On complexity in parity games (abstract)

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Parity games are infinite duration games played between Even and her opponent Odd on a finite graph labelled with integer priorities. The importance of these games is partially due to their connections to verification logics, in particular as the model-checking games for the modal μ calculus, L_μ. Despite extended efforts, the exact complexity of deciding the winner of a parity game remains an open problem. Most recently, it has been shown to be quasi-polynomial and fixed-parameter tractable.

Although a general polynomial algorithm remains elusive, many restricted classes of parity games can be shown to be solvable in polynomial time. This is the case for classes of games with a bounded number of priorities as well as for classes of games where the structure of the underlying graph is restricted: parity games of bounded tree-width, connectivity, DAG-width and entanglement are all solvable in polynomial time. Entanglement is particularly appealing for the analysis of parity games due to its relationship with the L_μ calculus and the fact that although not bisimulation invariant, bounded-entanglement classes of graphs are closed under taking bisimulation quotients.

These structural measures on the underlying graph of a parity game are orthogonal to the complexity of the priority assignement. However the complexity of a parity game is not solely down to its structure nor to the priority assignement, but to how both interact. For example, the cycles in a graph may be intertwined, causing high entanglement, yet unless the priorities are equally entangled within these cycles, this measure in not necessarily an accurate reflection of algorithmic complexity. This paper proposes capturing this intuition and parameterising how entangled both the cycles and priorities of a parity game are. We propose *parity-entanglement* as a new, L_μ describable, bisimulation invariant measure of complexity for parity games which takes into account both the structure of the underlying graph and how priorities are embedded into it.

Like entanglement, parity-entanglement is defined in game-theoretic terms, using a *parity-detective game* that is played on a parity game arena. Unlike the games that define entanglement and other similar measures, this game is infinite-length and uses a parity condition. Unlike entanglement, the resulting measure is bisimulation invariant. Parity games of bounded parity-entanglement are solvable in polynomial time.

Similarly to entanglement, parity-entanglement is closely related to L_μ. Indeed for fixed k and p, there are L_μ formulas describing i) the class of parity games of maximal priority p and parity-entanglement k, and ii) the winning regions of parity games of maximal priority p and parity-entanglement

k. Furthermore, the complexity of the latter, as measured by its alternation depth, depends only on k. This connections allows us to extend the definition of parity-entanglement to L_μ formulas and show that the L_μ alternation hierarchy collapses on formulas of bounded parity-entanglement: a L_μ formula of parity-entanglement k is equivalent to a formula of alternation-depth and parity-entanglement k.

Finally, we compare parity-entanglement with existing measures. The number of even priorities in a parity game is an upper bound on its parity-entanglement. There are fully connected games of parity-entanglement 1 with arbitrary high entanglement and priority assignement – such games form a novel class of parity games solvable in polynomial time. They are of particular interest as they demonstrate how non-trivial interactions between the cycles of a parity game and its parity assignement can make seemingly complex parity games algorithmically simple. We then show that entanglement and parity-entanglement are orthogonal: there are parity games of entanglement 2 with arbitrarily high pentanglement. The Rabin index, which also takes into account the structure of a parity game as well as its priority assignement, is agnostic to the ownership of positions while parity-entanglement depends solely on the regions induced by winning strategies – therefore, while the Rabin index of a parity game is an upper bound on the parity-entanglement of a game, games of parity-entanglement 1 can trivially have arbitrarily high Rabin-index.

In short, we introduce a new bisimulation invariant measure of complexity for parity games which performs an entanglement-type game-theoretic analysis that takes into account the priority assignement of the parity game. We show that solving parity games of bounded parity-entanglement is in P, compare parity-entanglement to other measures of complexity for parity games, and show that there is a rich connection between parity-entanglement and the L_μ alternation hierarchy.

A working draft is available on request.