A Debugging Scheme for Functional Logic Programs *

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Abstract. We present a general framework for the declarative debugging of functional logic programs, which is valid both for eager as well as lazy programs. We associate to our programs a semantics based on a (continuous) immediate consequence operator which models computed answers. Then we show that, given the intended specification of a program P, it is possible to check the correctness of P by a single step of the immediate consequence operator. We also present a more effective methodology which is based on abstract interpretation. By approximating the intended specification of the success set we derive a finitely terminating debugging method, which can be used statically. Our framework is parametric w.r.t. to the chosen approximation of the success set. We present one specific example of approximation which improves previous work. We also give discuss about how we plan to use inductive learning techniques in order to correct bugs. We provide an implementation of our debugging system which shows experimentally on a wide set of benchmarks that we are able to find some common errors in the user programs.

1 Introduction

Functional logic programming combines the avantages of functional languages (efficient and, for particular subclasses, optimal evaluation strategies) and logic languages (logical variables, built–in search). The operational semantics of integrated languages is usually based on *narrowing*, a combination of unification for parameter passing and reduction as evaluation mechanism which subsumes rewriting and SLD-resolution. Essentially, narrowing consists of the instantiation of goal variables, followed by a reduction step on the instantiated goal. Narrowing is complete in the sense of functional programming (computation of normal forms) as well as logic programming (computation of answers). Due to the huge search space of unrestricted narrowing, steadily improved strategies have been proposed (see [24] for a survey).

Finding program bugs is a long-standing problem in software construction. However, current debugging tools do not enforce program correctness adequately as they do not provide means to finding bugs in the source code w.r.t. the intended program semantics. In pure logic programming, [14, 13] defined a declarative framework for debugging which extends the methodology in [20, 34], based on using the immediate consequences operator T_P to identify program bugs, to diagnosis w.r.t. computed answers. The framework is goal independent, it is driven by a set of most general atomic goals, and does not require the determination of symptoms in advance.

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In this paper, one of the contributions is to develop a declarative diagnosis method w.r.t. computed answers which generalizes the ideas of [14] to the diagnosis of functional logic programs. The conditions which we impose on the programs which we consider allow us to define a framework for declarative debugging which works for both eager (*call–by–value*) narrowing as well as for lazy (*call–by–name*) narrowing. We associate a (continuous) immediate consequence operator to our programs. Then we show that, given the intended specification $\mathcal{I}$ of a program $\mathcal{R}$, we can check the correctness of $\mathcal{R}$ by a single step of this operator.

We also present an efficient methodology which is based on abstract interpretation. We proceed by approximating the intended specification of the success set. Following an idea inspired in [14, 13, 11], we use *over* and *under* specifications $\mathcal{I}^+$ and $\mathcal{I}^-$ to correctly over- (resp. under-) approximate the intended semantics. We then use these two sets respectively for the functions in the premises and the consequence of the immediate consequence operator, and by a simple static test we can determine whether some of the clauses are wrong.

Plan of the paper The rest of the paper is organized as follows. Section 2 briefly presents some preliminary definitions and notations. Section 3 first formulates a novel, generic immediate consequence operator $T_{\mathcal{R}}^{\varphi}$ for functional logic program $\mathcal{R}$ which is parametric w.r.t. the narrowing strategy φ which can be either eager or lazy. We then define a fixpoint semantics based on $T_{\mathcal{R}}^{\varphi}$ which correctly models the answers computed by a narrower which uses the narrowing strategy φ. It is necessary to introduce a flattening transformation w.r.t. the narrowing strategy φ which eliminates nesting calls and allows goals to run in the semantics by standard unification. We also formulate an operational semantics and we show the correspondence with the least fixpoint semantics. In section 4, we introduce the necessary general notions of incorrectness and insufficiency symptoms and uncovered calls. Section 5 provides an abstract semantics which correctly approximates the fixpoint semantics of $\mathcal{R}$. In Section 6, we present our method of abstract diagnosis. We also discuss how to incorporate inductive learning techniques in order to correct bugs: once a bug has been detected in a program, the inductive techniques allow us to correct it by inducing a new program which has the same computed answers semantics given in the specification. In Section 7, we present a prototype implementation of the method. Section 8 concludes.

2 Preliminaries

We briefly summarize some known results about rewrite systems [6, 30] and functional logic programming (see [24, 29] for extensive surveys). Throughout this paper, V will denote a countably infinite set of variables and Σ denotes a set of function symbols, or signature, each of which has a fixed associated arity. $\tau(\Sigma \cup V)$ and $\tau(\Sigma)$ denote the non-ground word (or term) algebra and the word algebra built on $\Sigma \cup V$ and Σ, respectively. $\tau(\Sigma)$ is usually called the Herbrand universe ($\mathcal{H}_\Sigma$) over Σ and it will be denoted by $\mathcal{H}$. $\mathcal{B}$ denotes the Herbrand base, namely the set of all ground equations which can be built with the elements of $\mathcal{H}$. A Σ-equation $s = t$ is a pair of terms $s, t \in \tau(\Sigma \cup V)$. We let $mgu(E)$ denote the *most general unifier* of the equation set E.

Terms are viewed as labelled trees in the usual way. Positions are represented by sequences of natural numbers denoting an access path in a term, where Λ denotes the empty sequence. $\bar{O}(t)$ denotes the set of nonvariable positions of a term t. $t_{|u}$ is the subterm at the position u of t. $t[r]_u$ is the term t with the subterm at the position u replaced with r. These notions extend to sequences of equations in a natural way. For

instance, the nonvariable position set of a sequence of equations $g \equiv (e_1, \ldots, e_n)$ can be defined as follows: $\overline{O}(g) = \{i.u \mid u \in \overline{O}(e_i), i = 1, \ldots, n\}$. By $Var(s)$ we denote the set of variables occurring in the syntactic object s, while $[s]$ denotes the set of ground instances of s. A *fresh* variable is a variable that appears nowhere else. The symbol $\tilde{}$ denotes a finite sequence of symbols. Identity of syntactic objects is denoted by $\equiv$.

We write $mgu(\{s_1 = t_1, \ldots, s_n = t_n\}, \{s'_1 = t'_1, \ldots, s'_n = t'_n\})$ to denote the most general unifier of the set of equations $\{s_1 = s'_1, t_1 = t'_1, \ldots, s_n = s'_n, t_n = t'_n\}$. We write $\theta_{\restriction s}$ to denote the restriction of the substitution θ to the set of variables in the syntactic object s.

A *conditional term rewriting system* (CTRS for short) is a pair $(\Sigma, \mathcal{R})$, where $\mathcal{R}$ is a finite set of reduction (or rewrite) rule schemes of the form $(\lambda \to \rho \Leftarrow C)$, λ, $\rho \in \tau(\Sigma \cup V)$, $\lambda \notin V$ and $Var(\rho) \subseteq Var(\lambda)$. The condition C is a (possibly empty) sequence $e_1, \ldots, e_n$, $n \geq 0$, of equations which we handle as a set (conjunction) when we find it convenient. Variables in C that do not occur in λ are called *extra-variables*. We will often write just $\mathcal{R}$ instead of $(\Sigma, \mathcal{R})$. If a rewrite rule has no condition, we write $\lambda \to \rho$. For CTRS $\mathcal{R}$, $r \ll \mathcal{R}$ denotes that r is a new variant of a rule in $\mathcal{R}$ such that r contains only *fresh* variables, i.e. contains no variable previously met during computation (standardized apart). Given a CTRS $\langle \Sigma, \mathcal{R} \rangle$, we assume that the signature Σ is partitioned into two disjoint sets $\Sigma = \mathcal{C} \uplus \mathcal{D}$, where $\mathcal{D} = \{f \mid (f(\tilde{t}) \to r \Leftarrow C) \in \mathcal{R}\}$ and $\mathcal{C} = \Sigma \setminus \mathcal{D}$. Symbols in $\mathcal{C}$ are called *constructors* and symbols in $\mathcal{D}$ are called *defined functions*.

