Phase Based Image Reconstruction in the Monogenic Scale Space

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Abstract. In this paper, we present an approach for image reconstruction from local phase vectors in the monogenic scale space. The local phase vector contains not only the local phase but also the local orientation of the original signal, which enables the simultaneous estimation of the structural and geometric information. Consequently, the local phase vector preserves a lot of important information of the original signal. Image reconstruction from the local phase vectors can be easily and quickly implemented in the monogenic scale space by a coarse to fine way. Experimental results illustrate that an image can be accurately reconstructed based on the local phase vector. In contrast to the reconstruction from zero crossings, our approach is proved to be stable. Due to the local orientation adaptivity of the local phase vector, the presented approach gives a better result when compared with that of the Gabor phase based reconstruction.

1 Introduction

In the past decades, signal reconstruction from partial information has been an active area of research. Partial information such as zero crossing, Fourier magnitude and localized phase are considered to represent important features of the original signal. Therefore, we are able to reconstruct the original signal based on only the partial information. The variety of results on signal reconstruction has a major impact on the research fields like image processing, communication and geophysics.

Reconstruction from zero crossings in the scale space is investigated by Hummel [1]. He has demonstrated that reconstruction based on zero crossings is possible but can be unstable, unless gradient values along the zero crossings are added. In [2], it is proved that many features of the original image are clearly identifiable in the phase only image but not in the magnitude only image, and reconstruction from Fourier phase is visually satisfying. However, the application of this approach is rather limited in practice due to the computational complexity. Behar et al. have stated in [3] that image reconstruction from localized phase

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only information is more efficient and faster than that from the global phase. The reconstruction errors produced by this method can be very small. However, compared with this approach, the way of image reconstruction presented in this paper is more easier and faster.

In this paper, we present an approach of image reconstruction from the local phase vector in the monogenic scale space. Image reconstruction is easy, fast, accurate and stable when compared with the above mentioned approaches. In [4], Felsberg and Sommer proposed the first rotationally invariant 2D analytical signal. As one of its features, the monogenic phase vector preserves most of the important information of the original signal. The local phase vector contains not only the local phase but also the orientation information of the original signal, which enables the evaluation of structure and geometric information at the same time. The embedding of local phase and local orientation into monogenic scale space improves the stability and robustness. However, in the Gaussian scale space, there is no common filter set which could evaluate the local orientation and local phase simultaneously. To show the advantage of our approach, we replace the Gaussian kernel with the Gabor filter for phase evaluation, the reconstruction results of these two approaches are compared in this paper.

2 The Monogenic Scale Space

The structure of the monogenic scale space [5] is illustrated in Fig.1. The Riesz transform of an image yields the corresponding figure flow, a vector field representing the Riesz transformed results. If we define $\mathbf{u} = (u_1, u_2)^T$ and $\mathbf{x} = (x_1, x_2)^T$, then the Riesz kernel in the frequency domain reads $H(\mathbf{u}) = -i\frac{\mathbf{u}}{|\mathbf{u}|}$ and the convolution mask of the Riesz transform is given by $\mathbf{h}(\mathbf{x}) = \frac{\mathbf{x}}{\mathbf{2\pi|x|^3}}$. The combination of the signal and its Riesz transformed result is defined as the monogenic signal. Let $f(\mathbf{x})$ represent the input signal, the corresponding monogenic signal thus takes the form: $\mathbf{f}_M(\mathbf{x}) = f(\mathbf{x}) + (\mathbf{h} * f)(\mathbf{x})$. The monogenic signal is a vector valued extension of the analytical signal, it is rotation invariant. The monogenic scale space is built by the monogenic signals at all scales, it can alternatively be regarded as the combination of the Poisson scale space and its harmonic conjugate. The Poisson scale space and its harmonic conjugate form the monogenic scale space, they are obtained as follows, respectively.

$$p(\mathbf{x}; s) = (f * P)(\mathbf{x}) \quad \text{where} \quad P(\mathbf{x}) = \frac{s}{2\pi(|\mathbf{x}|^2 + s^2)^{3/2}} \tag{1}$$

$$\mathbf{q}(\mathbf{x}; s) = (f * Q)(\mathbf{x}) \quad \text{where} \quad Q(\mathbf{x}) = \frac{\mathbf{x}}{2\pi(|\mathbf{x}|^2 + s^2)^{3/2}} \tag{2}$$

