OPTICAL FLOW COMPUTATION IN THE LOG-POLAR PLANE

Kostas Daniilidis and Volker Krüger
Computer Science Institute, Christian-Albrechts University Kiel
Preusserstr. 1-9, 24105 Kiel, Germany
email: {kd,vok}@informatik.uni-kiel.de

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ABSTRACT:
Vision systems with spatially homogeneous resolution are not able to provide a real time response in a dynamically changing environment. A reactive behavior necessitates selective sensing in space. Such a selection can be accomplished by the combination of a space-variant resolution scheme and a sensor with controllable degrees of freedom. The field of view is split into a homogeneous high resolution area - the fovea - and the periphery with decreasing resolution. Both in neurobiology and in robot vision, models of the resolution decrease towards the image boundaries have been established. The most convincing model is the theory of logarithmic polar mapping. In this paper we propose two new methods for the estimation of the optical flow and its spatial derivatives in the log-polar plane. We study analytically and experimentally the effects of the polar deformation and the decimation due to subsampling on the computation of optical flow.

1 Introduction

This paper is concerned with the computation of optical flow in image sequences obtained with the logarithmic polar transformation. The log-polar transformation is a model for space-variant resolution in the periphery of the image. Space-variant sensing arises as a necessity in systems which must be able to process simultaneously a central region of interest (fovea) in detail for recognition tasks and a wide-angle peripheral view for detecting events and new candidates for gaze change. Uniform resolution in the peripheral part would result to a computational burden unacceptable for real-time reactive behavior. The prerequisite for space-variant sensing is active sensing that means the capability to control the gaze direction. The biologically motivated log-polar transformation has a second significant advantage: It is a very rich representation regarding recognition tasks (rotation and scaling invariance) as well as navigational tasks (ego-motion and time to collision estimation, motion detection).

We first shortly describe the basic anatomy of the primate's space-variant sensing. The retina consists of three layers. In the first layer we find the photo-receptor-cells (PRC) that code the visual information in terms of nervous impulses. The PRC are connected either directly via bipolar-cells or indirectly via amacrin- bipolar- and horizontal-cells in the second layer to the retinal ganglion cells (RGC) in the third layer. An area of PRC on the retina that are combined by one single RGC is called receptive field (RF). PRC are nonuniformly spaced over the human retina. Their heightest density can be found within the fovea centralis, but their density decreases with increasing eccentricity. The spatial density of RFs on the human retina is related to the density of the PRC. The highest density of RFs is therefore found in the fovea where some RF even consist of only one PRC each. In the periphery the density of RF decreases whereas their center-size increases nearly linearly with eccentricity. This allows keeping the amount of visual information as received by $\approx 10^8$ cones and rods low enough to be processed by only $\approx 10^6$ RGC and optic nerve fibers. [Schwartz, 1977] proposed the complex logarithmic mapping for the retinotopic mapping of the RGC onto the first area of the visual cortex (V1). [Weiman and Chaikin, 1979] studied first the properties of the complex logarithmic transformation as a conformal mapping and they proposed a logarithmic spiral grid as a digitization scheme for both image synthesis and analysis.

The goal of this paper is to study what kind of information is still preserved after the log-polar transformation which can be used for motion tasks. We use the optical flow as an intermediate step and study analytically and experimentally the effects of the polar deformation in sec. 3 and the logarithmic subsampling in sec. 5. In particular,

- we prove that the polar transformation introduces fictitious gray-values curvature that leads to an erroneous elimination of the aperture problem,
- we propose two new methods for the optical flow estimation in the log-polar domain that are superior to methods directly transferred from the cartesian domain and we experimentally study their performance in a real sequence,
- we propose a basis for an analysis of the logarithmic subsampling that allows spectral techniques for the design of the necessary low-pass and gradient filters.

We use (x, y) for the cartesian coordinates and (ρ, η) for the polar coordinates in the plane. By denoting with $z = x + jy = \rho e^{j\eta}$ a point in the complex plane the complex logarithmic (or log-polar) mapping is defined as

$$w = ln(z) \tag{1}$$

for every $z \neq 0$ where $\mathrm{Re}(w) = \ln\rho$ and $\mathrm{Im}(w) = \eta + 2k\pi$. To exclude the periodicity of the imaginary part we constrain the range of $\mathrm{Im}(w)$ to $[0, 2\pi)$. The complex logarithmic mapping is a well-known conformal mapping preserving the angle of the intersection of two curves. It is trivial to show that every scaling and rotation about the origin in the z-plane is represented in the w-plane by a shift parallel to the real and imaginary axis, respectively.