Functional logic languages are extensions of functional languages with principles derived from logic programming [33]. The computation mechanism of functional logic languages is based on *narrowing*.

Since unrestricted narrowing has quite a large search space, several strategies to control the selection of redexes have been developed. A *narrowing strategy* (or *position constraint*) is any well-defined criterion which obtains a smaller search space by permitting narrowing to reduce only some chosen positions. A narrowing strategy φ can be formalized as a mapping that assigns a subset $\varphi(g)$ of $\overline{O}(g)$ to every goal g (different from $\top$) such that, for all $u \in \varphi(g)$, the goal g is narrowable at position u. An important property of a narrowing strategy φ is completeness, meaning that the narrowing constrained by φ is still complete. There is an inherited tradeoff coming from functional programming, between the benefits of outside-in evaluation of orthogonal, nonterminating rules and those of inner or eager evaluation with terminating, non orthogonal rules. A survey of results about the completeness of narrowing strategies can be found in [5, 16, 17, 24]. To simplify our notation, we let $I\!R_\varphi$ denote the class of CTRS's which satisfy the conditions for the completeness of the strategy φ.

We let $inn(g)$ (resp. $out(g)$) denote the narrowing strategy which assigns the position p of the leftmost-innermost (resp. leftmost-outermost) narrowing redex of g to the goal g. [1] We formulate a conditional narrower with strategy φ, $\varphi \in \{inn, out\}$, as the smallest relation $\leadsto_\varphi$ satisfying

$$\frac{u = \varphi(g) \ \wedge \ (\lambda \to \rho \Leftarrow C) \ll \mathcal{R}^{\varphi}{}_{+} \ \wedge \ \sigma = mgu(\{g_{\restriction u} = \lambda\})}{g \overset{\sigma}{\leadsto}_\varphi (C, g[\rho]_u)\sigma}.$$

[1] An *innermost* term is an operation applied to constructor terms, i.e., a term of the form $f(d_1, \ldots, d_k)$, where $f \in \mathcal{F}$ and for all $i = 1, \ldots, k$, $d_i \in \tau(\mathcal{C} \cup V)$. The leftmost-innermost position of g is the leftmost position of g which points to an innermost subterm. A position p is leftmost-outermost in a set of positions O if there is no $p' \in O$ with p' prefix of p, or $p' = q.i.q'$ and $p = q.j.q''$ and $i < j$.

For $\varphi \in \{inn, out\}$, ${\mathcal{R}^{\varphi}}_{+} = \mathcal{R} \cup \{Eq^{\varphi}\}$, where Eq^{φ} are the rules which model the equality on data terms:

$$\begin{array}{ll} c =_{\varphi} c \rightarrow true & \%\ c/0 \in \mathcal{C} \\ c(x_1, \ldots, x_n) =_{\varphi} c(y_1, \ldots, y_n) \rightarrow (x_1 =_{\varphi} y_1) \wedge \ldots \wedge (x_n =_{\varphi} y_n) & \%\ c/n \in \mathcal{C} \end{array}$$

Here $=_{\varphi}$ is the standard equality $=$ of terms whenever $\varphi = inn$, whereas for the case when φ is out, and nonterminating rules are considered, we need to distinguish the standard (non-strict) equality $=$, which is defined on partially determined or infinite data structures from the strict equality $\approx$, which is only defined on finite and completely determined data structures, and which gives to equality the weak meaning of identity of finite objects (e.g., see [32]). We also assume that equations in g and C have the form $s = t$ whenever we consider $\varphi = inn$, whereas the equations have the form $s \approx t$ when we consider $\varphi = out$. Note that a non–strict equation like $f(a) = g(a)$ is not an acceptable goal when $\varphi = out$.

Innermost narrowing is the foundation of several functional logic programming languages like SLOG [21], LPG [7, 8] and (a subset of) ALF [23]. Innermost narrowing corresponds to the eager evaluation strategies in functional programming. Modern functional logic languages like Curry [25], Escher [31] and Toy [12] are based on lazy evaluation principles, which delay the evaluation of function arguments until their values are needed to compute some result. This avoids unnecessary computations and allows one to deal with infinite data structures [24]. Needed narrowing is a complete lazy narrowing strategy which is optimal w.r.t. the length of the derivations and the number of computed solutions in inductively sequential programs. Informally, inductive sequentiality amounts to the existence of discriminating left-hand sides, i.e. typical functional programs. Needed narrowing can be easily and efficiently implemented by translating definitional trees into case expressions as proposed in [26], which also proves that there is a strong equivalence of needed narrowing derivations in the original program and leftmost-outermost narrowing derivations in the transformed program. A similar transformation is presented in [35], where inductively sequential programs are translated to *uniform* form, which has only flat rules with pairwise non-subunifiable left-hand sides, where the strong equivalence between needed narrowing and leftmost-outermost narrowing derivations also holds.

3 Denotation of a Functional Logic Program

In order to formulate a semantics for functional logic programs modeling computed answers, the usual Herbrand base has to be extended to the set of all (possibly) non-ground equations modulo variance [18, 19]. $\mathcal{H}_V$ denotes the *V-Herbrand universe* which allows variables in its elements, and is defined as $\tau(\Sigma \cup V)/_{\approx}$. For the sake of simplicity, the elements of $\mathcal{H}_V$ have the same representation as the elements of $\tau(\Sigma \cup V)$ and are also called terms. $\mathcal{B}_V$ denotes the *V-Herbrand base*, namely, the set of all equations $s = t$ modulo variance, where $s, t \in \mathcal{H}_V$. Note that the standard Herbrand base $\mathcal{B}$ is equal to $[\mathcal{B}_V]$. The preorder $\leq$ on $\tau(\Sigma \cup V)$ induces an order relation on $\tau(\Sigma \cup V)/_{\approx}$ (and therefore on $\mathcal{H}_V$). The ordering on $\mathcal{H}_V$ induces an ordering on $\mathcal{B}_V$, namely $s' = t' \leq s = t$ if $s' \leq s$ and $t' \leq t$. The power set of $\mathcal{B}_V$ is a complete lattice under set inclusion.

In the following, we recall a semantics $\mathcal{F}^{ca}_{\varphi}(\mathcal{R})$ for program $\mathcal{R}$ given in [1] such that the computed answer substitutions of any (possibly conjunctive) goal g can be derived from $\mathcal{F}^{ca}_{\varphi}(\mathcal{R})$ by unification of the equations in the goal with the equations in the denotation. We assume that the equations in the denotation are renamed apart. Equations in the goal have to be flattened first, i.e. subterms have to be unnested so that the term structure is directly accessible to unification.

