A New Extension of Linear Signal Processing for Estimating Local Properties and Detecting Features*

Michael Felsberg and Gerald Sommer

Christian-Albrechts-University of Kiel
Institute of Computer Science and Applied Mathematics
Cognitive Systems
Preußerstraße 1-9, 24105 Kiel, Germany
Tel: +49 431 560433, Fax: +49 431 560481
{mfe,gs}@ks.informatik.uni-kiel.de

Abstract. The analytic signal is one of the most capable approaches in one-dimensional signal processing. Two-dimensional signal theory suffers from the absence of an isotropic extension of the analytic signal. Accepting the fact that there is no odd filter with isotropic energy in higher dimensions, one tried to circumvent this drawback using the one-dimensional quadrature filters with respect to several preference directions. Disadvantages of these methods are an increased complexity, the loss of linearity and a lot of different heuristic approaches. In this paper we present a filter that is isotropic and odd, which means that the whole theory of local phase and amplitude can directly be applied to images. Additionally, a third local property is obtained which is the local orientation. The advantages of our approach are demonstrated by a stable orientation detection algorithm and an adaption of the phase congruency method which yields a superior edge detector with very low complexity.

1 Introduction

The analytic signal and the corresponding filters (quadrature filters) are well suited for detecting local properties and features of signals. The so called split of identity is the fundamental property of quadrature filters. The local amplitude of the filter response corresponds to a quantitative measure of a structure (including the contrast) and the local phase corresponds to a qualitative measure of a structure (step, peak, etc.).

Unfortunately, the analytic signal is only defined for one-dimensional signals. Quadrature filters consist of two filters: an even and an odd bandpass filter. For images, the fundamental problem is to find an odd filter with an isotropic energy distribution. Up to now, it has been commonly accepted that no such filter exists [10]. In the frequency domain, the corresponding problem is to find positive and negative frequencies in a two-dimensional domain which is also impossible according to [6].

Both statements are true if the odd filter is constrained to be scalar valued. Whereas if the filter is allowed to be vector valued, one can find an odd filter with isotropic

* This work has been supported by German National Merit Foundation and by DFG Graduiertenkolleg No. 357 (M. Felsberg) and by DFG So-320-2-2 (G. Sommer).

energy. The corresponding (vector valued) transfer function is also odd and has unit magnitude. It is therefore a multidimensional generalization of the sign-function. This approach is straightforward in the framework of geometric algebra [3]. Furthermore, it is related to the structure tensor, but without being nonlinear [4].

Using this 'real' two-dimensional analytic signal (the monogenic signal), it is easy to apply approaches based on the one-dimensional analytic signal to two-dimensional signals without sampling the orientation or using adaptive filtering (e.g. [7]). As an example, we applied the phase congruency [10, 11] in order to obtain an edge detector which is independent of the local contrast.

Additionally to the information of local amplitude and local phase, the monogenic signal includes geometric properties, e.g. the orientation of an intrinsically one-dimensional structure in the two-dimensional plane. In this paper, we also present a stable algorithm which estimates the local orientation of a signal.

Throughout this paper, we avoid to use terms of geometric algebra, because it is not very widely spread in the community, yet.

2 Theoretic Framework

As we pointed out in the introduction, no odd isotropic filter can be constructed for two dimensions if we are restricted to scalar valued filters. For vector valued filters, however, it is possible. Without going into theoretic details, we introduce the following filters in frequency domain:

$$H_1(u_1, u_2) = i\frac{u_1}{\sqrt{u_1^2 + u_2^2}} \quad \text{and} \quad H_2(u_1, u_2) = i\frac{u_2}{\sqrt{u_1^2 + u_2^2}} \ . \tag{1}$$

The vector $\boldsymbol{H} = (H_1, H_2)$ has unit length in any direction (i.e. it is isotropic) because $|(H_1, H_2)| = \sqrt{|H_1|^2 + |H_2|^2} = 1$ and it is odd[1] because a reflection through the origin yields $\boldsymbol{H}(-u_1, -u_2) = -\boldsymbol{H}(u_1, u_2)$.

