# ALF User's Manual

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## 1 Introduction

ALF (Algebraic Logic Functional programming language) is a language which combines functional and logic programming techniques. The foundation of ALF is Horn clause logic with equality which consists of predicates and Horn clauses for logic programming, and functions and equations for functional programming. Since ALF is a genuine integration of both programming paradigms, any functional expression can be used in a goal literal and arbitrary predicates can occur in conditions of equations. The operational semantics of ALF is based on the resolution rule to solve literals and narrowing to evaluate functional expressions. In order to reduce the number of possible narrowing steps, a leftmost-innermost basic narrowing strategy is used which can be efficiently implemented. Furthermore, terms are simplified by rewriting before a narrowing step is applied and also equations are rejected if the two sides have different constructors at the top. Rewriting and rejection can result in a large reduction of the search tree. Therefore this operational semantics is more efficient than Prolog's resolution strategy.

The ALF system is an efficient implementation of the combination of resolution, narrowing, rewriting and rejection. Similarly to Prolog, ALF uses a backtracking strategy corresponding to a depth-first search in the derivation tree. ALF programs are compiled into instructions of an abstract machine. The abstract machine is based on the Warren Abstract Machine (WAM) with several extensions to implement narrowing and rewriting. In the current implementation programs of this abstract machine are executed by an emulator written in C.

ALF has also a type and module concept which allows the definition of generic modules. A preprocessor checks the type consistence of the program and combines all needed modules into one flat-ALF program which is compiled into a compact bytecode representing an abstract machine program. The current implementation has the following properties:

- The machine code for pure logic programs without defined functions is identical to the code of the original WAM [War83], i.e., for logic programs there is no overhead because of the functional part of the language.
- Functional programs where only ground terms have to be evaluated are executed by deterministic rewriting without any dynamic search for subterms positions where the next rewriting step can be applied. The compiler computes these positions and generates particular machine instructions. Therefore such programs are also efficiently executed.
- In mixed functional and logic programs argument terms are simplified by rewriting before narrowing is applied and therefore function calls with ground arguments are automatically evaluated by rewriting and not by narrowing. This is more efficient because rewriting is a deterministic process. Hence in most practical cases the combined rewriting/narrowing implementation is more efficient than an implementation of narrowing by flattening terms and applying SLD-resolution [Han91, Han92].

This manual describes the syntax and semantics of ALF. Moreover, it contains a detailed description of the current implementation of the ALF system from a user's point of view. Details about the implementation technique (i.e., the abstract machine and compilation of ALF programs into this machine) can be found in [Han90, Han91]. The current system is available from the first author.

## 2 An introductory example

In this section we present an example of an ALF program to show the basic ideas of the language. More details about the language and a description of the basic modules can be found in subsequent sections.

An ALF program is a collection of modules including one main module which contains a sequence of goals to be proved. In the following we want to define a function which sorts a list of natural numbers by the insertion sort algorithm. In the beginning we must define the elementary data types. Therefore our first basic module contains the definition of natural numbers. Following the spirit of algebraic specifications, natural numbers are constructed by the constant 0 and a successor function. Moreover, this module defines an addition function on natural numbers and a predicate to compare natural numbers:

```
module nats.
    export 0, s, +, < .
    datatype nat = { 0 ; s(nat) }.
    func + : nat, nat -> nat infixleft 500.
    pred < : nat, nat infix.
rules.
    N + 0 = N.
    N + s(M) = s(N + M).
    0 < s(M).
    s(N) < s(M) :- N < M.
</pre>
```

end nats.

The addition function + is defined by two equations (in the style of algebraic specifications or functional languages with pattern-matching) and the comparison predicate < is defined by two Horn clauses as in logic languages. For readability the function + and the predicate < are defined as infix operators (keywords infixleft and infix) which allows an infix notation in the rules. Since functions can be used in nested expressions, the infix operator is defined to be left-associative (infixleft) with a precedence of 500 (see also Section 3.3).

Next we define our second elementary data type "lists over elements of an arbitrary type". In this module we use the Prolog notation for lists [CM87] which is a predefined feature of ALF:

```
module list(elem).
```

```
export [] , '.' , append , member.
datatype list = { '.'(elem,list) ; [] }.
func append: list, list -> list.
pred member : elem, list.
rules.
```

```
append([],L) = L.
append([H|T],L) = [H|append(T,L)].
```

```
member(E, [E|_]).
member(E, [_|L]) :- member(E, L).
```

end list.

The type of the list elements is a parameter to this module, i.e., list is a generic module. It exports the list contructors [] and '.', a function to concatenate two lists and a member predicate on lists.

Now it would be possible to define our sort function on lists but for convenience we define a further module which extends the module **nats** by defining some additional predicates to compare natural numbers:

module natord.

```
export 0 , s , + , < , > , =/= , =< , >= .
   use nats.
   pred >
               : nat, nat infix;
        =/=
              : nat, nat infix;
              : nat, nat infix;
        =<
               : nat, nat infix.
        >=
rules.
   N > M
           :-M < N.
   N = /= M :- N < M.
   N = /= M :- N > M.
   N = < N.
   N = < M : - N < M.
   N \ge N.
   N \ge M :- N \ge M.
```

#### end natord.

This module imports the module **nats** by the declaration "**use nats**" and exports all objects defined in module **nats** and the additional predicates defined in this module. Now we declare our main module containing the definition of the sort function on lists of naturals:

```
module isort.
```

```
export isort.
use natord;
    list(nat).
func isort : list -> list;
    insert: nat, list -> list.
rules.
isort([]) = [].
isort([E|L]) = insert(E,isort(L)).
insert(E,[]) = [E].
insert(E,[F|L]) = [E,F|L] :- E =< F.
insert(E,[F|L]) = [F|insert(E,L)] :- E > F.
```

end isort.

?- isort([3,1,5,4,1,3,2]) = L.

This module imports the modules **natord** and **list** where the generic list module is used with the actual sort parameter **nat** (imported from **natord**). The sort function **isort** is specified by two equations where the second equation uses the function **insert** to insert an element at the appropriate position in an ordered list. The definition of **insert** uses *conditional equations* which is a standard feature of ALF since ALF is a genuine integration of functional and logic languages. At the end of this main module the goal to be proved by the ALF system is appended. Note that natural numbers need not be written down as **s**-terms since ALF translates natural numbers like **3** into equivalent **s**-terms like **s**(**s**(**s**(**0**))).

If the main module and all imported modules are translated by the ALF system and the compiled program is executed, we get the answer

isort([3,1,5,4,1,3,2]) = [1,1,2,3,3,4,5]

which is a logical consequence of our program (see Section 4 for the operational semantics of ALF).

## 3 Syntax of ALF

This section defines the syntax of the functional-logic language ALF in two parts. Firstly, the context-free component of the syntax is defined by a grammar in EBNF style. In the second part various restrictions on the syntax are introduced in order to obtain well-formed programs satisfying the theoretical conditions for the completeness of the operational semantics (cf. Section 4).