Definition 1 (Flat goal w.r.t. φ). *A* flat *equation is an equation of the form* $f(d_1,\ldots,d_n) = d$ *or* $d_1 =_\varphi d_2$, *where* $d, d_1, \ldots, d_n \in \tau(\mathcal{C} \cup V)$ *are constructor terms. A* flat *goal is a set of flat equations.*

For the outermost strategy, $\varphi = out$, the only non–strict equations in a flat goal are of the form $f(\tilde{d}) = x$. These equalities are treated differently from those originally present in the bodies of program rules and in the goal (denoted by $\approx$). In particular, the clauses for $=$ must allow the elimination of $f(a) = x$, whenever $f(a)$ would not have been selected by narrowing (i.e., when its value is not required to reduce $g(f(a))$.

Any sequence of equations E can be transformed into an equivalent one, $flat_\varphi(E)$, which is flat. The *flattening* procedures for equation sets which produce flat goals w.r.t. *inn* and *out*, respectively, can be found in [10, 22].

It is known that the fixpoint semantics allows for the reconstruction of the top down operational semantics and allows for the (bottom-up) computation of a model which is completely independent from the goal.

Fixpoint Semantics Now we are ready to introduce a new, generic immediate consequence operator $T_\mathcal{R}^\varphi$ which models computed answers w.r.t. φ. For any program $\mathcal{R}$, we denote by $\Phi_\mathcal{R}$ the set of identical equations $f(x_1,\ldots,x_n) = f(x_1,\ldots,x_n)$, for each function symbol $f/n \in \mathcal{D}$. We let $\Im_\mathcal{R}^\varphi$ denote the set of the identical equations $c(x_1,\ldots,x_n) =_\varphi c(x_1,\ldots,x_n)$ for the constructor symbols c/n occurring in $\mathcal{R}$ only. As we will see, these *functional reflexivity axioms* play an important role in defining the fixpoint semantics of $\mathcal{R}$.

In non–strict languages, if the compositional character of meaning has to be preserved in presence of infinite data structures and partial functions, then non-normalizable terms, which may occur as subterms within normalizable expressions, also have to be assigned a denotation. Such a denotation is bound to the class of all partial results of the infinite computation along with the usual approximation ordering on them [22, 32] or, equivalently, the infinite data structure defined as the least upper bound of this class. Following [22, 32], we introduce a fresh constant symbol $\perp$ into Σ to represent the value of expressions which would otherwise be undefined.

Definition 2. *Let* $\mathcal{I}$ *be a Herbrand interpretation and* $\mathcal{R} \in I\!R_\varphi$. *Then,*

$$T_\mathcal{R}^\varphi(\mathcal{I}) = \Phi_\mathcal{R} \;\cup\; \Im_\mathcal{R}^\varphi \;\cup\; \{e \in \mathcal{B}_V \mid (\lambda \to \rho \Leftarrow C) \ll \mathcal{R}^\varphi,\;\; \{l = r\} \cup C' \subseteq \mathcal{I},\; mgu(flat_\varphi(C), C') = \sigma,\; mgu(\{\lambda = r_{|u}\}\sigma) = \theta,\;\; u \in \bar{O}^\varphi(r),\; e = (l = r[\rho]_u)\sigma\theta \;\}.$$

where $\mathcal{R}^\varphi = \mathcal{R}$ *if* $\varphi = inn$, *whereas* $\mathcal{R}^\varphi = \mathcal{R} \cup \{f(x_1,\ldots,x_n) \to \perp \mid f/n \in \mathcal{D}$ *and* $x_1,\ldots,x_n$ *are variables*$\}$ *if* $\varphi = out$.

The following proposition allows us to define the fixpoint semantics.

Proposition 1. *The* $T_\mathcal{R}^\varphi$ *operator is continuous on the complete lattice of Herbrand interpretations,* $\varphi \in \{inn, out\}$. *The least fixpoint* $lfp(T_\mathcal{R}^\varphi) = T_\mathcal{R}^\varphi{\uparrow}\omega$.

Definition 3. *The least fixpoint semantics of a program* $\mathcal{R}$ *in* $I\!R_\varphi$ *is defined as* $\mathcal{F}_\varphi(\mathcal{R}) = lfp(T_\mathcal{R}^\varphi)$, $\varphi \in \{inn, out\}$.

The following semantics is proven equivalent to the computed answer semantics below.

Definition 4. *We let* $\mathcal{F}_\varphi^{ca}(\mathcal{R})$ *denote* $\{l = r \in lfp(T_\mathcal{R}^\varphi) \mid r$ *does not contain any defined function symbol* $f/n \in \mathcal{D}\}$, $\varphi \in \{inn, out\}$.

A goal of the form $\Leftarrow x = y$, with $x, y \in V$ is called a *trivial goal*.

Theorem 1. (strong soundness and strong completeness)
Let $\mathcal{R}$ be a program in $I\!R_\varphi$ and g be a (non–trivial) goal according to φ. Then θ is an answer substitution computed by $\leadsto_\varphi$ for g in $\mathcal{R}$ iff there exist $g' \equiv (e_1, \ldots, e_n) \ll \mathcal{F}^{ca}_\varphi(\mathcal{R})$ such that $\theta = mgu(flat_\varphi(g), g')_{|Var(g)}$.

Example 1. Let us consider the program $\mathcal{R} = \{\mathtt{g(x)} \to \mathtt{0},\ \mathtt{f(0)} \to \mathtt{0},\ \mathtt{f(s(x))} \to \mathtt{f(x)}\}$. According to Definition 3, $\mathcal{F}^{ca}_{inn}(\mathcal{R}) = \{\mathtt{0} = \mathtt{0},\ \mathtt{s(x)} = \mathtt{s(x)},\ \mathtt{g(x)} = \mathtt{0},\ \mathtt{f(0)} = \mathtt{0},\ \mathtt{f(s(0))} = \mathtt{0}, \ldots, \mathtt{f(s^n(0))} = \mathtt{0}, \ldots\}$. Given the goal $g \equiv (\mathtt{y} = \mathtt{f(z)})$, innermost narrowing computes the answers $\{\ \{\mathtt{y/0, z/0}\},\ \{\mathtt{y/0, z/s(0)}\}, \ldots, \{\mathtt{y/0, z/s^n(0)}\}, \ldots\}$ in $\mathcal{R}$, which exactly coincides with the set of substitutions computed by unifying the flat goal $\mathtt{f(z)} = \mathtt{y}$ with the equations in $\mathcal{F}^{ca}_{inn}(\mathcal{R})$.

Example 2. Now consider the program $\mathcal{R} = \{\mathtt{from(x)} \to \mathtt{[x|from(s(x))]}, \mathtt{first([x|y])} \to \mathtt{x}\}$. According to Definition 3, $\mathcal{F}^{ca}_{out}(\mathcal{R}) = \{\mathtt{s(x)} \approx \mathtt{s(x)}, \mathtt{from(x)} = \bot,\ \mathtt{from(x)} = \mathtt{[x|}\bot\mathtt{]}, \ldots,\ \mathtt{from(x)} = \mathtt{[x|[s(x)|} \ldots \mathtt{[s^n(x)|}\bot\mathtt{]]]}, \ldots,\ \mathtt{first(x)} = \bot,\ \mathtt{first([x|y])} = \mathtt{x}\}$, with $n \in \omega$. Given the goal $g \equiv (\mathtt{first(from(s(x)))} \approx \mathtt{z})$, outermost narrowing only computes the answer $\{\mathtt{z/s(x)}\}$ in $\mathcal{R}$, which is also the only substitution which can be computed by unifying the flat goal $(\mathtt{first(y)} = \mathtt{w}, \mathtt{from(s(x))} = \mathtt{y}, \mathtt{w} \approx \mathtt{z})$ in $\mathcal{F}^{ca}_{out}(\mathcal{R})$.