If we reduce $\boldsymbol{H}$ to one dimension, we obtain $H(u) = i\frac{u}{|u|} = i\,\mathrm{sign}(u)$ which is the transfer function of the Hilbert transform (e.g. [8, 9]). The vector $\boldsymbol{H}(u_1, u_2)$ is the transfer function of a transform which is called the *Riesz transform* [15, 14]. The spatial representation of $\boldsymbol{H}$ is the convolution kernel of the Riesz transform and it reads

$$(h_1(x_1, x_2), h_2(x_1, x_2)) = \left(-\frac{x_1}{2\pi(x_1^2 + x_2^2)^{\frac{3}{2}}}, -\frac{x_2}{2\pi(x_1^2 + x_2^2)^{\frac{3}{2}}}\right) \ . \tag{2}$$

If we combine a signal f with its Riesz transform according to

$$f_M(x_1, x_2) = (\, f(x_1, x_2)\, ,\, (h_1 * f)(x_1, x_2)\, ,\, (h_2 * f)(x_1, x_2)\,) \tag{3}$$

[1] At this point, one has to define which signals are odd. If only one constraint of oddness should exist, the only possibility in 2D is the oddness with respect to a reflection through the origin. Oddness with respect to reflections in arbitrary lines can only be handled using spherical harmonics.

we obtain the multi-dimensional generalization of the analytic signal which is called the *monogenic signal*.

The local amplitude of the monogenic signal is the vector norm of f_M:

$$A_f = \sqrt{f^2 + (h_1 * f)^2 + (h_2 * f)^2} \ , \tag{4}$$

as in the case of the analytic signal. The next thing to be redefined is the phase. Obviously, for a triple we need two phases instead of one. Using the standard spherical coordinates, we obtain the equations

$$f = A_f \cos(\varphi) \tag{5}$$
$$(h_1 * f) = A_f \sin(\varphi)\cos(\theta) \tag{6}$$
$$(h_2 * f) = A_f \sin(\varphi)\sin(\theta) \ , \tag{7}$$

where $\varphi \in [0; 2\pi)$ and $\theta \in [0; \pi)$.

For an intrinsically one-dimensional signal[2] (i.e. $f(x_1, x_2) = g(x_1 n_1 + x_2 n_2)$, see [12]), one can prove that φ is identical to the local phase of $g(x)$ and that θ is the orientation[3] of the vector (n_1, n_2). Therefore, φ and θ are called the local phase and local orientation, respectively. The spherical coordinates are illustrated in Fig. 1.

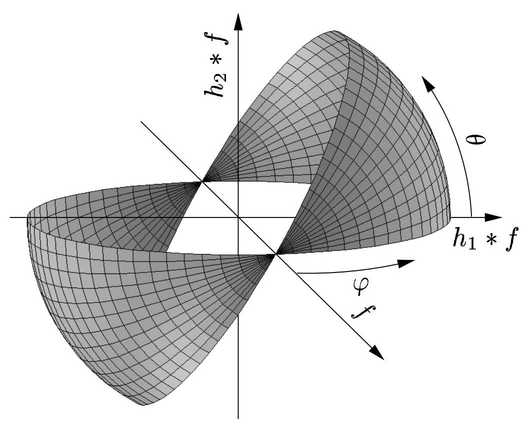

Fig. 1. Spherical coordinates

In practical applications, the infinite impulse response of the Hilbert kernel is reduced to a local filter mask by use of a bandpass filter. The resulting filters (the bandpass filter and the bandpass filtered Hilbert kernel) are called (a pair of) quadrature filters (see e.g. [9]). The same idea can be applied for the Riesz transform. The resulting filters form a triple of spherical quadrature filters (SQF): the radial bandpass filter and the radial bandpass filtered kernels h_1 and h_2.

---

[2] Basically, it is sufficient to discuss the behavior of a linear approach wrt. intrinsically one-dimensional signals, because every signal can be decomposed in i1D functions (e.g. Fourier transform).