We apply the log-polar mapping on the non-foveal part of a retinal image. Therefore, we define as the domain of the mapping the ring-shaped area $\rho_0 < \rho < \rho_{max}$ where ρ_0 and ρ_{max} are the radius of the fovea and the half-size of the retinal image, respectively. Furthermore, a hardware CCD-sensor with the log-polar property or a software implementation of the mapping needs a discretization of the w-plane -which we will call log-polar plane in contrast to the cartesian plane. By assuming that N_r is the number of cells in the radial direction and N_a is the number of cells in the angular direction the mapping from the polar coordinates (ρ, η) to the log-polar coordinates (ξ, γ) reads as follows (see also [Tistarelli and Sandini, 1992])

$$\xi = \log_a\left(\frac{\rho}{\rho_0}\right) \qquad \gamma = \frac{N_a}{2\pi}\eta \tag{2}$$

where the logarithmic basis a is obtained from the foveal radius ρ_0, the image radius ρ_{max} and the radial resolution N_r: $a = e^{\frac{1}{N_r} ln(\frac{\rho_{max}}{\rho_0})}$. From now on we will use only η ranging from 0 to 2π for the angular component of the motion field vector.

The mapping of the gray-value function $I(x, y)$ in the cartesian plane to the gray-value function $J(\xi, \eta)$ in the log-plane is by no means trivial. This issue concerns the software implementation of the log-polar mapping given a cartesian image. Every log-polar cell corresponds to a receptive field in the cartesian plane. The image $J(\xi, \eta)$ is the result of a space-variant filtering that affects all subsequent computations on the log-polar plane like spatiotemporal filtering appearing later in this paper. We will not delve in this issue here. It has been extensively studied in [Bolduc and Levine, 1994] but it still remains an open problem as we will see in section 5. In our implementation we used non-overlapping averaging receptive fields as implemented in the emulation of the space-variant sensor by [Tistarelli and Sandini, 1992].

2 Optical flow in the log-polar plane

We will use the notion of optical flow for the apparent velocity of gray-value structures in the image as opposed to the pure geometric definition of the motion field as the velocity of the projected scene points on the image Interpretations and actions concerning the scene are based on the motion field although only the optical flow field can be observed. This discrimination becomes more crucial here than in the cartesian plane due to the polar deformation and the logarithmic subsampling of the gray-value function. Using the coordinate transformation we are able to transform exactly the cartesian motion field on the log-polar plane. However, the deformation of the gray-value function causes new apparent shifts of the gray-value function or eliminates existing ones.

In this section, we first apply flow computation methods already existing for the cartesian plane to a log-polar image sequence. We denote by (u, v) the optical flow vector in the cartesian plane and by (u^l, v^l) the optical flow vector in the log-polar plane. The motion field vectors in the cartesian and log-polar plane are denoted by $(\dot{x}, \dot{y})$ and $(\dot{\xi}, \dot{\eta})$, respectively. We first compute the motion field vectors in the polar plane. The definition of the polar coordinates yields

$$\dot{\rho} = \dot{x}\cos\eta + \dot{y}\sin\eta \qquad \dot{\eta} = \frac{1}{\rho}(-\dot{x}\sin\eta + \dot{y}\cos\eta). \tag{3}$$

The radial component of the log-polar motion field can be easily obtained:

$$\dot{\xi} = \frac{1}{\ln a}\frac{\dot{\rho}}{\rho}. \tag{4}$$

To compute optical flow we will use methods based on the spatiotemporal derivatives of the image and on the Brightness Change Constraint Equation (BCCE) on the log-polar image $J(\xi, \eta)$:

$$J_\xi u^l + J_\eta v^l + J_t = 0, \tag{5}$$

where J_ξ, J_η, and J_t are the spatiotemporal derivatives of the image. A first method is the application of the BCCE in the neighborhood $(\xi + \delta\xi, \eta + \delta\eta)$ of every considered pixel (ξ, η) assuming that the optical flow is locally constant [Lucas and Kanade, 1981]. We give appropriate weights $w(\delta\xi, \delta\eta)$ to the application of BCCE at every pixel so that the influence is higher in the center of the neighborhood. The solution is obtained by minimizing