According to Theorem 1, $\mathcal{F}^{ca}_\varphi(\mathcal{R})$ can be used to simulate the execution for any (non–trivial) goal g, that is, $\mathcal{F}^{ca}_\varphi(\mathcal{R})$ can be viewed as a (possibly infinite) set of 'unit' clauses, and the computed answer substitutions for g in $\mathcal{R}$ can be determined by 'executing' $flat(g)$ in the program $\mathcal{F}^{ca}_\varphi(\mathcal{R})$ by standard unification, as if the equality symbol were an ordinary predicate. In the following, we show the relation between the semantics $\mathcal{F}^{ca}_\varphi(\mathcal{R})$ and a novel operational "computed answer" semantics $\mathcal{O}^{ca}_\varphi(\mathcal{R})$ which correctly models the behavior of single equations, which we introduce in the following.

Computed Answer Semantics The operational success set semantics $\mathcal{O}^{ca}_\varphi(\mathcal{R})$ of a program $\mathcal{R}$ w.r.t. narrowing semantics φ is defined by considering the answers computed for "most general calls", as shown by the following definition.

Definition 5. *Let $\mathcal{R}$ be a program in $I\!R_\varphi$. Then,*

$\mathcal{O}^{ca}_\varphi(\mathcal{R}) = \Im^\varphi_\mathcal{R} \cup \{(f(x_1, \ldots, x_n) = x_{n+1})\theta \mid (f(x_1, \ldots, x_n) =_\varphi x_{n+1}) \overset{\theta}{\leadsto}{}^*_\varphi \top$
where $f/n \in \mathcal{D}$, x_{n+1} and x_i are distinct variables, for $i = 1, \ldots, n$ }.

The equivalence between the operational and the least fixpoint semantics is established by the following theorem.

Theorem 2. *If $\mathcal{R} \in I\!R_{inn}$, then $\mathcal{O}^{ca}_{inn}(\mathcal{R}) = \mathcal{F}^{ca}_{inn}(\mathcal{R})$.*
If $\mathcal{R} \in I\!R_{out}$, then $\mathcal{O}^{ca}_{out}(\mathcal{R}) = \{l = r \in \mathcal{F}^{ca}_{out}(\mathcal{R}) \mid \bot$ does not occur in $r\}$

4 Diagnosis of declarative programs

We now introduce some basic definitions on the diagnosis of declarative programs [14]. As operational semantics we consider the set of computed answer substitutions.

Definition 6. *Let $\mathcal{I}$ be the specification of the intended computed answer semantics for $\mathcal{R}$.*

1. *$\mathcal{R}$ is partially correct w.r.t. $\mathcal{I}$, if $\mathcal{O}^{ca}_\varphi(\mathcal{R}) \subseteq \mathcal{I}$.*
2. *$\mathcal{R}$ is complete w.r.t. $\mathcal{I}$, if $\mathcal{I} \subseteq \mathcal{O}^{ca}_\varphi(\mathcal{R})$.*
3. *$\mathcal{R}$ is totally correct w.r.t. $\mathcal{I}$, if $\mathcal{O}^{ca}_\varphi(\mathcal{R}) = \mathcal{I}$.*

If a program contains errors, these are signalled by corresponding *symptoms*.

Definition 7. *Let $\mathcal{I}$ be the specification of the intended computed answer semantics for $\mathcal{R}$. An* incorrectness symptom *is an equation e such that $e \in \mathcal{O}^{ca}_{\varphi}(\mathcal{R})$ and $e \notin \mathcal{I}$. An* incompleteness symptom *is an equation e such that $e \in \mathcal{I}$ and $e \notin \mathcal{O}^{ca}_{\varphi}(\mathcal{R})$.*

In case of errors, in order to determine the faulty rules, we give the following definitions.

Definition 8. *Let $\mathcal{I}$ be the specification of the intended fixpoint semantics for $\mathcal{R}$. If there exists an equation $e \in T^{\varphi}_{\{r\}}(\mathcal{I})$ and $e \notin \mathcal{I}$, then the rule $r \in \mathcal{R}$ is incorrect on e.*

Therefore, the incorrectness of rule r is signalled by a simple transformation of the intended semantics $\mathcal{I}$.

Definition 9. *Let $\mathcal{I}$ be the specification of the intended fixpoint semantics for $\mathcal{R}$. An equation e is uncovered if $e \in \mathcal{I}$ and $e \notin T^{\varphi}_{\mathcal{R}}(\mathcal{I})$.*

By the above definition, an equation e is uncovered if it cannot be derived by any program rule using the intended fixpoint semantics.

Proposition 2. *If there are no incorrect rules in $\mathcal{R}$ w.r.t the specification of the intended fixpoint semantics, then $\mathcal{R}$ is partially correct w.r.t. the intended computed answer semantics.*

Proposition 2 shows a simple methodology to prove partial correctness. Completeness is harder: some incompleteness cannot be detected by comparing the specification of the intended fixpoint semantics $\mathcal{I}$ and $T^{\varphi}_{\mathcal{R}}(\mathcal{I})$. That is, the absence of uncovered equations doesn't allows us to derive that the program is complete. Let us consider the following counterexample:

Example 3. Consider the program $\mathcal{R} = \{\mathtt{f(x)} \rightarrow \mathtt{a} \Leftarrow \mathtt{f(x)} = \mathtt{a}\}$ and the specification $\mathcal{I} = \{\mathtt{a} = \mathtt{a}, \mathtt{f(x)} = \mathtt{f(x)}, \mathtt{f(x)} = \mathtt{a}\}$. Then $\mathcal{R}$ is not complete since $\mathcal{I} \nsubseteq \mathcal{O}^{ca}_{inn}(\mathcal{R}) = \{\}$. However $\mathcal{I} \subseteq T^{inn}_{\mathcal{R}}(\mathcal{I}) = \{\mathtt{a} = \mathtt{a}, \mathtt{f(x)} = \mathtt{f(x)}, \mathtt{f(x)} = \mathtt{a}\}$ and there are no uncovered equation.

The problem is related to the existence of several fixed points for the $T_{\mathcal{R}}$ operator. See [14] for details.