#### 3.1 Context free grammar

The syntax of ALF is similar to the syntax of Prolog [CM87] with additional constructions for type and module declarations. We use the following notations in the grammar rules: Terminals are character sequences enclosed in apostrophes,  $\{\cdots\}$  denotes a (possibly empty) repetition,  $[\cdots]$  denotes an optional part, and  $\cdots \mid \cdots$  marks choices.

```
\begin{array}{l} Program \longrightarrow \{Module\} \ Module \{Goal\} \\ Goal \longrightarrow `?-' \ LiteralList `.' \\ \\ Module \longrightarrow \\ ``module' \ ModuleName [`(' Parameter {`,' Parameter} `)'] `.' \\ \{ModuleInterface \mid ModuleDeclaration\} \\ [`rules.' \\ ModuleBody] \\ ``end' \ ModuleName `.' \\ \\ \\ ModuleInterface \longrightarrow \\ ``export' \ SymbolList `.' | \\ ``use' \ UseDeclaration {`;' UseDeclaration} `.' \\ \\ \\ \\ ModuleDeclaration \longrightarrow \end{array}
```

'datatype' DatatypeDeclaration {';' DatatypeDeclaration} '.' |

'func' FunctionDeclaration {';' FunctionDeclaration} '.' |
'pred' PredicateDeclaration {';' PredicateDeclaration} '.'

 $UseDeclaration \longrightarrow$ 

ModuleName ['(' Name {', ' Name } ')' ['=' ModuleName]] ['with' Renamings]

Renamings  $\longrightarrow$  Atom 'for' Atom {', 'Atom 'for' Atom}

DatatypeDeclaration → Ident | Ident '=' '{' ConstructorDeclaration {';' ConstructorDeclaration} '}'

- ConstructorDeclaration  $\longrightarrow$  FuncAtom ['(' SortList ')'] [OpType Precedence]
- FunctionDeclaration  $\longrightarrow$  FuncAtom ': [SortList '->'] Sort [OpType Precedence] ['partial']
- $PredicateDeclaration \longrightarrow PredAtom `:` [SortList] [PredOpType]$
- $OpType \longrightarrow$  'prefix' | 'postfix' | 'infixnot' | 'infixleft' | 'infixright'
- $PredOpType \longrightarrow$  'prefix' | 'postfix' | 'infix'
- $ModuleBody \longrightarrow \{FunctionRule \mid PredicateRule\}$
- FunctionRule  $\longrightarrow$  FunctionTerm '=' Term [':-' LiteralList] ['onlyrewrite' | 'onlynarrow'] '.'

 $PredicateRule \longrightarrow Literal [`:-' LiteralList] `.'$ 

LiteralList  $\longrightarrow$  EquationLiteral {', 'EquationLiteral}

 $EquationLiteral \longrightarrow Literal \mid Equation$ 

Literal  $\longrightarrow$  PredName '(' TermList ')' | [Term] PredAtom [Term]

Equation  $\longrightarrow$  Term '=' Term

 $TermList \longrightarrow Term \{`, `Term\}$ 

Term  $\longrightarrow$  Number | Variable | String | FunctionTerm | '(' Term ')'

FunctionTerm → FuncName '(' TermList ')' | [Term] FuncAtom [Term] | '(' FunctionTerm ')'

Variable  $\longrightarrow$  Uppercase {Letter | Digit | '\_' } | '\_' {Letter | Digit | '\_' }

Parameter → Ident | '(' 'func' FunctionDeclaration ')' | '(' 'pred' PredicateDeclaration ')' Name → [ModuleName '.'] Atom ModuleName → Ident | ''' PathName '/' Ident ''' PathName → {Character}  $PredName \longrightarrow Name$  $PredAtom \longrightarrow Atom$  $FuncName \longrightarrow Name$  $FuncAtom \longrightarrow Atom$ SortList  $\longrightarrow$  Sort {', 'Sort} Sort  $\longrightarrow$  [ModuleName '.'] Ident SymbolList  $\longrightarrow$  Symbol {', 'Symbol} Symbol  $\longrightarrow$  FuncAtom ['/' Number] | PredAtom ['/' Number]  $Precedence \longrightarrow Number$ String  $\longrightarrow$  "'{Character}'"' Atom  $\longrightarrow$  Ident | SpecialCharacters | ','{Character}',' Ident  $\longrightarrow$  Lowercase {Letter | Digit | '\_' }  $Character \longrightarrow \langle ASCII-Character \rangle$ SpecialCharacters  $\longrightarrow$  SpecialCharacter {SpecialCharacter} | `~' | `~' Number  $\longrightarrow$  Digit {Digit}  $Digit \longrightarrow 0' | 1' | 2' | 3' | 4' | 5' | 6' | 7' | 8' | 9'$ Letter  $\longrightarrow$  Uppercase | Lowercase Uppercase  $\longrightarrow$  'A' | 'B' | 'C' | ... | 'Z' Lowercase  $\longrightarrow$  'a' | 'b' | 'c' | ... | 'z'

#### 3.2 Syntactic restrictions

Syntactically correct ALF programs must satisfy the following conditions:

- Inside quoted atoms all ASCII characters are allowed. To include the quote itself, it has to be doubled, e.g., 'that''s a quoted atom'.
- White spaces, i.e., spaces, tabs and line feeds, are allowed at arbitrary positions in the program except
  - inside non-quoted atoms, numbers and variables
  - between the functor and the open parenthesis
  - after the period if it is not used as end-of-term period, where it *must* be followed by a white space

- Comments are marked with the character %. All text between % and the end of the line is ignored. Another way to mark a comment is to enclose it with /\* and \*/. Inside atoms these characters are not interpreted as comment characters.
- The following restrictions must hold for any ALF program and are checked by the ALF system:
  - The names after module at the beginning of a module and after end at the end must be the same.
  - In an application of a function, a predicate, or a constructor to terms the arity and the types of the terms must coincide with the declaration (cf. Section 3.4).
  - The identifiers in an export list must be distinct from each other and declared or imported in this module.
  - Every sort, constructor, function and predicate must be declared before its use.
- The keywords and symbols in Table 1 are reserved and should not be used to define new operators.

datatype	end	export	for	func
infix	infixleft	infixnot	infixright	module
onlynarrow	onlyrewrite	partial	postfix	pred
prefix	rules	use	with	,
;	=	:	->	/
•	:-	?-		

Table 1: Reserved words and symbols

#### 3.3 Lists and operators

In ALF lists are represented by the functor '.' of arity 2 where the first argument is the first element of the list and the second argument is the rest of the list. The empty list is represented by the constant []. Since lists are commonly used objects, there is an alternative notation which is shown in Table 2.