$$\sum_{\delta\xi,\delta\eta} w(\delta\xi, \delta\eta)^2 \left(J_\xi(\xi + \delta\xi, \eta + \delta\eta)u^l + J_\eta(\xi + \delta\xi, \eta + \delta\eta)v^l + J_t(\xi + \delta\xi, \eta + \delta\eta)\right)^2, \tag{6}$$

with respect to (u^l, v^l). If $\boldsymbol{\xi}_i = (\xi + \delta\xi, \eta + \delta\eta)$, $i = 1 \dots n$ are the n points of a neighbourhood and if

$$W = \operatorname{diag}(w(\boldsymbol{\xi_1}) \dots w(\boldsymbol{\xi_n})) \qquad A = (\nabla J(\boldsymbol{\xi_1}) \dots \nabla J(\boldsymbol{\xi_n}))^T \qquad b = -(J_t(\boldsymbol{\xi_1}) \dots J_t(\boldsymbol{\xi_n}))^T \qquad \boldsymbol{u^l} = (u^l, v^l)^T \tag{7}$$

then the above minimization problem is equivalent to the minimization of $\|W(A\boldsymbol{u^l} - b)\|$ which is solved by singular value decomposition.

The second method is an extension of the first one. It allows the linear variation of the flow inside the neighborhood (cf. similar methods in [Kearney et al., 1987]) enabling, thus, an estimation of flow as well as its spatial derivatives. The equation

$$J_\xi(\xi + \delta\xi, \eta + \delta\eta)(u^l + u^l_\xi\delta\xi + u^l_\eta\delta\eta) + J_\eta(\xi + \delta\xi, \eta + \delta\eta)(v^l + v^l_\xi\delta\xi + v^l_\eta\delta\eta) + J_t(\xi + \delta\xi, \eta + \delta\eta) = 0 \tag{8}$$

is applied for every $(\delta\xi, \delta\eta)$ inside a neighborhood yielding an overconstrained system with six unknowns. However, both these already known methods use assumptions about local constancy or affinity of the optical flow that do not reflect the harmonic variation of both flow components (3) with the angle η in the log-polar plane. We will delve into this problem in the next section. We finish this section giving abbreviations to the presented methods. As of now we will call LCT the method based on the Local Constancy of the flow in the Transformed image (polar or log-polar) and LAT to the method based on the Local Affinity of the flow in the transformed image.

3 The polar deformation

The first source of error in the log-polar optical flow field is due to the polar deformation of the gray-value function. The polar transformation maps straight edges into curved edges (see Fig. 2 (left)) enabling thus the computation of both components of the optical flow at points without curvature in the original cartesian image. This superficial elimination of the aperture problem introduces optical flow values with a large error regarding the expected motion field. In this section we will first transform the neighborhood–gradient approach into a second derivative method in order to study analytically the rank of the resulting linear system. Then we will prove the expected fact that the linear system in the polar plane has full rank even if the Hessian matrix in the cartesian plane is singular. The aperture problem is always eliminated by introducing some assumption on the local variation of the flow. In this sense, we assume the local constancy of the back-transformed flow in the cartesian plane. We, thus, face the aperture problem in the cartesian place. The resulting matrix of gray-value derivatives in the polar plane will be proved to have the same rank as the cartesian Hessian. We do not here consider the error due to subsampling introduced by the low-angular resolution of the log-polar plane.

We denote by $E(\rho, \eta)$ the gray-value function and by (u^p, v^p) the optical flow in the polar domain. Using the Brightness Change Constraint Equation and the assumption that the polar flow is locally constant and applying the Taylor expansion to the derivatives at the positions $(\rho + \delta\rho, \eta + \delta\eta)$ yields the overconstrained system

$$E_\rho u^p + E_\eta v^p + E_t = 0 \qquad E_{\rho\rho} u^p + E_{\eta\rho} v^p + E_{t\rho} = 0 \qquad E_{\rho\eta} u^p + E_{\eta\eta} v^p + E_{t\eta} = 0 \tag{9}$$

where we have omitted the resulting weights on each equation. We are interested in the 2×2 coefficient matrix of the second and third equation which is the Hessian of the polar gray-value function $E(\rho, \eta)$. By twice differentiating $E(\rho, \eta)$ it can be easily proved that