5 Abstract success set

The theory of abstract interpretation [15] provides a formal framework for developing advanced data-flow analysis tools. Abstract interpretation formalizes the idea of 'approximate computation' in which computation is performed with descriptions of data rather than with the data itself. The semantics operators are then replaced by abstract operators which are shown to 'safely' approximate the standard ones. In this section, starting from the fixpoint semantics in Section 3, we develop an abstract semantics which approximates the observable behavior of the program and is adequate for modular data-flow analysis, such as the analysis of unsatisfiability of equation sets or any analysis which is based on the program success set. We assume the framework of abstract interpretation for analysis of equational unsatisfiability as defined in [3]. Another approach to constructing an abstract term rewriting system is followed in [9]. We think that another approximation of the fixpoint semantics given in the previous section which is different from the one that we describe in this section can be characterized by following an approach similar to that in [9]. We recall some previous principal definition about the abstract domains and the associated abstract operators, see [1, 3, 4] for details. Then we describe the novel abstract immediate consequence operators $T^{\varphi\sharp}_{\mathcal{R}}$ which are able to approximate the operators $T^{\varphi}_{\mathcal{R}}$, and the abstract fixpoint semantics $\mathcal{F}^{\sharp}_{\varphi}(\mathcal{R})$ and $\mathcal{F}^{ca\sharp}_{\varphi}(\mathcal{R})$. In the following, we denote the abstract analog of a concrete object O by $O^{\sharp}$.

5.1 Abstract Domains and Operators

A *description* is the association of an *abstract domain* $(D, \leq)$ (a poset) with a *concrete domain* $(E, \leq)$ (a poset). When $E = Eqn$ or $E = Sub$, the description is called an *equation description* or a *substitution description*, respectively. The correspondence between the abstract and concrete domain is established through a 'concretization' function $\gamma : D \to 2^E$. We say that d *approximates* e, written $d \propto e$, iff $e \in \gamma(d)$.

Abstract substitutions are introduced for the purpose of describing the computed answer substitutions for a given goal. The domains for equations and substitutions are based in a notion of the abstract Herbrand universe $\mathcal{H}_V^\sharp = (\tau(\Sigma \cup V \cup \{\sharp\}), \preceq)$, which introduces an irreducible symbol $\sharp$, such that $\sharp \notin \Sigma$ (see [1, 3, 4].)

Our analysis is based on a form of simplified (abstract) program which always terminates and in which the query can be executed efficiently. Our notion of abstract program is parametric with respect to a loop-check. Two different instances can be found in [3, 4].

Definition 10. *A loop-check is a graph $\mathcal{G}_\mathcal{R}$ associated with a program $\mathcal{R}$, i.e. a relation consisting of a set of pairs of terms, such that: (1) the transitive closure $\mathcal{G}_\mathcal{R}^+$ is decidable and (2) Let $\overset{\circ}{t} = t'$ be a function which assigns to a term t some node t' in $\mathcal{G}_\mathcal{R}$. If there is an infinite sequence:*

$\langle \Leftarrow G_0, \theta_0 \rangle \rightsquigarrow \langle \Leftarrow G_1, \theta_1 \rangle \rightsquigarrow \ldots$

then $\exists i \geq 0$. $\langle \overset{\circ}{t_i}, \overset{\circ}{t_i} \rangle \in \mathcal{G}_\mathcal{R}^+$, where $t_i = e_{|u}\theta_i$, $e \in G_i$ and $u \in \bar{O}(e)$. (we refer to $\langle \overset{\circ}{t_i}, \overset{\circ}{t_i} \rangle$ as a 'cycle' of $\mathcal{G}_\mathcal{R}$.)

A program is abstracted by simplifying the right-hand side and the body of each clause. This definition is given inductively on the structure of terms and equations. The main idea is that terms whose corresponding nodes in $\mathcal{G}_\mathcal{R}$ have a cycle are drastically simplified by replacing them by $\sharp$. We use this definition in a iterative manner. We first abstract a concrete rule r obtaining $r^\sharp$ (we select a rule with direct recursion if any; otherwise we choose any rule in the program). Then we replace r by $r^\sharp$ in $\mathcal{R}$, and recompute the loop–check before proceeding to abstract the next rule. Each concrete rule is considered only once in the abstraction process.

Definition 11 (abstract rule). *Let $\mathcal{R}$ be a program and let $r = (\lambda \to \rho \Leftarrow C) \in \mathcal{R}$. Let $\mathcal{G}_\mathcal{R}$ be a loop-check for $\mathcal{R}$. We define the abstraction of r as follows: $r^\sharp = (\lambda \to sh(\rho, \mathcal{G}_\mathcal{R}) \Leftarrow sh(C, \mathcal{G}_\mathcal{R}))$ where the shell $sh(x, \mathcal{G})$ of an expression x according to a loop-check $\mathcal{G}$ is defined inductively*

$$sh(x, \mathcal{G}) = \begin{cases} x & \text{if } x \in V \\ f(sh(t_1, \mathcal{G}), \ldots, sh(t_k, \mathcal{G})) & \text{if } x \equiv f(t_1, \ldots, t_k) \text{ and } \langle \overset{\circ}{x}, \overset{\circ}{x} \rangle \notin \mathcal{G}^+ \\ sh(l, \mathcal{G}) = sh(r, \mathcal{G}) & \text{if } x \equiv (l = r) \\ sh(e_1, \mathcal{G}), \ldots, sh(e_n, \mathcal{G}) & \text{if } x \equiv e_1, \ldots, e_n \\ \sharp & \text{otherwise} \end{cases}$$

We can now formalize the abstract semantics.

5.2 Bottom-up Abstract Semantics

We define an abstract fixpoint semantics in terms of the least fixpoint of a continuous transformation $T_\mathcal{R}^{\varphi\sharp}$ based on abstract unification and the operation of abstraction of a program. The idea is to provide a finitely computable approximation of the concrete denotation of the program $\mathcal{R}$. In the following, we define the abstract transformation $T_\mathcal{R}^{\varphi\sharp}$.

Definition 12. (abstract Herbrand base, abstract Herbrand interpretation) *The abstract Herbrand base of equations $\mathcal{B}_V^\sharp$ is defined as the set of equations over the abstract Herbrand universe $\mathcal{H}_V^\sharp$. An abstract Herbrand interpretation is any element of $2^{\mathcal{B}_V^\sharp}$.*

We can show that the set of abstract interpretations is a complete lattice w.r.t. $\subseteq$. An abstract trivial equation is an equation $\sharp = X$, $X = \sharp$ or $\sharp = \sharp$.

Definition 13. *Let $\mathcal{R}$ be a program in $I\!R_\varphi$, $\mathcal{G}_\mathcal{R}$ be a loop-check for $\mathcal{R}$ and $\mathcal{R}^\sharp$ be the abstraction of $\mathcal{R}$ using $\mathcal{G}_\mathcal{R}$ where we also drop any abstract trivial equation from the body of the rules if there exist. Let $\mathcal{I}$ be an abstract interpretation. Then,*

$$T_\mathcal{R}^{\varphi\sharp}(\mathcal{I}) = \Phi_\mathcal{R} \ \cup \ \Im_\mathcal{R}^\varphi \ \cup \ \{e \in \mathcal{B}_V^\sharp \mid (\lambda \rightarrow \rho \Leftarrow C) \ll \mathcal{R}^{\varphi\sharp}, \ \ \{l = r\} \cup C' \subseteq \mathcal{I}, \\ mgu^\sharp(flat(C), C') = \sigma, \\ mgu^\sharp(\{\lambda = (r_{|u})\}\sigma) = \theta, \\ u \in \bar{O}(r), \ \ e = (l = r[\rho]_u)\sigma\theta \ \}.$$

where $\mathcal{R}^{\varphi\sharp} = \mathcal{R}^\sharp$ if $\varphi = inn$, whereas $\mathcal{R}^{\varphi\sharp} = \mathcal{R}^\sharp \cup \{f(x_1, \ldots, x_n) \rightarrow \bot \mid f/n \in \mathcal{D}$ and $x_1, \ldots, x_n$ are variables$\}$ if $\varphi = out$

Proposition 3. *The $T_\mathcal{R}^{\varphi\sharp}$ operator is continuous on the complete lattice of abstract interpretations.*

We can define $\mathcal{F}_\varphi^\sharp(\mathcal{R})$ and $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$ in a way similar to the concrete constructions of $\mathcal{F}_\varphi(\mathcal{R})$ and $\mathcal{F}_\varphi^{ca}(\mathcal{R})$, as in Section 3.