Notation	equivalent to
$[elem \mid list]$	.(elem, list)
[elem]	.(elem, [])
$[a_1, a_2, \dots, a_n]$	$(a_1, (a_2, \ldots, (a_n, [])))$
$[a_1, a_2, \ldots, a_n \mid list]$	$(a_1, (a_2, \ldots, (a_n, list)))$

Table 2: List notations in ALF

As in Prolog [CM87] it is also possible to define an operator notation for constructors, functions and predicates. The keywords prefix and postfix declare a name as a prefix and a postfix operator, respectively (in Prolog: type fy and yf), and the keywords infixleft, infixright and infixnot declare the name as a (left-associative, right-associative, non-associative) infix operator (in Prolog: type yfx, xfy and xfx). The number after the operator type is the precedence of the operator which must be number from 1 to 1200. Binary predicates can only be of type infixnot (which is the meaning of infix) and have the same precedence as the equality symbol =. The operator definitions are significant while reading the rules of a module. The operators which are predefined in the rule part of a module are summarized in Table 3

Operator name	Type	Precedence
onlyrewrite	postfix	1200
onlynarrow	postfix	1200
:-	infixnot	1199
?-	prefix	1199
end	prefix	1150
,	infixright	1000
=	infixnot	700
	infixnot	400

Table 3: Predefined operators in ALF clauses

#### 3.4 The type system

The functional-logic language ALF has a type system which is comparable to *many-sorted* equational logic [Pad88]. Therefore the programmer must declare the data type together with their constructors and the argument and result types of each function and predicate occurring in the program. A data type is declared with its constructors by a **datatype** declaration of the form

datatype  $s = \{ c_1(s_{11}, \ldots, s_{1n_1}), \ldots, c_k(s_{k1}, \ldots, s_{kn_k}) \}.$ 

Such a declaration defines a new data type s (called the *sort*) and *constructors*  $c_1, \ldots, c_k$  where the *i*-th constructor  $c_i$  has s as the result type and terms of type  $s_{i1}, \ldots, s_{in_i}$  as arguments. All constructors of the same result type must be declared by one **datatype** declaration.

In ALF each object must be declared before its use. In order to allow the definition of mutually recursive data types, it is possible to declare a sort without its constructors, e.g.,

```
datatype s.
```

declare the sort s without its constructors. Now it is possible to use s in subsequent data type declarations and then to define the constructors of s. For instance, the following declarations define two mutually recursive data types s1 and s2:

```
datatype s1.
datatype s2 = { a ; c(s1) }.
datatype s1 = { b ; d(s2) }.
```

The types of the variables used in the rules need not be declared but are inferred by the type checker. Overloading is allowed under the restriction that all overloading conflicts can be resolved by a top down pass through the terms. If an overloading conflict cannot be resolved, the ALF system prints a warning message. In this case the programmer must solve the conflict by specifying the function or predicate symbol together with the appropriate module name (if the reason for this conflict is the declaration of the symbol in two different modules) or by using a new name for one of the symbols. Qualification is done by prefixing the function or predicate name with the module name and a period: For instance, if the function f is defined in the modules m1 and m2 which are imported into the main module, we can refer to the function f of module m2 by the notation "m2.f".

The language ALF offers also a kind of *parametric polymorphism* by allowing parameterized modules. For instance, a module containing a definition of lists may be parameterized by the sort of the elements (see Section 5.2). It is also allowed to have functions or predicates as module parameters. E.g., a generic module for sorting lists of element have two parameters: the sort of the elements and a binary predicate to compare two elements of this sort (see also Section 5.4). But note that this concept of generic modules has nothing to do with parametric polymorphism: it is a purely syntactic feature, i.e., if a parameterized module is used in another module, the text of the parameterized module is copied with the actual parameters (however, ALF performs some optimizations that avoids text copying if it is unnecessary).

#### 3.5 The module concept

In order to implement large applications, ALF has a module concept to structure large programs. A module is a collection of data type, function and predicate definitions together with a list of clauses specifying the semantics of functions and predicates. If the module does not contain an export declaration, all objects defined in this module are exported, otherwise only those constructors, functions and predicates are exported which are listed in an export declaration. Sorts are implicitly exported if an argument or result type of an exported object contains these sorts.

A module can be imported into another module by a **use** declaration. In this case all exported objects of the imported module are visible inside the module. Note that if the module should export some of the imported objects, it is necessary to list these imported objects in the export declaration of the module, otherwise they are not visible outside this module. Example:

```
module m1.
    export f.
    datatype s = { ··· }.
    func f: s -> s;
        g: s -> s.
rules.
    % equations for f and g
end m1.
module m2.
    export f, p.
    use m1.
    pred p: s, s.
rules.
    % clauses for p
end m2.
```

Module m1 exports the function f and sort s, i.e., function g is invisible outside m1. Module m2 exports the imported function f, the imported sort s and the predicate p.

In order to have the complete specification of a function or predicate available in one module, all defining clauses of a function or predicate must be stored in one module. Hence it is not allowed to distribute the defining clauses of one function or predicate over several modules. Therefore the ALF system outputs an error message if a module contains a clause where the function or predicate in the left-hand side is a parameter (see below) or imported from other modules. For instance, it is not allowed to write a clause " $f(\cdots) = \cdots$ " inside module m2 in the example above.

If a module **m** imports two other modules **ma** and **mb**, it is possible that name conflicts occur because **ma** and **mb** exports objects with identical names. In order to avoid such name conflicts, ALF allows a renaming of objects when modules are imported. For instance, if both **ma** and **mb** exports a function **f**, then we can rename these functions in the import declaration:

```
module m.
...
use ma with fma for f.
use mb with fmb for f.
...
```

Now the function **f** of module **ma** is identified by the name **fma** in the body of module **m**. Another solution to avoid name conflicts is to qualify names by the dot notation (cf. Section 3.4): If the function **f** is not renamed in the **use** declaration, then we can refer by the notation "**ma.f**" to the function **f** of module **ma**.

ALF modules can be parameterized by sorts, functions and predicates (see Section 3.4). Examples for such generic modules can be found in Sections 5.2, 5.3 and 5.4. If a generic module is imported and instantiated with actual parameters and we want to refer to this instantiated module (e.g., to avoid name conflicts using the dot notation), it is necessary to have a unique identifier for this module. Therefore it is allowed to specify a new name for this instantiated module inside the **use** declaration. For instance, if we import the generic list module (Section 5.2) with the actual sort parameter **nat**, we can write

use list(nat) = natList.

and refer to the function append for lists of naturals by the notation "natList.append". An alternative solution is to rename the function append in the use declaration:

use list(nat) with natListAppend for append.

(it is also allowed to combine the renaming of a generic module with the renaming of some objects of this module). Note that this renaming is only necessary if the type checker of ALF cannot resolve name conflicts in subsequent clauses.