$$\begin{pmatrix} E_{\rho\rho} & E_{\eta\rho} \\ E_{\rho\eta} & E_{\eta\eta} \end{pmatrix} = \frac{\partial(x,y)}{\partial(\rho,\eta)}^T \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix} \frac{\partial(x,y)}{\partial(\rho,\eta)} + I_x \begin{pmatrix} \frac{\partial^2 x}{\partial\rho^2} & \frac{\partial^2 x}{\partial\rho\partial\eta} \\ \frac{\partial^2 x}{\partial\rho\partial\eta} & \frac{\partial^2 x}{\partial\eta^2} \end{pmatrix} + I_y \begin{pmatrix} \frac{\partial^2 y}{\partial\rho^2} & \frac{\partial^2 y}{\partial\rho\partial\eta} \\ \frac{\partial^2 y}{\partial\rho\partial\eta} & \frac{\partial^2 y}{\partial\eta^2} \end{pmatrix}, \tag{10}$$

with

$$\frac{\partial(x,y)}{\partial(\rho,\eta)} = \begin{pmatrix} \frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\eta} \\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\eta} \end{pmatrix}.$$

All the matrices with derivatives of (x, y) with respect to (ρ, η) have full rank up to two values of η for the second derivatives. Hence, the singularity of the cartesian Hessian does not lead to the singularity of the polar Hessian which also depends on the cartesian gradient. It is plausible to suppose that the smallest singular value of the coefficient matrix – used as a confidence measure – will be higher in the system (9) than in the equivalent system in the cartesian plane. This fact causes the acceptance of erroneous optical flow values.

We proceed by substituting the assumption of local flow constancy with local flow constancy before applying the polar transformation. We differentiate the Brightness Change Constraint Equation assuming that the spatial derivatives of (u, v) vanish and we obtain a system with new coefficient matrix

$$\begin{pmatrix} E_{\rho\rho} & E_{\eta\rho} - \frac{E_\eta}{\rho} \\ E_{\rho\eta} - \frac{E_\eta}{\rho} & E_{\eta\eta} + \rho E_\rho \end{pmatrix} = \frac{\partial(x,y)}{\partial(\rho,\eta)}^T \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix} \frac{\partial(x,y)}{\partial(\rho,\eta)} \tag{11}$$

which proves that the singularity of the cartesian Hessian matrix is the necessary and sufficient condition for the singularity of the new coefficient matrix. Hence, the second derivative system with coefficient matrix (11) does not introduce erroneous values of the optical flow like the system (9). Based on this fact we are going to construct in the next section a gradient-sampling method like (6).

4 Assumptions concerning the cartesian plane

In this section we transfer the assumptions about constancy and affinity of the *cartesian* flow to the application of the BCCE in the neighborhood pixels $(\xi + \delta\xi, \eta + \delta\eta)$

$$\begin{pmatrix} J_\xi(\xi + \delta\xi, \eta + \delta\eta) & J_\eta(\xi + \delta\xi, \eta + \delta\eta) \end{pmatrix} \begin{pmatrix} u^l \\ v^l \end{pmatrix} = -J_t(\xi + \delta\xi, \eta + \delta\eta) \tag{12}$$

allowing the log-polar flow to vary

$$\begin{pmatrix} u^l \\ v^l \end{pmatrix} = \frac{1}{\rho_0 a^{\xi+\delta\xi}} \begin{pmatrix} \frac{\cos(\eta+\delta\eta)}{\ln a} & \frac{\sin(\eta+\delta\eta)}{\ln a} \\ -\sin(\eta+\delta\eta) & \cos(\eta+\delta\eta) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \tag{13}$$

We expect that the resulting system does not have full rank at the positions where the aperture problem appears in the cartesian plane and in general that the resulting coefficient matrix has lower singular values than the coefficient matrix A in (7). We will call this method the Local Constancy in the Cartesian image (LCC) method. The next step is straightforward. We allow the cartesian flow to vary linearly in the local neighborhood. Combining Cartesian local linear variation with equations (4) and (5) for the Log-polar BCCE, we get

$$J_t = -\frac{1}{\rho a^{\xi+\delta\xi}} \begin{pmatrix} J_\xi & J_\eta \end{pmatrix} \begin{pmatrix} \frac{\cos(\eta+\delta\eta)}{\ln a} & \frac{\sin(\eta+\delta\eta)}{\ln a} \\ -\sin(\eta+\delta\eta) & \cos(\eta+\delta\eta) \end{pmatrix} \begin{pmatrix} u + u_x\delta x + u_y \delta y \\ v + v_x \delta x + v_y \delta y \end{pmatrix} \tag{14}$$

We call this method the Local Affinity in the Cartesian image (LAC) method. By setting $\xi = 0$ and $\eta = 0$ we obtain the equations derived by [Tunley and Young, 1994] as a special case.