Definition 14. (abstract least fixpoint semantics) *The abstract least fixpoint semantics of a program $\mathcal{R}$ is $\mathcal{F}_\varphi^\sharp(\mathcal{R}) = lfp(T_\mathcal{R}^{\varphi\sharp})$. Let $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R}) = \{l = r \in \mathcal{F}_\varphi^\sharp(\mathcal{R}) \mid$ r does not contain any defined function symbol $f/n \in \mathcal{D}\}$, $\varphi \in \{inn, out\}$*

The following theorem states that $\mathcal{F}_\varphi^\sharp(\mathcal{R})$ and $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$ are finitely computable.

Theorem 3. *There exists a finite positive number k such that $\mathcal{F}_\varphi^\sharp(\mathcal{R}) = T_\mathcal{R}^{\varphi\sharp} \uparrow k$, $\varphi \in \{inn, out\}$.*

¿From a semantics viewpoint, given a program $\mathcal{R}$, the fixpoint semantics $\mathcal{F}_\varphi(\mathcal{R})$ (resp. $\mathcal{F}_\varphi^{ca}(\mathcal{R})$) is approximated by the corresponding abstract fixpoint semantics $\mathcal{F}_\varphi^\sharp(\mathcal{R})$ (resp. $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$). That is, we can compute an abstract approximation of the concrete semantics in a finite number of steps. The correctness of the abstract fixpoint semantics with respect to the concrete semantics is proved by the following:

Theorem 4. *There exists a finite positive number k such that $T_\mathcal{R}^{\varphi\sharp} \uparrow k \propto T_\mathcal{R}^\varphi \uparrow \omega$.*

Corollary 1. *$\mathcal{F}_\varphi^\sharp(\mathcal{R}) \propto \mathcal{F}_\varphi(\mathcal{R})$ and $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R}) \propto \mathcal{F}_\varphi^{ca}(\mathcal{R})$.*

The semantics $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$ collects goal-independent information about success patterns of a given program. The relation between the abstract fixpoint and the concrete operational semantics (computed answer substitutions) is given by the following theorem. Roughly speaking, given a goal $\Leftarrow G$, we obtain a description of the set of the computed answers of $\Leftarrow G$ by abstract unification of the equations in $flat_\varphi(G)$ with equations in the approximated semantics $\mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$.

Theorem 5 (strong completeness). *Let $\mathcal{R}$ be a program in $I\!R_\varphi$ and $\Leftarrow g$ be a (non–trivial) goal. If θ is a computed answer substitution for $\Leftarrow g$ in $\mathcal{R}$ w.r.t. φ, then there exists $g' \equiv e_1, \ldots, e_m \ll \mathcal{F}_\varphi^{ca\sharp}(\mathcal{R})$ such that $\theta' = mgu^\sharp(flat_\varphi(g), g')$ and $\theta' \preceq \theta$.*

Example 4. Let us consider the program $\mathcal{R} = \{\ \mathtt{g(0)} \to \mathtt{0},\ \mathtt{g(c(x))} \to \mathtt{c(g(x))}, \mathtt{f(0)} \to \mathtt{0},\ \mathtt{f(c(x))} \to \mathtt{x} \Leftarrow \mathtt{g(c(x))} = \mathtt{c(x)}\}$ and the strategy $\varphi = inn$. Let us consider the loop-check [4] $\mathcal{G}_{\mathcal{R}} = \{\langle \mathtt{g(x)}, \mathtt{g(x)}\rangle\}$, and let $\overset{\circ}{t}= t'$ be the (partial) function which, given a graph, assigns to a term t some node t' in the graph such that t' unifies with t, if one such node t' exists. Then, the abstraction of the program $\mathcal{R}$ is $\mathcal{R}^\sharp = \{\ \mathtt{g(0)} \to \mathtt{0},\ \mathtt{g(c(x))} \to \mathtt{c(\sharp)},\ \mathtt{f(0)} \to \mathtt{0},\ \mathtt{f(c(x))} \to \mathtt{x} \Leftarrow \mathtt{g(c(x))} = \mathtt{c(x)}\}$.
$\mathcal{F}_{inn}(\mathcal{R}) = \{\ \mathtt{0} = \mathtt{0},\ \mathtt{g(x)} = \mathtt{g(x)},\ \mathtt{f(x)} = \mathtt{f(x)},\ \mathtt{c(x)} = \mathtt{c(x)},\ \mathtt{g(0)} = \mathtt{0},\ \mathtt{f(0)} = \mathtt{0},\ \mathtt{g(c(x))} = \mathtt{c(g(x))}, \ldots,\ \mathtt{g(c^n(x))} = \mathtt{c^n(g(x))}, \ldots,\ \mathtt{g(c(0))} = \mathtt{c(0)}, \ldots,\ \mathtt{g(c^n(0))} = \mathtt{c^n(0)}, \ldots, \mathtt{f(c(0))} = \mathtt{0}, \ldots,\ \mathtt{f(c^n(0))} = \mathtt{c^{n-1}(0)}, \ldots\}$ and the corresponding fixpoint abstract semantics is the finite set
$\mathcal{F}^\sharp_{inn}(\mathcal{R}) = \{\ \mathtt{0} = \mathtt{0},\ \mathtt{g(x)} = \mathtt{g(x)},\ \mathtt{f(x)} = \mathtt{f(x)},\ \mathtt{c(x)} = \mathtt{c(x)},\ \mathtt{g(0)} = \mathtt{0},\ \mathtt{f(0)} = \mathtt{0},\ \mathtt{g(c(x))} = \mathtt{c(\sharp)},\ \mathtt{f(c(x))} = \mathtt{x}\}$, and $\mathcal{F}^{ca\sharp}_{inn}(\mathcal{R}) = \{\ \mathtt{0} = \mathtt{0},\ \mathtt{c(x)} = \mathtt{c(x)},\ \mathtt{g(0)} = \mathtt{0},\ \mathtt{f(0)} = \mathtt{0},\ \mathtt{g(c(x))} = \mathtt{c(\sharp)},\ \mathtt{f(c(x))} = \mathtt{x}\}$ which approximates the program success set. Given a goal $\Leftarrow g \equiv\ \Leftarrow \mathtt{f(g(x))} = \mathtt{y}$, innermost conditional narrowing computes the infinite set of substitutions $\{\{x/\mathtt{0},\ \mathtt{y/0}\}, \{\mathtt{x/c(0)},\ \mathtt{y/0}\}, \{\mathtt{x/c^2(0)},\ \mathtt{y/c(0)}\}, \ldots,$ $\{\mathtt{x/c^n(0)},\ \mathtt{y/c^{n-1}(0)}\}\}$. The abstract substitutions returned by abstract unification of the equations in the flattened goal $\Leftarrow \mathtt{f(z)} = \mathtt{y},\ \mathtt{g(x)} = \mathtt{z}$ with $\mathcal{F}^{ca\sharp}_{inn}(\mathcal{R})$ are $\{\{x/\mathtt{0},\ \mathtt{y/0}\}, \{\mathtt{x/c(x')}, \mathtt{y/\sharp}\}\}$, which approximate the computed answers of $\Leftarrow g$.