An important requirement to ALF modules is the *persistency condition*: all objects which are imported in a module (where parameters are also considered as imported objects) are not modified inside this module. In order to ensure this, it is not allowed to redefine imported sorts or to specify equations or clauses for imported functions or predicates inside the module. These conditions are checked by the ALF system.

Usually all modules are stored in one directory in the file system, but it is also possible to distribute the modules in several directories. For details see remarks at beginning of Section 7.2.

#### 3.6 Semantic restrictions

The following restrictions are necessary in order to ensure the completeness of the operational semantics (cf. Section 4) (however, the implemented operational semantics is incomplete because a depth-first search strategy is used instead of a breadth-first one). Some of these restrictions are checked by the ALF system, but some restrictions cannot be automatically checked and therefore

the programmer is responsible for these conditions. Fortunately, there are other tools for checking these conditions (completion procedures) and a connection of the ALF system with one of these tools is under development.

- The top symbol of the left-hand side of a (conditional) equation (program rule) has to be a function symbol and not a constructor (checked by the ALF system). Hence equations between constructors are not allowed. This ensures that two constructor terms are equal w.r.t. the defined equational theory if they are identical. This property is useful for an early detection of a failure if the two sides of a goal equation are not unifiable.
- The term rewriting relation generated by the equational rules must be canonical, i.e., confluent and terminating. Confluence is necessary for the completeness of the narrowing procedure and termination is required in order to perform the simplification of the goal before a narrowing step in finite time. Moreover, all conditional equations must be *reductive*, i.e., the conditions must be smaller than the left-hand side w.r.t. some termination ordering (otherwise basic conditional narrowing may be incomplete as Middeldorp and Hamoen [MH92] have pointed out).
- Occurrences of function symbols are not allowed in the head of a relational clause for a predicate, i.e., the argument terms should only contain variables and constructors (checked by the ALF system). This is necessary to ensure completeness of the combined resolution/narrowing procedure.
- No extra-variables are allowed in equational rules, i.e., all variables in the right-hand side of a defining equation and the condition must also occur in the left-hand side. Otherwise narrowing may be incomplete [GM86]. However, there are a lot of interesting programs with extra-variables where ALF's operational semantics is complete [Han91]. Therefore the ALF system allows extra-variables in conditions as well as in right-hand side. In this case the programmer must be aware of the potential incompleteness due to the extra-variables.
- A function must be reducible on all ground constructor terms if it is not declared as partial. If the function is declared as partial, then the innermost reflection rule (cf. Section 4) can be applied to this function. This is necessary because of ALF's innermost strategy: E.g., if the function pop is not reducible on the constructor empty, then the equation pop(empty) = X has the only solution X/pop(empty). This solution is only computable by an innermost strategy if the term pop(empty) is not evaluated but considered as an irreducible term. This is done by the innermost reflection rule which is only generated for partially defined functions. If the function is totally defined, i.e., reducible on all ground constructor terms, then the innermost reflection rule is unnecessary and therefore the function can be compiled in a more efficient way. Hence all functions are assumed to be totally defined (because this is the normal case). If the definition of a function is not complete for all constructor argument terms (which cannot be checked by the ALF system), then the user must declare it as partial. As a special case a constant function c must be declared as partial if c is an irreducible term.

## 4 Semantics

The *declarative semantics* of ALF is the well-known Horn clause logic with equality as to be found in [Pad88]. The *operational semantics* of ALF is based on resolution for predicates and rewriting

and innermost basic narrowing for functions. In order to give a precise definition of the operational semantics, we represent a goal by a skeleton and an environment part [Höl88]: the *skeleton* is a goal composed of terms and literals occurring in the original program, and the *environment* is a substitution which has to be applied to the goal in order to obtain the current goal. The initial goal G is represented by the pair  $\langle G; id \rangle$  where *id* is the identity substitution. We define the following inference rules to derive a new goal from a given one (if  $\pi$  is a position in a term t, then  $t/\pi$  denotes the subterm of t at position  $\pi$  and  $t[\pi \leftarrow s]$  denotes the term obtained by replacing the subterm  $t/\pi$  by s in t): Let  $\langle L_1, \ldots, L_n; \sigma \rangle$  be a given goal  $(L_1, \ldots, L_n$  are the skeleton literals and  $\sigma$  is the environment).

1. If  $L_1$  is an equation s = t and there is a mgu  $\sigma'$  for  $\sigma(s)$  and  $\sigma(t)$ , then the goal

$$< L_2, \ldots, L_n ; \sigma' \circ \sigma >$$

is derived by **reflection**.

2. If  $L_1$  is not an equation and there is a new variant  $L \leftarrow C$  of a program clause and  $\sigma'$  is a mgu for  $\sigma(L_1)$  and L, then the goal

$$< C, L_2, \ldots, L_n ; \sigma' \circ \sigma > 0$$

is derived by **resolution**.

- 3. Let  $\pi$  be a leftmost-innermost position in the first skeleton literal  $L_1$ , i.e., the subterm  $L_1/\pi$  has a defined function symbol at the top and all argument terms consist of variables and constructors (cf. [Fri85]).
  - (a) If there is a new variant  $l = r \leftarrow C$  of a program clause and  $\sigma(L_1/\pi)$  and l are unifiable with mgu  $\sigma'$ , then the goal

$$< C, L_1[\pi \leftarrow r], L_2, \dots, L_n ; \sigma' \circ \sigma >$$

is derived by innermost basic narrowing,

(b) If x is a new variable and  $\sigma'$  is the substitution  $\{x \leftarrow \sigma(L_1/\pi)\}$ , then the goal

$$< L_1[\pi \leftarrow x], L_2, \dots, L_n ; \sigma' \circ \sigma >$$

is derived by **innermost reflection** (this corresponds to the elimination of an innermost redex [Höl88]).

4. If  $\pi$  is a non-variable position in  $L_1$ ,  $l = r \leftarrow C$  is a new variant of a program clause and  $\sigma'$  is a substitution with  $\sigma(L_1/\pi) = \sigma'(l)$  and the goal  $\langle C; \sigma' \rangle$  can be derived to the empty goal without instantiating any variables from  $\sigma(L_1)$ , then the goal

$$< L_1[\pi \leftarrow \sigma'(r)], L_2, \ldots, L_n ; \sigma >$$

is derived by **rewriting** (thus rewriting is only applied to the first literal, but this is no restriction since a conjunction like  $L_1, L_2, L_3$  can also be written as an equation  $and(L_1, and(L_2, L_3)) = true).$ 

Start: Apply <i>rewriting</i> as long as possible (from innermost to outermost positions).			
If the goal is not <i>rejected</i> then:			
Narrow: If possible, apply the <i>innermost basic narrowing</i> rule and go to Start.			
If possible, apply the <i>innermost reflection</i> rule and goto Narrow.			
If the first literal of the goal is an equation			
then: If possible, apply the <i>reflection</i> rule and go to Start.			
else: If possible, apply the <i>resolution</i> rule and go to Start.			
Otherwise: fail (and try an alternative proof)			

5. If  $L_1$  is an equation and the two sides have different constructors at the same outermost position (a position not belonging to arguments of functions), then the whole goal is **rejected**, i.e., the proof fails.