5 The logarithmic subsampling

In this section we present first steps towards understanding the effects of the logarithmic subsampling on the computation of the spatiotemporal derivatives. We restrict our study on an 1D gray-value function $g(\rho)$ on the discrete domain $\rho = \rho_0..\rho_{max}$ and its logarithmically subsampled version $\gamma(\xi)$ defined on $\xi = 0..N$ as if both were the radius and its log-polar map of a 2D-image without polar subsampling. We introduce an intermediate function $\lambda(\xi)$ [Porat and Zeevi, 1988] obtained by exact coordinate transformation $\lambda(\xi) = g(\rho_0 b^\xi)$ where the logarithm's basis b is chosen in such a way that the discrete original signal is transformed without loss. This means that the basis b must be less equal than the coordinates ratio $b = \rho_{max}/(\rho_{max} - 1)$ so that even the gray-value of the most peripheral pixel is exactly transformed. The lossless signal is sparse and has a dimension $M = \ln(\rho_{max}/\rho_0)/\ln b$ much greater than the original signal so we interpolate the intermediate valueless pixels with the neighboring values. The logarithmic signal $\gamma(\xi)$ is then obtained by the three steps of linear shift-invariant low pass filtering, subsampling, and shrinking as in a layer transition step in a regular pyramid. The subsampling interval is M/N where M is the dimension of the lossless signal as above and $N = \ln(\rho_{max}/\rho_0)/\ln a$ is the final resolution of the logarithmic signal $\gamma(\xi)$,

The introduction of the lossless image enables the study of the decimation effects with spectral methods. First, we become able to design the appropriate low-pass filters to suppress the energy above half of the subsampling frequence (future work). Second, considering the optical flow u_λ in the lossless signal as the flow with the lowest error we can use it as the reference for the error introduced by subsampling in the flow u_γ. If we use a differential technique as in the previous sections the flow of the 1D lossless and subsampled signals is $u_\lambda = -\frac{\lambda_t}{\lambda_\xi}$ and $u_\gamma = -\frac{\gamma_t}{\gamma_\xi}$, respectively.

The temporal gradient γ_t of the logarithmic signal is the response of the lossless temporal gradient to the low-pass filtering step. To study the spatial gradient we first point out that the gradient is a bandpass e.g. the first derivative of a Gaussian here. We assume that the low pass filter has the appropriate antialiasing characteristics. The spectrum of the lossless image is stretched out by the subsampling and shrinking steps whereas the spectrum of the spatial gradient is the same before and after subsampling. The error in the spatial gradient depends on the amplification or attenuation of frequency contributions according to the contributions under the frequency support of the gradient. If the central frequency of the gradient in the lossless image is larger than the low pass cut frequence then applying the same spatial gradient after subsampling will lead to decreasing of the response. We conjecture that this decreasing of γ_ξ might be the reason for the systematic overestimation of the length of the optical flow γ_t/γ_ξ. We illustrate these facts in Fig. 1. The lossless signal is the stretched signal in Fig. 1 (left), the shrinked one is after low-pass filtering and subsampling. We show in the middle and right of Fig. 1 the magnitude spectrum, the transfer function of the gradient, and the spectrum of the response for the lossless and the signal after subsampling, respectively.

6 Experimental comparison

In this section we present results that compare the four flow computation methods described and show the effect of the polar deformation before the logarithmic subsampling. In all figures the log-polar images are drawn such that the η-axis is the horizontal axis and the ξ-axis is the vertical axis pointing downwards. To interpret the log-polar images we note that the angle η is measured beginning counterclockwise from the y-axis that is pointing downwards. So moving horizontally in the log-polar plane we first see the transformed lower right quadrant, then the transformed upper right quadrant and so on. The compression rates obtained by the log-polar transformation are about 1:25 for all analyzed sequences. The local constancy and the local affinity assumption are applied in 5×5 neighborhoods. The least squares problems are solved with the Singular Value Decomposition and a threshold is applied to the smallest singular value as a reliability criterion.