[3] Note the difference between direction and orientation (e.g. [5]). If the *direction* of (n_1, n_2) is greater than π, we can replace it by $(-n_1, -n_2)$ and simultaneously replace $g(x)$ by $g(-x)$. The latter yields a negation of the local phase, which is consistent with spherical coordinates.

A good choice for the bandpass filter is a lognormal filter (which is a Gaussian filter if considered in logarithmic scale). For the applications in the following section we always used the radial bandpass filter with the transfer function

$$B(u_1, u_2) = \exp\left(-\frac{(\log(\sqrt{u_1^2+u_2^2}/2^k))^2}{2(\log(w))^2}\right) \quad , \tag{8}$$

where k indicates the octave (the center frequency) and the bandwidth-parameter w is either 0.55 or 0.41 (about two and three octaves). The lognormal filter possesses an crucial advantage compared to the Gaussian bandpass filter: one can construct filters of arbitrary bandwidth without introducing a DC component [10]. Therefore, we use the following triple of SQF: $(\,B\,,\,H_1B\,,\,H_2B\,)$.

In Fig. 2, the bandpass characteristic and the isotropy of these spherical quadrature filters is illustrated (for isotropy of filters see [1, 13]). The modulated ring is a typical image for testing the isotropy of an operator. So is the Siemens star, but additionally, the latter gives information about the passbands.

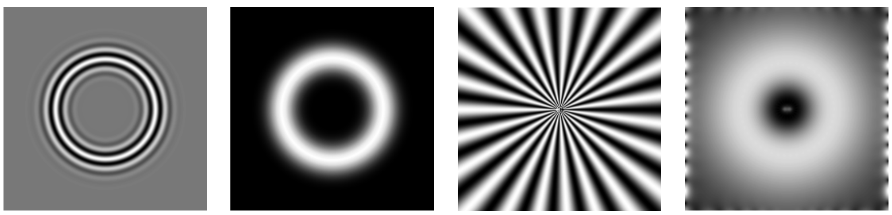

Fig. 2. Modulated ring (outer left), filter response (middle left), Siemens star (middle right), filter response (outer right)

3 Applications

As already mentioned, the monogenic signal contains information about the local orientation of an image. It can be obtained by

$$\theta = \operatorname{atan1}\left(\frac{h_2 * f}{h_1 * f}\right) \quad , \tag{9}$$

where $\operatorname{atan1}$ is the modified arc tangent which results in angles in $[0;\pi)$.

Though the analytic and the monogenic signal are both very stable approaches, the estimation of the local orientation by use of (9) becomes ill-conditioned if the local phase is nearly zero (or 2π, see (5)-(7)). The confidence to the measurement of θ is approximated by $\sin^2(\varphi)$. Therefore, singular values of θ can be eliminated by the following non-linear filter:

$$\theta_s = \arg(\,\mathrm{box}_k * (f_1 + if_2)^2)/2 = \arg(\,\mathrm{box}_k * (\sin^2(\varphi)\exp(i2\theta)))/2 \quad , \tag{10}$$

where box_k is the $k \times k$ box-filter ($k = 3$ is sufficient) and $f_j = (h_j * f)/A_f$.

The images in Fig. 4 illustrate the results of the described algorithm. The image of a circle (Fig. 3) is good for testing the performance of orientation estimating operators because all orientations are included and the correctness can easily be verified by evaluating the orientation on the circle, which must be a (piecewise) linear function. While the arc tangent of the partial derivatives (obtained from an optimized Sobel operator [9]) and the direct estimation of θ according to (9) show a similar quantitative error, the result of (10) is nearly perfect (error less than $0.3°$).

