Low-cost Junction Characterization Using Polar Averaging Filters

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Abstract

Junction characterization is a very costly task since junctions are multi-scalar and multi-oriented structures. Besides, high directional selectivity is also required to deal with complex junctions. Steerable filters attenuate this computational burden by approximating a filter at an arbitrary scale and orientation with a finite set of so-called basis filters. But due to the uncertainty principle high orientational selectivity can be only achieved by applying a large number of basis filters. In this paper we present a kind of filter with small spatial support to alleviate this effort. The mask of our filter is determined not by Fourier transform, but directly according to the coordinates. Hence, masks with high orientational selectivity can be obtained without increasing any extra computation. Moreover, only averaging values in the masks are calculated. The expensive 2D convolution of normal filtering is therefore avoided.

1 Introduction

Junctions of gray-value lines or edges are rare events in images carrying important information for many image processing tasks like point matching in object recognition, point tracking in motion analysis and attentive coding.

In order to use junctions for such tasks we must be able to characterize them by means of signatures and to classify them in junction categories. Junctions are local structures with multiple intrinsic scales and orientations [2]. A signature characterizing such a junction can be only obtained by applying a filter in different scales and orientations. This results in enormous computational effort. To reduce such a complexity the concept of steerability has been introduced [4, 1]. Steerability is based on the approximation by a Fourier series. The response of a filter at an arbitrary scale and orientation can be approximated as a linear combination of a finite set of so-called basis filters. Usually the number of basis filters is much less than that of scale changes and rotations. Therefore, the computational cost is highly reduced.

Although steerability provides us with a solid mathematical theory, the complexity of its implementation remains high. Due to the uncertainty principle the product of orientational resolution of a steerable filter and its corresponding spectral bandwidth has a lower bound. Therefore, in order to achieve a high orientational selectivity we have to apply a large number of basis filters with each of them having supports covering the whole neighborhood of the corresponding keypoint [5, 6].

But why is it necessary to cover the neighbor so many times? In principle we need to "scan" the neighbor only *once* to obtain the required information for characterization. This is the motivation of our polar averaging filter. The goal is to reduce the computational cost by applying a rotated mask with small spatial support. Furthermore, we try to accomplish all filtering by avoiding 2D convolution which is computationally expensive if the filter is non-separable and the computer does not contain a dedicated convolver.

In the following we first describe the new polar averaging filter in detail. Then the complexity of our approach is compared with the steerable wedge filter [7]. Finally we show the performance of both filters in real experiments and give the conclusion about the new filter.

2 Polar Averaging Filter

Our method applies a local polar mapping at a keypoint. Then, it estimates averaging values over the rectangular support in polar coordinates as shown in figure 1. The positions of local maxima show the orientation of lines and the positions of steepest descent/ascent indicate the orientation of edges. Because in natural images edges are more important than lines we further apply a 1D derivative filter with respect to the angle to get the required information. In this paper we use the first derivative of an 1D Gaussian function G_1 with tap size S. Thus, for a circular neighborhood of a keypoint $f(x, y)$ the impulse response yields:

$$g(\theta_i) \stackrel{def}{=} \frac{1}{\mathcal{N}(f(\rho,\phi))} \sum_{\rho=R_{\min}}^{R_{\max}} \sum_{\phi=\theta_i-\frac{W}{2}}^{\theta_i+\frac{W}{2}} f(\rho,\phi)$$

$$h(\theta) \stackrel{def}{=} \mid G_1(\theta) * g(\theta) \mid$$

where $\mathcal{N}(\ )$ means the number of pixels in the masks and $\theta, \theta_i \in [0, 2\pi]$. The maxima of $g(\theta)$ and $h(\theta)$ indicate the existence of lines and edges, respectively. The meaning of other parameters are shown in figure 1. As an example we present synthetic image results in figure 2.

The support of the filter does not contain the offset-keypoint so that confusion is avoided due to artifact structures close to the keypoint. The advantages of introducing $R_{\min}$ can be seen in figure 3.

