightarrow{k \in \mathcal{E}} s \supset p' \]  
(suspend2)

\[ \uparrow p \xrightarrow{E' \uparrow k} \uparrow p' \]  
(shift)

Note: It might be a bit surprising that in (trap2), the braces (trap scope) remain in the program derivative when an internal exception is propagated up. However, this works fine: the \( \downarrow k \) operator keeps lowering the trap completion code, and as soon as we reach the trap scope corresponding to the exception, everything reduces to nothing. See for example \{ \{ \text{s?!s}; 3; \text{s?2} \}; \text{s?3} \}:

\[ \{ \{ \text{s?1}; 3; \text{s?2} \}; \text{s?3} \}\]

\[ \{ \{ \text{s?1}; 3; \text{s?2} \}; \text{s?3} \} \]

Definition: Program \( P \) is logically reactive (resp. logically deterministic) w.r.t. an input event \( I \) if there exists at least (resp. at most) one program transition \( P \xrightarrow{O} P' \) for some output event \( O \) and program derivative \( P' \).  

Definition: Program \( P \) is logically correct if it is logically reactive and logically deterministic.

How about \( (s?!s, 0) \)?  
And how about \( (s?0, !s) \)?
Reactivity and Determinism

- I/O determinism still leaves room for internal non-determinism
  - Consider $(s?1.s.0 \teil s)$
  - Forbidden in constructive semantics
- Definition: Program $P$ is strongly deterministic for an input event $I$ iff
  - $P$ is reactive and deterministic for this event, and
  - there exists a unique proof of the unique transition $P \xrightarrow{O} P'$.

Summary (2/3)

- $P$ is logically reactive w. r. t. input $I$ if there is at least one logically coherent global status
- $P$ is logically deterministic w. r. t. $I$ if there is at most one logically coherent global status
- $P$ is logically correct w. r. t. $I$ if $P$ is both logically reactive and deterministic
- $P$ is logically correct if $P$ is logically correct w. r. t. all possible input events

Summary (1/3)

- The intuitive semantics specifies what should happen when executing a program
- However, also want to guarantee that exactly one possible execution exists that satisfies the intuitive semantics
- The Logical Coherence Law specifies that a signal $S$ is present in a tick if and only if an "emit $S$" statement is executed in this tick
- Logical Correctness requires that there exists exactly one status for each signal that respects the coherence law

Summary (3/3)

- There exist several semantics for the Esterel language—one important distinction is between logical and constructive semantics, the latter being a refinement of the former
- We started discussing the logical behavioral semantics, expressed in Plotkin's Structural Operational Semantics, with basic transition rules and deduction rules
- We formally defined reactivity, determinism, logical correctness, and strong determinism
To Go Further


- Gérard Berry, Preemption in Concurrent Systems, In Proceedings FSTTCS 93, Lecture Notes in Computer Science 761, pages 72-93, Springer-Verlag, 1993,
  http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.42.1557