Figure 1: Operational semantics of ALF

The complete operational semantics of ALF is shown in Figure 1. The innermost reflection rule must only be applied to *partial* functions, i.e., functions which are not reducible for all ground terms of appropriate sorts [Höl88]. The attribute *basic* of a narrowing step emphasizes that a narrowing step is only applied at an occurrence of the original program and not at occurrences introduced by substitutions [Hul80]. The restriction to basic occurrences is important for an efficient implementation of narrowing and rewriting (see below). The rewriting rule has the disadvantage that terms from the environment part can be moved to the skeleton part, but it has been shown that such terms can be safely moved back to the environment part [NRS87]. Therefore environment terms are never moved to the skeleton part in our implementation.

This operational semantics is sound and complete if the term rewriting relation generated by the conditional equations is canonical and the condition and the right-hand side of each conditional equation do not contain *extra-variables* [Höl88]. If these restrictions are not satisfied, it may be possible to transform the program into an equivalent program for which this operational semantics is complete. For instance, Bertling and Ganzinger [BG89] have proposed a method to transform conditional equations with extra-variables such that narrowing and reflection will be complete. Therefore we allow extra-variables in conditional equations. For instance, our operational semantics is complete for the following set of equations defining quicksort, which can be proved by the CEC completion system [BG89] (we omit the definition of =< and >):

```
conc([],L) = L.
conc([E|R],L) = [E|conc(R,L)].
split(E,[]) = ([],[]).
split(E,[F|L]) = ([F|L1],L2) :- E > F, split(E,L) = (L1,L2).
split(E,[F|L]) = (L1,[F|L2]) :- E =< F, split(E,L) = (L1,L2).
qsort([]) = [].
qsort([E|L]) = conc(qsort(L1),[E|qsort(L2)]) :- split(E,L) = (L1,L2).
```

(',' is defined as an infix operator for building pairs of lists). Note that this is not a valid K-LEAF or BABEL program since the extra-variables L1 and L2 occur in the right-hand side of the defining equations. In order to avoid the extra-variables one has to replace the last equation by

qsort([E|L]) = conc(qsort(split1(E,L)),[E|qsort(split2(E,L))])

and redefine the split function. This solution is less efficient (because the list L must be processed twice) and simplification orderings fail to prove the termination of the rewrite relation [BG89]. These drawbacks may be accepted, but there are other examples where the use of extra-variables cannot be avoided with simple transformations. The function last computes the last element of a given list. It can be explicitly defined or, if conc is defined as above, by the simple conditional equation

last(L) = E :- conc(L1, [E]) = L.

In this case last(L) is evaluated by searching the right instantiations of L1 and E (note that there is at most one solution if L is given). The use of extra-variables gives us the full power of logic programming inside functional programming. Hence ALF allows extra-variables in conditional equations. If such a conditional equation is applied in a rewrite step, only the first solution to the extra-variables is considered. This is sufficient because all equations are required to be confluent.

It is also possible to specify additional equational clauses which are only used for rewriting. For instance, Fribourg [Fri85] has shown that the addition of inductive axioms for rewriting is useful to reduce the search space. In this case the proved goals are valid with respect to the least Herbrand model but may be invalid in the class of all models. Therefore an ALF-program consists of three groups of clauses:

- 1. Relational clauses which define all predicates except "=".
- 2. Conditional equations used for narrowing.
- 3. Conditional equations used for rewriting.

Usually, all conditional equations in an ALF-program are used for narrowing and rewriting, but the programmer can specify that some equations should only be applied for narrowing or rewriting, respectively (this is the meaning of the suffix onlynarrow and onlyrewrite in equational rules). For instance, the inductive axiom rev(rev(L)) = L can be used for rewriting to reduce the search space (the function rev reverses all elements in a list). To use it as a narrowing rule makes no sense since this would expand the search space.

Similarly to Prolog, the program clauses in ALF are ordered and the different choices for clauses in a computation step are implemented by a backtracking strategy. Note that backtracking is only necessary in the resolution and narrowing rule but not in rewriting since simplification by rewriting produces unique terms independently of the chosen clauses (because of the confluence of the term rewriting relation).

In order to demonstrate the improved efficiency of this operational semantics in comparison to Prolog's computation strategy, consider the following equations for the concatenation function on lists:

conc([],L) = L conc([E|R],L) = [E|conc(R,L)]

If  ${\tt a}$  and  ${\tt b}$  are constructors, then the goal

conc(conc([a|V],W),Y) = [b|Z]

is simplified by rewriting to the goal

[a|conc(conc(V,W),Y)] = [b|Z]

which is immediately rejected since a and b are different constructors. The equivalent Prolog goal

append([a|V],W,L), append(L,Y,[b|Z])

causes an infinite loop for any order of literals and clauses [Nai87]. More details about the advantages of rewriting and rejection in combination with narrowing can be found in [Fri85] and [Han92].

## 5 Examples

In this section we present several examples for ALF programs and ALF modules defining basic data types.

#### 5.1 Natural numbers

Our first example is the definition of the basic ALF module **nats** defining natural numbers and two functions and a predicate on naturals:

module nats.

```
export 0, s, + , *, < .
   datatype nat = \{0; s(nat)\}.
   func + : nat, nat -> nat infixleft 500;
         * : nat, nat -> nat infixleft 400.
   pred < : nat, nat infix.
rules.
   N + 0
            = N.
   0 + N
            = N
                       onlyrewrite.
   N + s(M) = s(N + M).
   s(N) + M = s(N + M) onlyrewrite.
   N * 0
            = 0.
   0 * N
            = 0
                        onlyrewrite.
   N * s(M) = N + N * M.
   s(M) * N = N + N * M onlyrewrite.
        < s(_).
   0
   s(N) < s(M) :- N < M.
```

end nats.

Note that the commutative laws for addition and multiplication are only used for rewriting. Using these rules also for narrowing would expand the search space. Since the onlyrewrite rules are inductive consequences of the other rules, it is allowed to use such rules for rewriting if we are interested in answers valid w.r.t. the initial model (least Herbrand model). See [Fri85] for a discussion of the use of additional rewrite rules.