In the above formulas, P and Q indicate the Poisson kernel and the conjugate Poisson kernel, respectively. At scale zero, the conjugate Poisson kernel is exactly the Riesz kernel. The Poisson scale space $p(\mathbf{x}; s)$ is obtained from the original image by Poisson filtering, its harmonic conjugate is the conjugate Poisson scale space $\mathbf{q}(\mathbf{x}; s)$, which can be formed by the figure flows at all scales. The unique

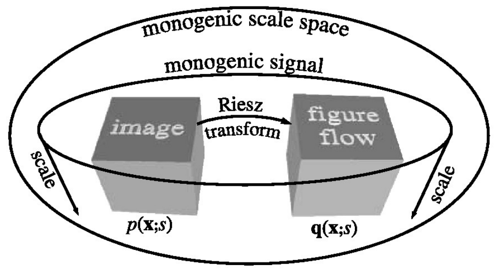

Fig. 1. The structure of the Monogenic Scale Space [5]

advantage of the monogenic scale space, compared with the Gaussian scale space, is the figure flow being in quadrature phase relation to the image at each scale. Therefore, the monogenic scale space is superior to the Gaussian scale space if a quadrature relation concept is required.

3 Important Features of the Monogenic Scale Space

As an analytical scale space, the monogenic scale space provides very useable signal features including local amplitude and local phase vector. The local phase vector contains both the local phase and local orientation information, which enables the simultaneous estimation of structural and geometric information. The local amplitude represents the local intensity or dynamics, the local phase indicates the local symmetry and the local orientation describes the direction of highest signal variance. Let $p(\mathbf{x};s)$ and $\mathbf{q}(\mathbf{x};s)$ represent the Poisson scale space and its harmonic conjugate, the logarithm of the local amplitude, namely the local attenuation in the monogenic scale space reads:

$$A(\mathbf{x};s) = \log(\sqrt{|\mathbf{q}(\mathbf{x};s)|^2 + (p(\mathbf{x};s))^2}) = \frac{1}{2}\log(|\mathbf{q}(\mathbf{x};s)|^2 + (p(\mathbf{x};s))^2) \tag{3}$$

The local orientation and the local phase are best represented in a combined form, namely, the local phase vector $\mathbf{r}(\mathbf{x};s)$. It is defined as the following form:

$$\mathbf{r}(\mathbf{x};s) = \frac{\mathbf{q}(\mathbf{x};s)}{|\mathbf{q}(\mathbf{x};s)|}\arctan(\frac{|\mathbf{q}(\mathbf{x};s)|}{p(\mathbf{x};s)}) \tag{4}$$

Whenever an explicit representation of phase or orientation is needed, the local orientation can be extracted from $\mathbf{r}$ as the orientation of the latter, and the local phase is obtained by projecting $\mathbf{r}$ onto the local orientation. The local phase vector thus denotes a rotation by the phase angle around an axis perpendicular to the local orientation. In the monogenic scale space, the local phase and orientation information are scale dependent, which means the local phase and orientation information can be correctly estimated at an arbitrary scale simultaneously. Unlike the monogenic scale space, there is no common filter set in the

Gaussian scale space which enables the estimation of phase and orientation at the same time. The evaluation of phase in that traditional framework is possible when the Gaussian kernel is replaced by the Gabor filter, and by using Gaussian derivatives, orientation can be evaluated. However, the Gabor filter and Gaussian derivatives are not compatible in the Gaussian scale space, the phase and orientation obtained from them are simply a collection of features, these two features can not be evaluated simultaneously in the Gaussian framework.

4 Image Reconstruction in the Monogenic Scale Space

It is reported in [6] that the local attenuation and the phase response of a minimum phase filter form a Hilbert pair. Under certain conditions, this could also be generalized to 2D. For a 2D signal with an intrinsic dimension of one, if the scale space representation has no zeros in the half space with $s \geq 0$, then the local attenuation and the local phase vector form a Riesz triplet [5]

$$\mathbf{r}(\mathbf{x}; s) \approx (\mathbf{h} * A)(\mathbf{x}; s) \tag{5}$$

where $\mathbf{h}$ refers to the Riesz kernel. In practice, images are in general not globally intrinsical 1D signal. However, they commonly have lots of intrinsically 1D neighborhoods which makes the reconstruction from the local phase vector available. In most practical applications zeros occur in the positive half-space, but as we can see from [5], the influence of the zeros can mostly be neglected. To recover the amplitude information from only the phase vector information, we take the inverse Riesz transform of the local phase vector. By definition, the Riesz transform of the local phase vector is DC free. This means that the transformed output has no DC component. Consequently, the DC-free local attenuation in the scale space is approximated by the following form