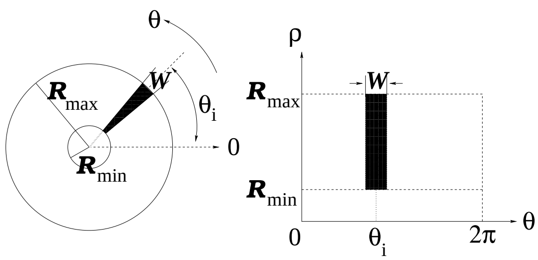

Figure 1: Mask centered at angle θ_i. **left:** Mask in original Cartesian coordinates. Keypoint is at the center of the circle. **Right:** Mask with θ and ρ as Cartesian coordinates. Radial boundaries of the mask are fixed by $R_{\max}$ and $R_{\min}$, W is the angle width of the mask.

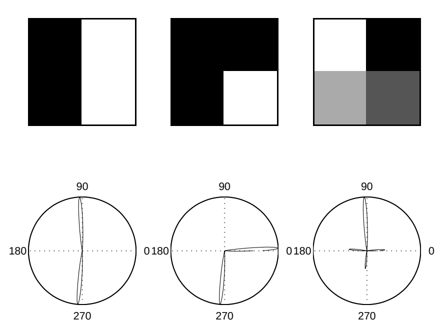

Figure 2: **Top:** Synthetic edge junctions. **Bottom:** Polar plots of $h(\theta)$. The local maxima show the orientation of edges. Here G_1 with tap size $S = 11$ is used. $W = 8°$, $R_{\min} = 3$, $R_{\max} = 9$. The small deviations in orientations are due to the fact that an edge can only be presented by two pixels in the grid, while we can not set the center of a mask between two pixels.

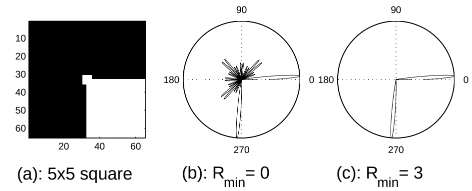

Figure 3: **(a):** An edge junction corrupted by a small white square. **(b):** With $R_{\min} = 0$ the output possesses many local maxima. **(c):** With $R_{\min} \neq 0$ the confusing structures are not corrupting the angular plot. $W = 8°$, $R_{\max} = 7$, $S = 11$.

3 Complexity Comparison

Many kinds of steerable filters are used to analyze junctions. Since the steerable wedge filter [7] represents basic attributes of steerable filters and is computationally more effective than normal steerable filters due to its polar separability, we choose this filter for comparison.

Steerable filter methods are based on convolution. The computational cost is proportional to the number of basis filters. In order to implement the steerable wedge filter [7] with N basis filters of mask size $P \times P$ we require $N(2P + \frac{360°}{\delta\psi})$ multiplications and $N(2P - 1) + \frac{360°}{\delta\psi}(N-1)$ additions, where $\delta\psi$ is the sampling interval in the angle domain. Normally $\delta\psi = 1°$ is fine enough to characterize *all* possible junctions.

In the polar averaging filter, the complexity is reduced in two-fold. First, we apply filters with much smaller support than the basis filters in [7]. Secondly, due to averaging we have up to the Gaussian derivative with respect to the angle only additions and about 360 multiplications (for $\delta\psi = 1°$). Precisely, we need $\frac{360°}{\delta\psi}S + \frac{360°}{\delta\psi}$ multiplications and $\frac{W}{\delta\psi}\pi(R_{\max} - R_{\min} + 1)^2 + \frac{360°}{\delta\psi}(S-1)$ additions to employ our method, where S is the tap size of 1D derivative filter. As shown in figure 4, to achieve the equivalent orientational selectivity, 90 basis filters are required in the steerable wedge filter method [7], yielding 37980 multiplications and 37530 additions ($N = 90, P = 31$) totally. The effort of the polar averaging filter is only 4320 multiplications and 6817 additions ($W = 4°, R_{\min} = 0, R_{\max} = 15, S = 11$).

It should be noticed that the local polar mapping can be done "off-line" since it is a transform between coordinates and is therefore valid for all different images. The resulting table-look-up is of negligible complexity in comparison to the averaging step.

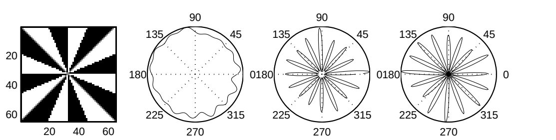

Figure 4: **(a):** A so-called "Siemens star" with 16 edges spanning evenly the orientation space. **(b):** Polar plot of the result using the steerable wedge filter [7] composed of 46 basis filters with 31-tap size. The edges are hardly discernible. **(c):** The same as in (b) but using 90 basis filters. **(d):** Results of polar averaging filter. The orientations of the edges are clearly presented. $W = 4°$, $R_{\min} = 0$, $R_{\max} = 15$, $S = 11$.