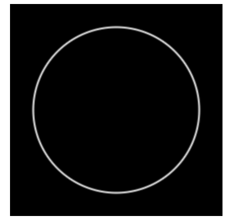

Fig. 3. circle

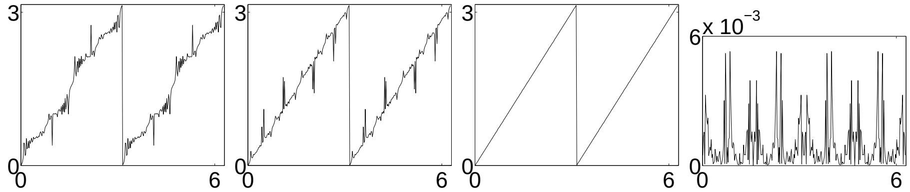

Fig. 4. Orientation estimation (from left to right): response of optimized Sobel operator, orientation of SQF, response of (10), error of (10)

Another application for the spherical quadrature filters is the detection of edges and lines. For this application, we have to distinguish between two cases:

1. the images have uniform contrast, e.g. binary images, hand writing, or
2. the images have arbitrary local contrast (and we want also to detect edges of low contrast).

In the first case, we can simply take the local amplitude as a measure for edges and lines. Since the contrast is constant, we can take a fixed value for thresholding, e.g. 50% of the maximal contrast.

The second case is much more difficult. If we take the local amplitude as a measurement, the problem is how to fix the threshold – if it is too low, we get a lot of false-positives, if it is too high, we miss the edges with low local contrast.

Since the phase information is independent of the local amplitude, one can use the so called *phase congruency* for detecting edges independently of the local contrast (see e.g. [10]). The idea is the following (1D): quadrature filters are applied for different scales. The responses of the filters are drawn head to tail. The quotient of the length of the resulting vector and the length of the path is a value between zero and one and it is called the phase congruency.

$$PC = \max_{\bar{\varphi}} \frac{\sum_n A_n \cos(\varphi_n - \bar{\varphi})}{\sum_n A_n} \tag{11}$$

A phase congruency of value one means that there is an edge (or a line), a phase congruency of value zero means that there is no structure.
The phase congruency approach has several problems:

1. if the local amplitude is nearly zero for all scales, the phase congruency becomes unstable (noise and numerical errors),
2. if the local amplitude of one scale is much higher than in the other scales, the congruency is always close to one (missing frequency spread),
3. the phase congruency is a function proportional to the sum of cosines of the phase differences and the cosine function is (nearly) one for a relatively wide range of angles (nonlinearity wrt. the phase), and
4. the approach becomes much more complicated and numerically more complex for the 2D case (sampling of orientations).

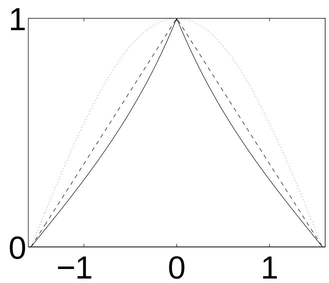

Fig. 5. Approximation of a linear function with cosine and sine function, cosine: dotted, linear function: dashed, approximation: solid

The last problem is solved by our approach of the 2D monogenic signal. The first problem can be solved by introducing an additional constant in the denominator of (11) and by subtracting the estimated noise energy (see [10]). The second problem is solved in [11] by the introduction of a function which measures the frequency spread. The third problem can be solved by replacing the cosine function with some function of the cosine and the absolute value of the sine (12), which is nearly linear in wide range of angles (see Fig. 5).

If we take the spherical quadrature filter responses of two different scales we can calculate the cosine of the angle between these vectors by the scalar product and the absolute value of the sine by the magnitude of the cross product. If the first SQF response is denoted by f_1 and the second one is denoted by f_2, we obtain

$$PC = \frac{\cos(\Delta\varphi)}{|\sin(\Delta\varphi)| + 1} = \frac{f_1 \cdot f_2}{|f_1 \times f_2| + |f_1||f_2|} \quad , \tag{12}$$

which also takes into account the frequency spread because of the products.

Since our bandpass filter covers three octaves, we can cover six octaves with two filters. In our opinion, this is enough for detecting structures which can be called edges or lines. Therefore, we can replace the sums in (11) by (12). The resulting measurement responds to nearly all edges and lines in an images with values close to one.