#### 5.2 Polymorphic lists

The next example is the basic module **list** which contains the definition of lists of elements of an arbitrary type:

```
module list(elem).
   export [] , '.' , append , length , member .
   use nats.
   datatype list = { '.'(elem,list) ; [] }.
   func append: list, list -> list;
        length: list -> nat.
   pred member : elem, list.
rules.
   append([],L)
                 = L.
   append([H|T],L) = [H|append(T,L)].
   length([])
               = 0.
   length([_|L]) = s(length(L)).
   member(E, [E|_]).
   member(E,[_|L]) :- member(E,L).
end list.
```

#### 5.3 Stacks

The following module defines stacks of an arbitrary type. Note that the functions pop and top are partial since they are not reducible on the argument term empty (cf. page 3.6):

module stackmod(elem).

```
export empty, push, top, pop, isEmpty, isNotEmpty.
  datatype stack = { empty ; push(elem,stack) }.
  func pop
                  : stack -> stack partial;
        top
                  : stack -> elem
                                    partial.
  pred isEmpty
                 : stack;
        isNotEmpty: stack.
rules.
  pop(push(X,S)) = S.
  top(push(X,S)) = X.
   isEmpty(empty).
   isNotEmpty(push(X,S)).
end stackmod.
```

#### 5.4 A generic sort module

The following module contains a sort function on lists which is defined by the quicksort algorithm. It is a generic module which has the sort of the list elements and a comparison predicate on these elements as parameters:

```
module qsortpoly(elem, (pred lesseq: elem, elem)).
```

```
use list(elem).
func qsort : list -> list.
pred split : elem, list, list, list.
rules.
qsort([]) = [].
qsort([E|L]) = append(qsort(L1),[E|qsort(L2)]) :- split(E,L,L1,L2).
split(E,[],[],[]).
split(E,[E1|L],[E1|L1],L2) :- lesseq(E1,E), split(E,L,L1,L2).
split(E,[E2|L],L1,[E2|L2]) :- lesseq(E,E2), split(E,L,L1,L2).
end qsortpoly.
```

Note that the equations use the feature of *extra variables* of ALF (compare Section 4).

## 6 Predefined functions and predicates

The current implementation of ALF has several functions and predicates which cannot be defined by Horn clauses but have a fixed interpretation with some side effects during the evaluation. Such functions and predicates are called *predefined*. The type definitions of these objects are contained in the modules **string**, **term**, **operator**, **streamio** and **system**. This section shows the declaration part of these modules (without **export** and **use** declarations) and gives an informal description of the definitions.

If the predefined functions and predicates require some arguments, then these arguments must be bound to ground terms when the function or predicate is evaluated. Otherwise an error will be reported.

#### 6.1 The module string

```
module string.
datatype string.
use list(string) with stringList for list.
func conc_string : string, string -> string.
pred string_nat : string, nat;
    string_chars : string, stringList;
    compare_string: string, string.
...
```

Figure 2: Module string

The module string (Figure 2) contains the necessary definitions to use string constants in ALF programs. A sequence of characters enclosed in double quotes is a term (constant) of type string. This module defines the sort string and operations on this builtin type.

The function conc\_string returns the concatenation of the argument strings. E.g., conc\_string("ab","cd") will be evaluated to the constant term "abcd".

The predicate string\_nat compares a natural number with its string representation and is used to convert a string into a natural number and vice versa. If the first argument of a call to string\_nat is a string representation of a natural number, then the value of the natural number (represented in ALF by an s-term, cf. Section 5.1) is unified with the second argument. If the first argument is a string which is not a valid representation of a natural number, then the predicate call fails. If the second argument is the ALF representation of a natural number (cf. Section 5.1), then a string representing this number is generated and unified with the first argument. E.g., string\_nat("3",s(s(s(0)))) is provable, but string\_nat("ab",N) fails.

The predicate string\_chars can be used to decompose a string into its characters or to concatenate a list of strings into a single string. If the second argument of a call to string\_chars is a list of strings, then these strings are concatenated and the resulting string is unified with the first argument. If the second argument is an unbound variable, then the first argument must be a string which will be decomposed into its list of characters (where each character is represented by a string of length 1) and then the second argument will be bound to this list. For instance, a call of string\_chars(S,["ab","cde"]) binds S to the string "abcde", and a call of string\_chars("abc",L) binds L to the list ["a","b","c"]. Note that if the second argument is not an unbound variable when calling string\_chars, then it *must* be a list of strings (without variables), otherwise a run-time error will occur.

The predicate compare\_string compares two strings. The literal compare\_string(S,T) is provable iff string S is less than string T in lexicographic order. For instance, compare\_string("alf", "alfcompile") is provable while compare\_string("beta", "alpha") fails.

#### 6.2 The module term

#### Figure 3: Module term

The definition of terms is necessary to implement the input and output operations for universal terms in ALF (similar to Prolog's read and write predicates [CM87]). The module term (Figure 3) defines a data type term which is the format of objects which are returned by the function read and which are expected as arguments to the predicate write (see Section 6.4). E.g., if the term

"f(g(X),1)." is on the input stream, the function read returns the ALF term
 compound("f",[compound("g",[var("X")]),integer(s(0))])

The meaning of the individual constructors of sort term are:

- compound represents a structured term where the first argument is the functor of the term
   and the second argument is the list of arguments of the structure. For instance,
   compound("f",[const("a"),const("b")]) represents the term "f(a,b)".
- const represents an atomic constant where the argument is the name of the atom. For instance, const("alf") and const("ALF") represent the atoms "alf" and "'ALF'", respectively. Note that atoms beginning with an upper-case character must always be quoted, otherwise they will be interpreted as a variable.
- var represents a variable where the argument is the name of the variable. For instance, the variable
   "List" is represented by the term var("List").
- str represents a string which is a sequence of characters enclosed in double quotes (cf. Section 6.1).
  For instance, the string constant ""a string"" is represented by str("a string").
- list represents a list which is written using the list notation (see Section 3.3). The first argument
  is the first element of the list and the second argument is the rest of the list. For instance, the
  list "[a]" is represented by list(const("a"),const("[]")). Note that a binary structure
  with the functor "." is not represented as a list, i.e., the term ".(a,b)" is represented by
  compound(".",[const("a"),const("b")]).
- integer is the representation of an integer constant where the argument is the value of the constant.
  For instance, "2" is represented by integer(s(s(0))).
- end\_of\_file is a special object which is returned by read if the input stream has ended.
- read\_error is another special object which is returned by read if a syntax error has occurred while reading the input stream.

#### 6.3 The module operator

module operator.

#### Figure 4: Module operator

In order to allow a flexible syntax with operator definitions (similar to Prolog [CM87]) as input to the predefined function **read**, it is necessary to declare operator tables which are parameters to read. Hence the read function assumes that one parameter is a list of operator declarations which are terms of type operator w.r.t. module operator (Figure 4). The constructors infixleft, infixnot, prefix and postfix are infix operators which construct the elements of a list of type operatorList. For instance,

["+" infixleft 500, "+" prefix 500, "\*" infixleft 400]

is an operator table where the atom + is an infix and a prefix operator and \* is an infix operator. The precedence of \* is lower than the precedence of + which means that a term of the form 1+2\*3 is interpreted as 1+(2\*3).