We first tested the Local Constancy methods on the polar transform of an image sequence consisting of black squares on white background moving with uniform velocity of (1,1). The polar transformed image is shown on the left of Fig. 2. The images on the middle and left of Fig. 2 show the smallest singular value of the linear systems using the Local Constancy assumption in the Polar (LCT) and the Cartesian (LCC) image, respectively. As we already expected from eq. 13 the LCC method produces a system with lower singular values. Thus, the aperture problem should not be overcome by introducing non-existing gray-value curvature.

To test the methods proposed in sections 2 and 4 we used the real sequence "Marbled Block" [1] [Otte and Nagel, 1994] with known ground-truth values for the motion field. The original image of the sequence and its log-polar transform are

[1] Created by Michael Otte at University of Karlsruhe and FhG-IITB, Germany.

shown in Fig. 3 left and middle, respectively. In Fig. 3 (right) we show the flow field computed with the LAC method in sec. 4. The error measures used are the relative error and the angle between $(u, v, 1)$ and $(\hat{u}, \hat{v}, 1)$ where (u, v) and $(\hat{u}, \hat{v})$ are the ground-truth and estimated flow, respectively. Furthermore, we compare not only to the transformed ground-truth flow but also to the transformation (3) of a flow estimated as usual in the cartesian plane. The error with respect to the latter should be considered as a lower bound for the error expected. The density is the fraction of the estimates with smallest singular value above a threshold which varies for the four estimation techniques.

We first present (Tab.1) the angle- and relative errors for the *polar* transform of the sequence obtained with angular resolution of 512 samples per 360 degrees and radial resolution equal to the original (256). As expected the error is lower when we compare the polar estimates to the transformed cartesian estimates. Regarding the local constancy assumption applied on the polar (LCT) and the cartesian (LCC) plane the errors are about the same for the same density. However, this density is achieved for appropriately chosen high threshold for the LCT method. The superiority of the LCC method is shown if we compare it to the performance of the LCT method with the same threshold (LCT-thr).

	Transformed ground truth			**Transform of the cartesian estimate**		
Technique	av. ang. err.	av. rel. err.	density	av. ang. err.	av. rel. er	density
LCT	5.64729	18.74557	0.54977	3.68909	12.36976	0.54977
LCT-th	7.26917	23.65154	0.77566	3.78853	13.56477	0.77566
LCC	6.02655	19.11052	0.48914	3.63529	12.04593	0.48914

Table 1: Error statistics for the polar transform of the "Marbled Block" sequence (see text for explanation).

The log-polar transform of the "Marbled Block" sequence is obtained with angular resolution of 128 samples/360 degrees and radial resolution of 45 samples for the radial range [32..356]. It should be noted that the angular resolution is one-fourth of the angular resolution of the polar transform, therefore the error is due to both the logarithmic and the polar subsampling. The errors with respect to the transformed ground-truth values are not significantly higher than to the transformed estimates what means that the effects of noise and poor gray-value structure in the original are inferior to the subsampling effects. The errors are shown for about the same density in order to compare the performance at points where the coefficient matrices are regular in all methods. We are thus strict to the methods using assumptions in the cartesian plane. Even with such a high relative error we can use the log-polar transform for 3D-analysis. We show in [Daniilidis, 1994] that the 3D-translation direction can be computed with only 5 degrees error.

	Transformed ground truth			**Transform of the cartesian estimate**		
Technique	av. ang. err.	av. rel. er	density	av. ang. err.	av. rel. er	density
LCT	5.34357	34.53929	0.71908	4.26153	32.02543	0.71908
LCC	5.79178	38.03186	0.56144	4.80384	34.88554	0.56144
LAT	5.30501	34.45435	0.75163	4.29278	32.22566	0.75163
LAC	5.10992	34.02044	0.67358	4.28657	32.28787	0.67358

Table 2: Error statistics for the log-polar transform of the "Marbled Block" sequence (see text for explanation).