$$A(\mathbf{x}; s) - \overline{A}(\mathbf{x}; s) \approx -(\mathbf{h} * \mathbf{r})(\mathbf{x}; s) \tag{6}$$

where $\overline{A}(\mathbf{x}; s)$ indicates the DC component of the local attenuation that should be calculated beforehand. Hence, the original image reconstruction based on the local phase vector reads

$$f(\mathbf{x}) = \exp(\overline{A}(\mathbf{x}; 0))\exp(-(\mathbf{h} * \mathbf{r})(\mathbf{x}; 0))\cos(|\mathbf{r}(\mathbf{x}; 0)|) + C_{DC} \tag{7}$$

where C_{DC} denotes a further DC correction term corresponding to a gray value shift. To reconstruct a real image, we use only the real part of the local phase vector $\cos(|\mathbf{r}(\mathbf{x}; 0)|)$. The above introduction indicates that image reconstruction from the local phase vector can be easily and quickly implemented, no iterative procedure is needed.

To investigate the image reconstruction in the monogenic scale space, a scale pyramid structure is employed. The differences of monogenic signals at adjacent scales are first computed as the bandpass decomposition at different frequencies in the monogenic scale space. The information of different bandpasses forms a

Laplacian pyramid. Local phase vectors of the corresponding bandpass information are considered as the partial information. Signals can thus be reconstructed in the scale space by a coarse to fine way. Let $g^{(s)}$ denote the representation of the image in the pyramid at scale s, then the one scale higher representation reads $g^{(s+1)}$. By interpolation, $g^{(s+1)}$ is expanded as $\widehat{g}^{(s+1)} = T_I g^{(s+1)}$, where T_I refers to the operation of interpolation and $\widehat{g}^{(s+1)}$ has the same size of $g^{(s)}$. The difference of adjacent scales can then be computed as

$$l^{(s)} = g^{(s)} - \widehat{g}^{(s+1)} = g^{(s)} - T_I g^{(s+1)} \tag{8}$$

where $l^{(s)}$ can be regarded as a bandpass decomposition of the original image. Based on only the local phase vector of the intermediate representation, the reconstruction at different scales can be implemented as follows

$$\widetilde{l}^{(s)} = \exp(\overline{A}(\mathbf{x};s))\exp(-(\mathbf{h} * \mathbf{r})(\mathbf{x};s))\cos(|\mathbf{r}(\mathbf{x};s)|) + C_{DC} \tag{9}$$

where $\widetilde{l}^{(s)}$ describes the reconstructed result at a certain scale. By means of a coarse to fine approach, all the scale space images can be combined together to make the final reconstruction of the original image. Starting from the most coarse level, the recovery of one scale lower image takes the following form

$$\widetilde{g}^{(s)} = \widetilde{l}^{(s)} + T_I \widetilde{g}^{(s+1)} \tag{10}$$

This is an iterative procedure. It will end until s goes to zero, hence, $\widetilde{g}^{(0)}$ indicates the final reconstruction.

5 Experimental Results

In this section, we present some experiments to check the performance of image reconstruction based on the local phase vector in the monogenic scale space. Three images used for the experiment are shown in Fig.2. Image reconstruction

Fig. 2. Test images, from left to right, are *lena*, *bird*, and *circles (synthetic image)*

in the monogenic scale space is illustrated in Fig.3. Although we use pyramid structures for scale space reconstruction, the results shown in Fig.3 at different scales are scaled to the same size as the original one. The top row shows the original image and the corresponding absolute error multiplied by 8. Bottom row demonstrates the reconstructed results in the monogenic scale space. The left image in the bottom row is the final result, which is reconstructed by a coarse to fine way. The final reconstruction has a normalized mean square error (NMSE) of 0.0018 when compared with the original one. This demonstrates that image reconstruction can be implemented accurately from the local phase vector.

Fig. 3. Image reconstruction in the monogenic scale space. The original image and the absolute error image multiplied by 8 are shown in the top row. Bottom row: right three images demonstrate the intermediate reconstruction in the monogenic scale space, the left image indicates the final result.