#### 6.4 The module streamio

```
module streamio.
   datatype fileMode = { input ; output ; append }.
   datatype stream.
   func open: string, fileMode -> stream;
        read: operatorList -> term;
        read: stream, operatorList -> term;
        read_string: string;
        read_string: stream -> string;
        get: nat;
        get: stream -> nat.
   pred write: operatorList, term;
        write: stream, operatorList, term;
        write_string: string;
        write_string: stream, string;
        put: nat;
        put: stream, nat;
        nl;
        nl: stream;
        close: stream.
   . . .
```

#### Figure 5: Module streamio

The module streamio (Figure 5) defines the input and output mechanism of ALF: The I/O functions can take an argument of type stream to select the stream to read from or write to. If it is omitted, the standard input/output streams are used which are usually connected to the user terminal. The function open connects an external file with a new stream object. It takes the name of the file (type string) and an argument of type fileMode with the following meaning:

- input The file is opened for reading. If the file does not exist, the function fails, i.e., the literal containing this function cannot be proved.
- output The file is opened for writing. If the file already exists, the previous contents are destroyed. Otherwise a new file is created.

append The file is opened for writing. The previous contents of an existing file are not destroyed, but the new items are appended to the file. If the file does not exist, a new file is created.

A call to the predicate close closes the stream.

The argument of read and write of type operatorList is a list of operator definitions (see Section 6.3). The function read reads a term according to these operator definitions from the input stream and returns the corresponding object of type term (see Section 6.2) as the result. The predicate write prints an object of type term according to the operator list parameter on the output stream.

read\_string and write\_string are used to read and write precisely one string. The function read\_string reads a line from the input stream and returns this line (without the line feed character) as a string. The predicate write\_string prints a single string on the output stream.

It is also possible to get and put single characters. The function get reads the next character from the input stream and returns the corresponding ASCII value of this character. The predicate put writes the character with the ASCII value of the argument to the output stream. nl is equivalent to put(10), i.e., nl puts a newline character on the output stream.

#### 6.5 The module system

```
module system.
    pred call_unix: string.
    ...
```

Figure 6: Module system

The module system (Figure 6) defines a predicate to call the operating system from ALF. If the literal call\_unix(S) is proved, then the Unix operating system is called with the string S as argument. For instance, call\_unix("ls > files") produces a directory listing in file files.

## 7 How to use the current system

This section describes how the current implementation of the ALF system is used under a UNIX environment. In order to use this implementation, the environment variable ALFHOME must be set to the name of the directory where the ALF system has been installed, e.g., by typing the command

#### setenv ALFHOME /usr/local/ALF

if "/usr/local/ALF" is the name of the installation directory of the ALF system. Furthermore, the directory "\$ALFHOME/bin" must be included in the path variable.

ALF programs are translated into a compact bytecode by a compiler written in Prolog. There are several versions of this compiler available running under different Prolog systems (currently Quintus-Prolog 3.1.1, Sicstus-Prolog 2.1 and SB-Prolog 3.1). The Quintus- and Sicstus-Prolog versions of the compiler are faster than the SB-Prolog version. Therefore you should only use the SB-Prolog version of the ALF compiler if you have no Quintus- or Sicstus-Prolog license for your machine. In order to tell the ALF system which version of the ALF compiler you want to use, you

must set the environment variable ALFPROLOG to an appropriate value i.e., you have to include one of the commands

setenv ALFPROLOG quintus # for the Quintus-Prolog version
setenv ALFPROLOG sicstus # for the Sicstus-Prolog version
setenv ALFPROLOG sbprolog # for the SB-Prolog version

in your .cshrc-file. Ask the person who has installed the ALF system at your site for the ALF compiler versions which are available on your machine.

#### 7.1 Files

An ALF program is a collection of modules. Each module must be stored in a separate file. The file with the main module contains also one or more goals at the end which should be proved by the ALF system. Usually, the main module together with the goal and all imported modules is compiled by the ALF system into a compact bytecode which will be executed by an emulator. The compilation is divided into the following steps:

- 1. In the first step the main module and all imported modules are checked for syntactic errors and type consistence. The result of the first step is a program where all types are deleted and all modules are copied into one large program, the so-called "flat-ALF" program.
- 2. In the second step the flat-ALF program is translated into a sequence of instructions of an abstract machine tailored for executing such programs very efficiently. The machine is called A-WAM (since it is an extension of the Warren Abstract Machine for executing ALF programs).
- 3. In the last step the A-WAM instructions are translated into a compact bytecode representing the compiled ALF program. This bytecode is the input to the A-WAM emulator.

It is possible to compile ALF programs in one step into the bytecode or also in several separate steps. Therefore the different versions of a program are distinguished by the following file name suffixes:

- .alf Files containing ALF modules. Module m must be stored in the file "m.alf". At the end of the main module the goals to be proved are appended.
- .fal Files containing flat-ALF programs. The flat-ALF program corresponding to the main module m (and all imported modules) is stored in the file "m.fal".
- .sow Files containing A-WAM programs. Hence "m.sow" contains a compiled version of the flat-ALF program in "m.fal".
- .byt Files containing bytecodes of A-WAM programs. Thus the ALF program with main module m is compiled into the bytecode stored in "m.byt".

#### 7.2 Compiling ALF programs

All modules modules of an ALF program must be stored in the directory where the ALF system is called (the *current directory*) or in the *global module directory* "\$ALFHOME/modules". If a module is imported, the corresponding file is searched in the current directory and then, if not found, in the global module directory. Usually, the global module directory contains frequently used

and predefined modules like nats (page 18), list (page 18), string (page 20), term (page 21), operator (page 22), streamio (page 23), and system (page 24).

It is also possible to import a module which is stored in another directory. In this case the module name must be prefixed by the path name. Note that a module name in a use declaration must always be an atom. Therefore the module name must be quoted if it is prefixed by the path name. For instance, the module defaults in the directory /usr/lib/ALF is imported by the following use declaration:

use '/usr/lib/ALF/defaults'.

The simplest way to compile and execute an ALF program is to execute the command

alfrun file

where *file* is the name of the main module. This command compiles the source program into bytecode (stored in the file "*file*.byt") and executes the bytecode in case of a successful compilation. It is also possible to compile and execute an ALF program by separate commands which are described in the following. This is useful if the various options of the bytecode emulator (see Section 7.3) should be applied.

An ALF program is compiled into bytecode by the command

alfcompile file

where *file* is the name of the main module. If all translation steps are successful, the bytecode is stored in the file *"file.byt"*, otherwise some error messages are printed and the program is not compiled into bytecode. If only the first translation step should be performed (syntax check, type check in all modules), it is sufficient to execute the command

alfcheck file

In this case only the compiled flat-ALF program is written into the file "file.fal" if the source program was correct. Now you can translate the flat-ALF program step by step by executing the command

alf

which calls the interactive ALF compiler. If the prompt "?-" appears, you can execute the following commands (do not forget to type the period after the command!):

**fs**(*prog1*,*prog2*). Translate the flat-ALF program in "*prog1*.fal" into the A-WAM code sequence in "*prog2*.sow".

**sb**(*prog1*,*prog2*). Translate the A-WAM code sequence in "*prog1*.sow" into the bytecode in "*prog2*.byt".

**fb**(*prog1*,*prog2*). Translate the flat-ALF program in "*prog1*.fal" into the bytecode in "*prog2*.byt" (without creating an A-WAM code sequence file).