4 Experiments

In real images junctions are often corrupted by noise. Experiment in figure 5 shows that the polar averaging filter is robust against noise. In addition, the detection and localization of keypoints are not always precise [3]. Therefore, it is necessary to study the behavior of the new filter with respect to the offset variations. Our filter is also stable in this case (figure 6). In figure 7 and 8 we show the performance of the steerable wedge filter and our polar averaging filter in real edge junctions. The keypoints are not always at centers of the masks. Both of filters are stable with respect to these offsets. Our filter characterizes the directions of junctions more distinctively but is relatively sensitive to high frequency components due to 1D derivative.

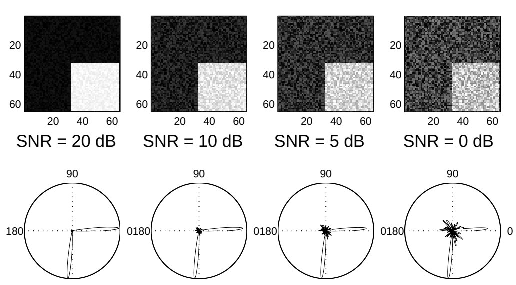

Figure 5: **Top:** A synthetic edge junction disturbed by four increasing levels of random noise. **Bottom:** Corresponding results using the polar averaging filter. Even in the very noisy case ($\mathcal{SNR} = 0\ dB$) we can still match the junction. $W = 10°$, $R_{\min} = 0$, $R_{\max} = 9$, $S = 11$.

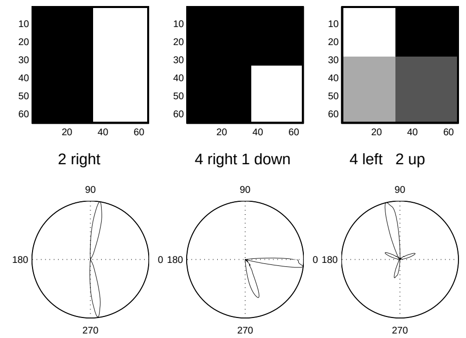

Figure 6: **Top:** Different deviation of the keypoints from centers of masks. **Bottom:** Polar plots of $h(\theta)$. $W = 15°, R_{\min} = 3, R_{\max} = 15, S = 17$.

5 Conclusion

The main task of image analysis can be expressed as the question "what is where". Usually different kinds of filters with different shapes are applied to extract required information. Therefore, convolution is a standard method. In order to optimize filters many methods of signal reconstruction such as Fourier series and wavelets are further introduced.

But these may be not necessary to characterize junctions. It is known that images are represented in a regular grid, i.e., a kind of topological structure. Applying filters can be viewed as converting one structure into another more suitable structure for extracting required information. With this view of structure we can obtain the orientational information much more straightforwardly by directly rearranging the structure from Cartesian coordinates to polar coordinates. The conversion avoids the cumbrous 2D convolution and represents the required information more distinctively. The polar averaging filter simplifies image analysis. It can be further applied to estimate multiple motions and to cope with scale problems.

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References

[1] E.P. Simoncelli, W.T. Freeman, E.H. Adelson and D.J. Heeger. Shiftable multi-scale transforms. *IEEE Trans. Information Theory*, 38(2):587–607, 1992.