6 Abstract diagnosis

An efficient debugger can be based on the notion of over-approximation and under-approximation for the intended fixpoint semantics that we have introduced. The basic idea is to consider two sets to verify partial correctness: $\mathcal{I}^+$ which overapproximates the intended semantics $\mathcal{I}$ (that is, $\mathcal{I} \subseteq \gamma(\mathcal{I}^+)$) and $\mathcal{I}^-$ which underapproximates $\mathcal{I}$ (that is, $\mathcal{I} \supseteq \gamma(\mathcal{I}^-)$). We can then use such sets as shown in Table 1, where the concrete operator $T^\varphi_{\mathcal{R}}$ (w.r.t. the program $\mathcal{R}$) is applied once to $\mathcal{I}^-$ (resp. $\mathcal{I}^+$) to check correctness (resp. completeness.)

Semantic Property	Requirement
$\mathcal{R}$ is partially correct w.r.t. $(\mathcal{I}^+, \mathcal{I}^-)$	$\not\exists e. e \in T^\varphi_{\mathcal{R}}(\mathcal{I}^-)$ and $e \notin \mathcal{I}^+$
$\mathcal{R}$ is complete w.r.t. $(\mathcal{I}^+, \mathcal{I}^-)$	$\not\exists e. e \in \mathcal{I}^-$ and $e \notin T^\varphi_{\mathcal{R}}(\mathcal{I}^+)$
$\mathcal{R}$ is incorrect w.r.t. $(\mathcal{I}^+, \mathcal{I}^-)$	$\exists e. e \in T^\varphi_{\mathcal{R}}(\mathcal{I}^-)$ and $e \notin \mathcal{I}^+$
$\mathcal{R}$ is incomplete w.r.t. $(\mathcal{I}^+, \mathcal{I}^-)$	$\exists e. e \in \mathcal{I}^-$ and $e \notin T^\varphi_{\mathcal{R}}(\mathcal{I}^+)$

Table 1. Sufficient Conditions for Correctness and Completeness

Proposition 4. *If there exists an equation e such that $e \notin \mathcal{I}^+$ and $e \in T^\varphi_{\{r\}}(\mathcal{I}^-)$, then the rule $r \in \mathcal{R}$ is incorrect on e.*

Proposition 5. *If there exists an equation e such that $e \notin T^\varphi_{\mathcal{R}}(\mathcal{I}^+)$ and $e \in \mathcal{I}^-$, then the equation e is uncovered.*

In the following, by abuse we let $\mathcal{I}$ denote the program that specifies the intended semantics. In the following, we consider $\mathcal{I}^+ = lfp(T^{\varphi\sharp}_{\mathcal{I}})$, i.e. we consider the abstract success set that we have defined in previous section as overapproximation of the success set of a program. We can consider any of the sets defined in the works of [11, 13] as underapproximation of $\mathcal{I}$. Alternatively, we can simply take the set which results from a finite number of iterations of the $T^\varphi_{\mathcal{I}}$ function (the concrete operator). Let us illustrate this method by an example.

Example 5. Let us consider the following program, which should order a list of integers using the algorithm of insertion sort. For the abstraction we assume that the loop check is done hierarchically.

$$\begin{aligned}
\mathtt{sort}([\mathtt{X}]) &\to [].\\
\mathtt{sort}([\mathtt{X}|\mathtt{Xs}]) &\to \mathtt{Ys} \Leftarrow \mathtt{sort}(\mathtt{Xs}) = \mathtt{Zs}, \mathtt{insert}(\mathtt{X},\mathtt{Zs},\mathtt{Ys}) = \mathtt{true}.\\
\mathtt{insert}(\mathtt{X},[],[\mathtt{X}]) &\to \mathtt{true}.\\
\mathtt{insert}(\mathtt{X},[\mathtt{Y}|\mathtt{Ys}],[\mathtt{Y}|\mathtt{Zs}]) &\to \mathtt{true} \Leftarrow \mathtt{X} > \mathtt{Y} = \mathtt{true}, \mathtt{insert}(\mathtt{X},\mathtt{Ys},\mathtt{Zs}) = \mathtt{true}.\\
\mathtt{insert}(\mathtt{X},[\mathtt{Y}|\mathtt{Ys}],[\mathtt{X},\mathtt{Y}|\mathtt{Ys}]) &\to \mathtt{true} \Leftarrow \mathtt{X} =< \mathtt{Y} = \mathtt{true}.
\end{aligned}$$

The specification $\mathcal{I}$ is given by the (quite) inefficient program which uses the naive sorting algorithm which computes the permutations of the input list.

$$\begin{aligned}
\mathtt{sort}(\mathtt{Xs}) &\to \mathtt{Ys} \Leftarrow \mathtt{perm}(\mathtt{Xs}) = \mathtt{Ys}, \mathtt{ord}(\mathtt{Ys}) = \mathtt{true}.\\
\mathtt{ord}([]) &\to \mathtt{true}.\\
\mathtt{ord}([\mathtt{X}]) &\to \mathtt{true}.\\
\mathtt{ord}([\mathtt{X},\mathtt{Y}|\mathtt{Xs}]) &\to \mathtt{true} \Leftarrow \mathtt{X} =< \mathtt{Y} = \mathtt{true}, \mathtt{ord}([\mathtt{Y}|\mathtt{Xs}]) = \mathtt{true}.\\
\mathtt{perm}(\mathtt{Xs}) &\to [\mathtt{Z}|\mathtt{Zs}] \Leftarrow \mathtt{select}(\mathtt{Z},\mathtt{Xs},\mathtt{Ys}) = \mathtt{true}, \mathtt{perm}(\mathtt{Ys}) = \mathtt{Zs}.\\
\mathtt{perm}([]) &\to [].\\
\mathtt{select}(\mathtt{X},[\mathtt{X}|\mathtt{Xs}],\mathtt{Xs}) &\to \mathtt{true}.\\
\mathtt{select}(\mathtt{X},[\mathtt{Y}|\mathtt{Xs}],[\mathtt{Y}|\mathtt{Zs}]) &\to \mathtt{true} \Leftarrow \mathtt{select}(\mathtt{X},\mathtt{Xs},\mathtt{Zs}) = \mathtt{true}.
\end{aligned}$$