### 7.3 Executing and debugging ALF programs

The bytecode of an ALF program is executed by an emulator written in C. After the translation of an ALF program into the bytecode, the emulator is called by

awam [-odt] [-H <heap size>] [-L <local stack size>] [-T <trail size>] [-O <occurrence stack size>]

```
[--occur-check] [--debug] [--trace]
[--heap-size=<heap size>] [--ls-size=<local stack size>]
[--trail-size=<trail size>]
[--occ-size=<occurrence stack size>]
filename[.byt]
```

where the following options are equivalent:

-o = --occur-check -d = --debug -t = --trace

There is also a statistical version of the A-WAM emulator which is executed by calling awamS instead of awam. If the awamS emulator is terminated, statistics about the used memory are printed.

The options have the following meaning (the last argument is always the name of the bytecode file):

- **occur-check** Unification is performed with occur check. Without this option the A-WAM does not perform an occur check.
- debug The emulator executes the bytecode in debug mode (see below).
- **trace** The emulator executes the bytecode in trace mode, i.e., all executed A-WAM instructions are printed on the standard output stream.
- heap-size Set the heap size to the given number of bytes (default: 2 Mbytes). If an execution stops with the error message "Heap overflow", restart the emulator with a larger heap size.
- **Is-size** Set the local stack size to the given number of bytes (default: 2 Mbytes). If an execution stops with the error message "Local-Stack overflow", restart the emulator with a larger local stack size.
- trail-size Set the trail size to the given number of bytes (default: 400 Kbytes). If an execution stops with the error message "Trail overflow", restart the emulator with a larger trail size.
- **occ-size** Set the occurrence stack size to the given number of bytes (default: 200 Kbytes). If an execution stops with the error message "Occurrence-Stack overflow", restart the emulator with a larger occurrence stack size.

In the *standard mode* the emulator computes a solution to the first given goal by interpreting the A-WAM instructions. If a solution is found, the instantiated goal is written on the standard output and the emulator waits for an input. If the input is empty, the emulator is terminated. If the input is ";", the next solution is computed or the next goal is solved (if there is no alternative solution to the current goal).

In the *trace mode* the emulator prints all executed A-WAM instructions on the standard output stream.

In the *debug mode* the emulator stops at particular A-WAM instructions which corresponds to the evaluation of a function or predicate. Hence a behaviour similarly to the common Prolog debuggers is obtained. If a predicate is called, the emulator prints the string CALL together with the currently called predicate (with instantiated argument terms). If a function should be evaluated by rewriting, the emulator prints the string REWRITE together with the current function call. If a function is evaluated by narrowing, the emular prints the string NARROW in a similar way. If the emulator has printed out the current call, it waits for one of the following inputs:

- **h** help: Display the table of possible inputs.
- **a** abort: Stop the emulator.
- **n** nodebug: Turn off debug mode.
- $|\langle \mathbf{cr} \rangle$  or **c** creep: Single-step execution: Stop at the next call.
- **s** skip: Don't show the entire evaluation of this call. Stop at the next literal (in case of predicates) or at the next function call outside this expression (in case of functions).
- **o** occurrence stack: Show all expressions stored on the occurrence stack.
- **t** term start: Show the current literal or argument of the literal where the functional evaluation is performed.
- **w** write: Set the output mode of the emulator to *write*. In *write* output mode the entire current call is printed without any depth limit (in contrast to the *print* output mode below).
- **p** print: Set the output mode of the emulator to *print*. In *print* output mode the current call is printed with a given depth limit (which is specified by the option '<', see below). This is useful to prevent that very large terms are shown in their entirety.
- $\leq n$  set depth limit: Set the depth limit of the *print* output mode of the emulator to n, i.e., subterms with a depth greater than n are printed as ... The default depth limit is 10.

**d** toggle trace mode: Turn on/off the trace mode of the emulator.

The following example shows a small session with the A-WAM emulator. The main module is the following:

```
module test.
  datatype abc = { a ; b ; c ; d }.
  use list(abc).
end test.
?- append(L,[c,d]) = [a,b,c,d].
```

The A-WAM emulator is called in the debug mode (inputs from the user are printed in **bold** face):

```
awam -d test
A-WAM -- Bytecode Interpreter, Version 0.5.9
REWRITE: append(_H0,[c,d]) ? t
append(_H0,[c,d]) = [a,b,c,d]
REWRITE: append(_H0,[c,d]) ? <cr>
NARROW: append(_H0,[c,d]) ? <cr>
REWRITE: append(_H20,[c,d]) ? t
```

```
[_H19|append(_H20,[c,d])] = [a,b,c,d]
REWRITE: append(_H20,[c,d]) ? <cr>
NARROW: append(_H20,[c,d]) ? t
[_H19|append(_H20,[c,d])] = [a,b,c,d]
NARROW: append(_H20,[c,d]) ? d
[ Trace mode is on ]
NARROW: append(_H20,[c,d]) ? <cr>
L97: switch-on-term(L106,L109,L119,L0)
L106: try-me-else(L118,2)
L109: get-nil(X1)
L110: get-variable(X3,X2)
L112: put-value-occ(X3)
L113: pop-occ
L114: call-rewriting(ao)
L115: rebuild-occ-stack
L116: reject
L118: trust-me-else-fail
L119: get-list(X1)
L120: unify-variable(X3)
L121: unify-variable(X4)
L122: get-variable(X5,X2)
L124: put-value(X4,X1)
L126: put-value(X5,X2)
L128: put-list-occ
L129: write-value(X3)
L130: write-variable(X6)
L131: load-occ(X6)
L132: call-rewriting(append/2)
REWRITE: append(_H27,[c,d]) ? d
[ Trace mode is off ]
REWRITE: append(_H27,[c,d]) ? t
[_H19,_H26|append(_H27,[c,d])] = [a,b,c,d]
REWRITE: append(_H27,[c,d]) ? <cr>
NARROW: append(_H27,[c,d]) ? <cr>
Elapsed time: 0.00s
append([a,b],[c,d]) = [a,b,c,d]
```

## 8 Restrictions and bugs

- The current implementation of the ALF emulator has no garbage collector.
- The type checking of terms is performed top down which has the consequence that not all overloading conflicts can be resolved. In this case the user must specify the module to resolve the conflict (cf. Section 3.4).
- The reader has problems to read operators in the standard prefix notation (e.g., to read 3+5 in the notation +(3,5)).

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