7 Conclusion

The log-polar transform of an image sequence provides both a very efficient coding and a useful representation for motion tasks. To compute the optical flow as an intermediate step for solving such tasks we have to overcome the problems of the polar deformation and the irregular subsampling of the image. As we are trying to exploit this flow for 3D-motion estimation we are interested in what amount of motion information is still preserved after the log-polar transform. According to a theoretical analysis we showed that the polar transform introduces erroneous flow values due to the fictitious gray-value curvature in the polar image. In the analysis of subsampling we proposed the introduction of the lossless logarithmic image in order to enable classical spectral techniques. We indicated as probable error sources the aliasing and the characteristics of the gradient. The next step consists of the design of appropriate filters to attenuate the aliasing effects and to reduce the error in the computation of the spatial gradient.

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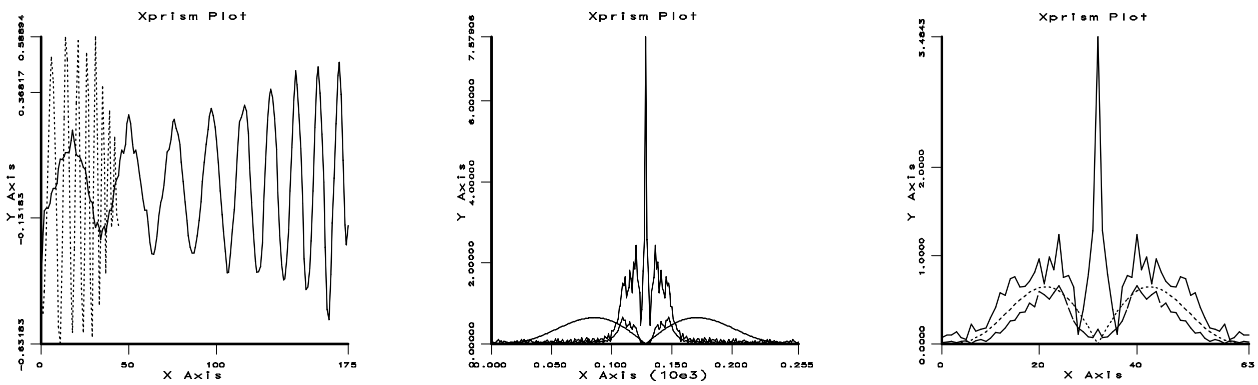

Figure 1: The lossless and the subsampled signal (left), the magnitude spectra of the signal, the gradient filter, and the filter response, for the lossless signal (middle) and the signal after subsampling(right).

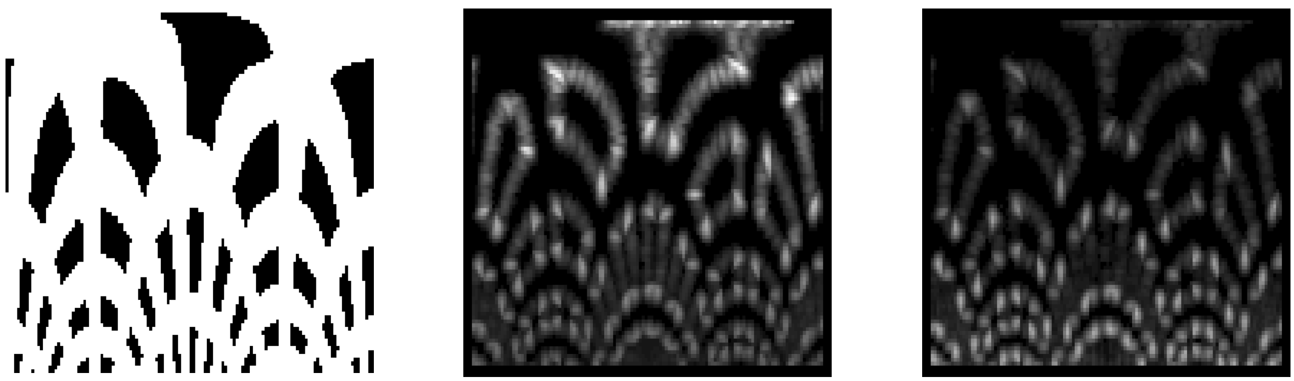

Figure 2: The polar transform of an image with black rectangles (left), and the smallest singular value of the linear system resulting from the LCT (middle) and the LCC (right) assumptions, respectively. The horizontal axis is the angular axis and the vertical axis is the radial axis.

Figure 3: The cartesian original image 512x512 pixels(left), its log-polar transform 77x128 pixels shown magnified (middle), and the computed optical flow (right) of the "Marbled Block" sequence. The horizontal axis is the angular axis and the vertical axis is the logarithmic radial axis.