A successful reconstruction from partial information requires a stable output. To investigate the performance of reconstruction from the local phase vector, we conduct another experiment by adding noise to contaminate the input images and checking the outputs. In this experiment, the *bird* image and the *lena* image are used as the noise contaminated inputs, outcomes are shown in Fig.4. The NMSEs increase when the signal noise ratio (SNR) is reduced. However, for both cases, our approach results in limited reconstruction errors even the SNR is set to zero. The results indicate that reconstruction based on the local phase vector is a stable process, hence, the local phase vector can be regarded as stable representation of the original signal. In contrast to this, reconstruction from only zero crossings is proved to produce unstable results [1] unless the gradient data along the zero crossings are combined for reconstruction.

There is no common filter set in the Gaussian framework to evaluate the phase and orientation simultaneously. However, phase information can be estimated when the Gaussian kernel is replaced by the Gabor filter. To show the advantage of our approach, we compare the results of our method with that of the Gabor phase based case. A certain orientation must be assigned to the Gabor filter beforehand. In this case, the orientation is independent with the scale space, local orientation estimation does not change when the scale is changed. Superior to the Gabor phase, the monogenic phase vector enables the estima-

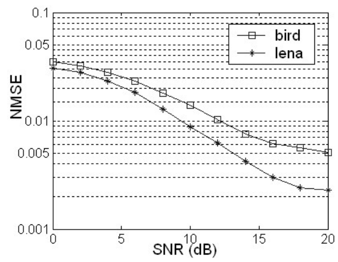

Fig. 4. Normalized mean square error with respect to signal noise ratio

tion of structural and geometric information simultaneously at each scale space. In the monogenic scale space, local phase vector and local attenuation form a Riesz triplet, which means that the amplitude can be easily recovered from the local phase vector simply by using inverse Riesz transform. Unfortunately, the Gabor phase and the local amplitude do not have the property of orthogonality. Hereby, we have to employ an iterative algorithm to reconstruct the image based on local Gabor phases. The iterative reconstruction procedure is similar to the Gerchberg Saxton algorithm [7]. By alternatively imposing constrains in the spatial and frequency domains, an image could be reconstructed in an iterative way. The comparison results are illustrated in Fig.5, four channels with orientations of 0^0, 45^0, 90^0 and 135^0 are considered, the corresponding normalized mean square errors are 0.0812, 0.0833, 0.0815, 0.0836. It is obvious that Gabor phase only preserves the information at the given orientation, however, the monogenic phase results in an accurate and isotropic outcome with an NMSE of 0.0014. Due to the rotation invariant property of the monogenic signal, signals can be well reconstructed in the isotropic way.

6 Conclusions

In this paper, we have presented an approach to reconstruct an image in the monogenic scale space based on the local phase vector. According to the estimated local structural and geometric information, an image can be easily and quickly reconstructed in the monogenic scale space by a coarse to fine way. Experimental results show that accurate reconstruction is available. In contrast to the reconstruction from zero crossings, a stable reconstruction can be achieved based on the local phase vector. Furthermore, the very nice property of local orientation adaptivity can result in a much better reconstruction when compared with that of the orientation selective Gabor phase.

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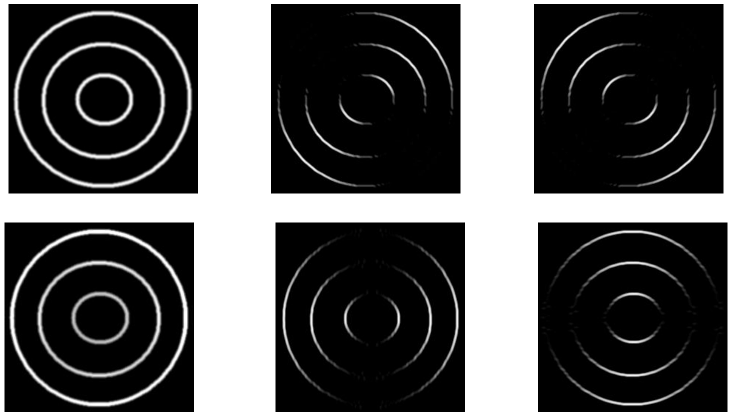

Fig. 5. Upper row: images from left to right are the original image and the reconstructed results based on Gabor phases with orientations of 45^0 and 135^0, the corresponding NMSEs are 0.0833 and 0.0836. Bottom row: the Left one shows the reconstruction based on the monogenic phase, it has a NMSE of 0.0014. The middle and the right images are the results from Gabor phases with orientations of 0^0 and 90^0, the NMSEs are 0.0812 and 0.0815, respectively.

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