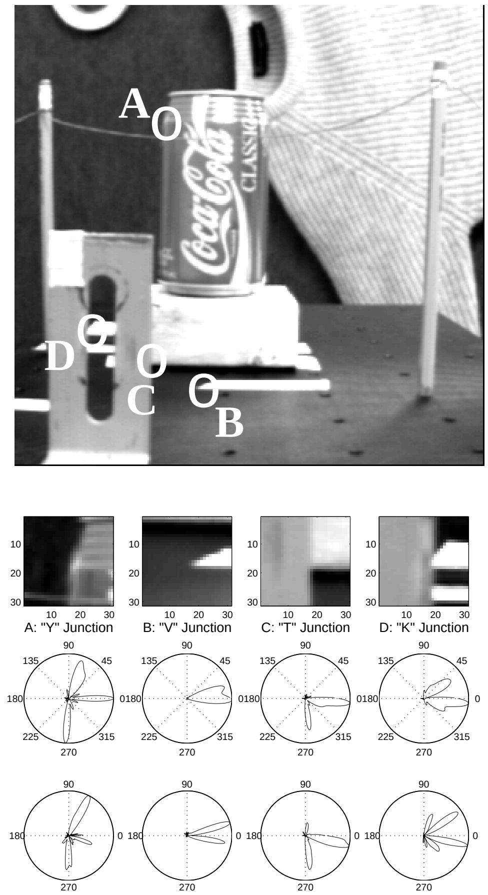

Figure 7: Comparison between steerable wedge filter [7] and polar averaging filter. **Row 1:** An image of the NASA sequence with four kinds of marked junctions. **Row 2:** Junctions in detail. The keypoints are not always at centers of the masks. **Row 3:** Polar plots using the steerable wedge filter [7] composed of 46 basis filters with 31 tap size. **Row 4:** Polar plot using the polar averaging filter. $W = 10°, R_{\min} = 3, R_{\max} = 15, S = 17$. Both methods are stable with respect to the offsets of keypoints. Our method presents higher orientational selectivity with lower cost.

[2] D. H. Ballard and L. E. Wixson. Object recognition using steerable filters at multiple scales. In *IEEE Workshop on Qualitative Vision*, pages 2–10, 1993.

[3] W. Foerstner. A framework for low level feature extraction. In *European Conf. on Computer Vision*, volume II, pages 383–394, Stockholm, Sweden, May 2-6, J.O. Eklundh (Ed.), Springer LNCS 801, 1994.

[4] W.T. Freeman and E.H. Adelson. The design and use of steerable filters. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 13:891–906, 1991.

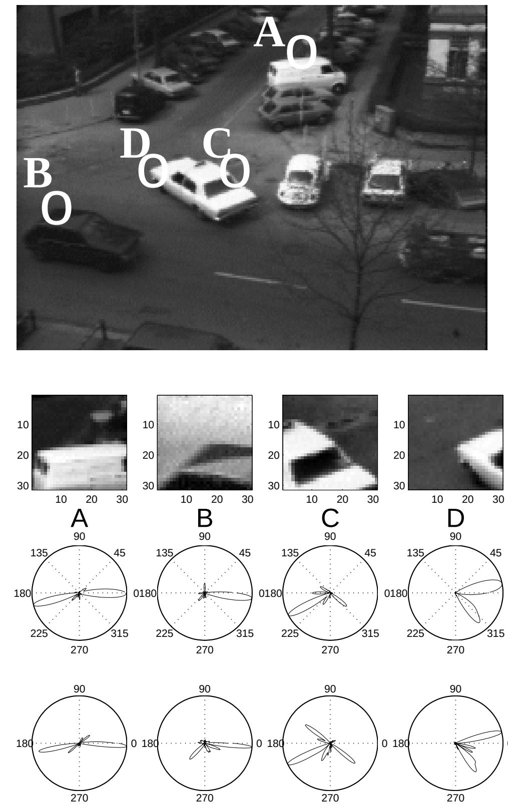

Figure 8: **Row 1:** The image "taxi" with four marked junctions. **Row 2:** Junctions in detail. **Row 3:** Polar plots using the steerable wedge filter [7] composed of 46 basis filters with 31 tap size. **Row 4:** Polar plot using the polar averaging filter. $W = 10°, R_{\min} = 3, R_{\max} = 15, S = 17$.

[5] M. Michaelis and G. Sommer. Junction classification by multiple orientation detection. In *European Conf. on Computer Vision*, volume I, pages 101–108, Stockholm, Sweden, May 2-6, J.O. Eklundh (Ed.), Springer LNCS 800, 1994.

[6] P. Perona. Deformable kernels for early vision. In *Conference on Computer Vision and Pattern Recognition*, pages 222–227, Maui, Hawaii, June 3-6, 1991.

[7] E. P. Simoncelli and H. Farid. Steerable wedge filters for local orientation analysis. *IEEE Trans. Image Processing*, 5(9):1377–1382, 1996.