Then $\mathcal{I}^\sharp =$

$$\begin{aligned}
\mathtt{sort}(\mathtt{Xs}) &\to \mathtt{Ys} \Leftarrow \mathtt{perm}(\mathtt{Xs}) = \mathtt{Ys}, \mathtt{ord}(\mathtt{Ys}) = \mathtt{true}.\\
\mathtt{ord}([]) &\to \mathtt{true}.\\
\mathtt{ord}([\mathtt{X}]) &\to \mathtt{true}.\\
\mathtt{ord}([\mathtt{X},\mathtt{Y}|\mathtt{Xs}]) &\to \mathtt{true} \Leftarrow \mathtt{X} =< \mathtt{Y} = \mathtt{true}, \sharp = \mathtt{true}.\\
\mathtt{perm}(\mathtt{Xs}) &\to [\mathtt{Z}|\mathtt{Zs}] \Leftarrow \mathtt{select}(\mathtt{Z},\mathtt{Xs},\mathtt{Ys}) = \mathtt{true}, \sharp = \mathtt{Zs}.\\
\mathtt{perm}([]) &\to [].\\
\mathtt{select}(\mathtt{X},[\mathtt{X}|\mathtt{Xs}],\mathtt{Xs}) &\to \mathtt{true}.\\
\mathtt{select}(\mathtt{X},[\mathtt{Y}|\mathtt{Xs}],[\mathtt{Y}|\mathtt{Zs}]) &\to \mathtt{true} \Leftarrow \sharp = \mathtt{true}.
\end{aligned}$$

The overapproximation $\mathcal{I}^+$ is given by the following set of equations (After four iterations of the $T_{\mathcal{I}}^{inn\sharp}$ operator, we get the fixpoint):

$\{\mathtt{sort}([\mathtt{A} \mid \mathtt{B}) = [\sharp], \ \mathtt{perm}([\mathtt{X} \mid \mathtt{Y}) = [\sharp|\sharp], \mathtt{sort}([]) = [], \ \mathtt{select}(\mathtt{X},[\mathtt{Y}|\mathtt{Z}],[\mathtt{Y}|\sharp]) = \mathtt{true}, \ \mathtt{select}(\mathtt{X},[\mathtt{X}|\mathtt{Y}],\mathtt{Y}) = \mathtt{true}, \mathtt{perm}([]) = [], \ \mathtt{ord}([\mathtt{X}]) = \mathtt{true}, \ \mathtt{ord}([]) = \mathtt{true}, \ \mathtt{select}(\mathtt{X},\mathtt{Y},\mathtt{Z}) = \mathtt{select}(\mathtt{X},\mathtt{Y},\mathtt{Z}), \ \mathtt{X} =< \mathtt{Y} = \mathtt{X} =< \mathtt{Y}, \ [\mathtt{X}|\mathtt{Y}] = [\mathtt{X}|\mathtt{Y}], \ [] = [], \ \mathtt{ord}(\mathtt{X}) = \mathtt{ord}(\mathtt{X}), \ \mathtt{true} = \mathtt{true}, \ \mathtt{perm}(\mathtt{X}) = \mathtt{perm}(\mathtt{X}), \ \mathtt{sort}(\mathtt{X}) = \mathtt{sort}(\mathtt{X})\}$

Now the system detects that the program rule $\mathtt{sort}([\mathtt{X}]) \to []$ is wrong. If the programmer replaces it with the correct equation $\mathtt{sort}([\mathtt{X}]) \to [\mathtt{X}]$, the program becomes correct and complete according to our conditions.

7 The System for Declarative Debugging Buggy

The basic rules presented so far have been implemented by a prototype system Buggy [2], which is written in SICStus Prolog and available at
`http://www.dsic.upv.es/users/elp/soft.html`.
It includes a parser for a conditional functional logic language which can be executed by lazy narrowing, innermost narrowing, and (innermost) basic narrowing. The module which computes the abstract program is an improving of a previous implementation (reported in [1]) by iteratively recomputing the loop check after abstracting each single rule. The debugger requires the user to fix some parameters, such as the narrowing strategy and the number n of iterations for approximating the success set. Then the errors are automatically found by the debugger and the user has to indicate the corrections to be made on the wrong rules.
The Buggy main screen allows the user to follow the following options.

1. `File options`: It contains the classical options for opening, loading and saving a file, as well as cleaning and exiting the system.
2. `Edit`: it is possible to load, edit, and visualize the program to be debugged on the screen, as well as its intended semantics. The program is incrementally parsed while it is loaded.
3. `Debug`: the debugger starts debugging the input program w.r.t. the intended semantics. The user has to say how many iterations to apply for approximating from below the intended semantics. The errors are shown one by one to the user who is required to propose the corrections which are in turn tested. The final correct program is shown to the user who can save it.
4. `Help`: contains additional information about the system.

We have tested our debugging methodology over several benchmarks. We have considered programs such as `append` for computing the concatenation of two input lists, `last` which returns the last element of a list, `knapsack` which returns a set of elements of the input list whose weight sum is equal to an input integer value, `fibonacci` which computes the Fibonacci numbers, `fact` which computes the factorial of a positive number, `sort` which uses the insertion sort for ordering an input list of integers. For all these programs by using the intended semantics we were able to find the errors which were inserted in the program. The final programs have passed the tests of correctness and completeness.

Let us finally discuss the use of inductive learning techniques in connection to our debugging methodology. The framework for the induction of functional logic programs (IFLP) defined in [28] is based on an "inverse *narrowing*" operator which can be seen as a kind of *folding* transformation which derives an hypothesis (set of programs) starting from some evidence (positive and negative examples). The process of learning begins with a (pairwise) generalization of the examples which are successively transformed to produce new equations. If the induced equations cover more examples than the initial ones (and do not derive any of the counter–examples), then they are selected for the hypothesis. The process stops when some optimality criterium is reached [27].

However, currently the IFLP framework only considers the (ground) Herbrand semantics of the programs. In order to correctly infer code (w.r.t. the computed answer semantics) from the evidences which can be gathered during the process of locating errors in the program, i.e. the equations e which are used in the comparison between $T^{\varphi}_{\mathcal{R}}(\mathcal{I}^-)$ and $\mathcal{I}^+$ (resp. $T^{\varphi}_{\mathcal{R}}(\mathcal{I}^+)$ and $\mathcal{I}^-$), first we have to extend the IFLP framework as to support the removal of incorrect answers (distilled from the negative examples) and the addition of missing answers (from the positive examples) when revising the program. In this process, the special symbol $\sharp$ is handled as any other variable.

8 Conclusions

We have presented an approach to declarative debugging of functional logic programs w.r.t. the set of computed answers. We have defined a declarative debugging method which has similarities to others which have been proposed in the literature (such as [11, 13]), but which is original w.r.t. the definition and use of the semantic equations for making the error diagnosis and is useful for both eager as well as for lazy languages. We have presented a novel fixpoint semantics for functional logic programs, which is parametric w.r.t. the narrowing strategy. This semantics characterizes the set of computed answers in a bottom-up manner. Thus, it is a suitable basis for dataflow analyses

based on abstract interpretation. We present one example of abstraction of this fixpoint semantics which yields an approximated finite (goal-independent) description of the success patterns of the program and which can then be used in combination with our debugging equations to obtain an efficient and terminating debugging system. In this paper, we have discussed the successful experiments which have been performed with a prototypical implementation of our debugging system which is publicly available. We believe that it is possible to extend our system in several ways; for instance, by integrating other dataflow analyses for approximating term rewriting systems [9]. We are currently investigating the integration of our prototype with the inductive system FLIP (`http://www.dsic.upv.es/users/elp/soft.html`) in order to inductively synthezising the revised program.

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