The images in Fig. 6 show the results of the algorithm of Kovesi (which is much better than conventional edge detectors e.g. the Canny detector [11]) and of our method (upper threshold: 0.75, lower threshold: 0.35). Our results are similar to those of the Kovesi approach. There are three main differences: first, our approach uses only a rough estimation of the noise energy. Therefore, a few structures with very low energy are missed by our method. Nevertheless, some edges are only found by our detector (e.g. in the first column, the left roof). On the other hand, our algorithm responds less to noise. Secondly: the amount of artifacts. Our algorithm produces less artificial edges and no 'double corners' (e.g. top of chimney). In this context, note that an edge detector which does not make use of semantic information or high level knowledge can only detect

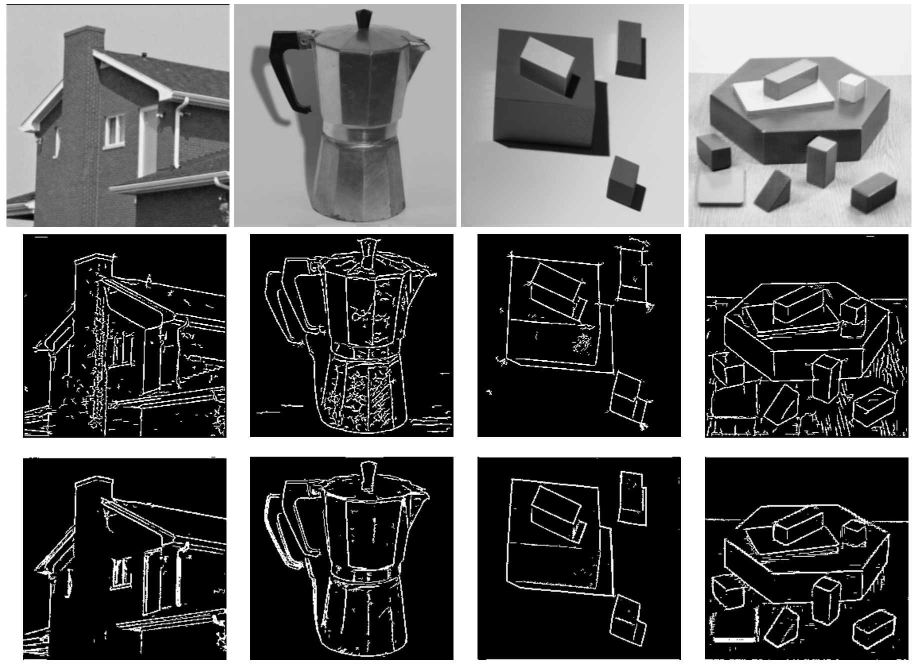

Fig. 6. Upper row: original images, middle row: phase congruency (Kovesi), bottom row: phase congruency (SQF), 2nd and 3rd image from INRIA-Syntim ©

edges which are present in the local gray value information. Finally, our algorithm is more than sixteen times faster[4].

4 Conclusion

We have shown that the monogenic signal and the spherical quadrature filters can be used to apply the approach of phase congruency to images without sampling the orientation. The resulting algorithm is an edge detector with very low complexity and which is independent of the local contrast. The approach has been compared to others in a qualitative way, a quantitative investigation [2] will follow.

Furthermore, we have presented a simple and stable algorithm which evaluates the local orientation of an image using the monogenic signal. Since the monogenic signal posses all the properties of the analytic signal and since it is linear, it is easy to convert the known one-dimensional approaches which make use of the analytic signal, to two dimensions.

[4] Elapsed times on PII, 233MHz, MatLab, 256x256 without thresholding: 60 s (Kovesi), 3.7 s (our approach).

Similar results can be obtained for example for texture analysis and corner detection which will be topics of our further investigations. We hope that we have convinced the reader of the capability of the monogenic signal. Furthermore, since the new theory can be seamlessly embedded into the framework of geometric algebra, we belief that the latter is superior to complex numbers for multi-dimensional signal processing.

References

[1] BRADY, J. M., AND HORN, B. M. P. Rotationally symmetric operators for surface interpolation. *Computer Vision, Graphics, and Image Processing 22*, 1 (April 1983), 70–94.

[2] CANNY, J. A computational approach to edge detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence 8*, 6 (November 1986), 679–698.

[3] FELSBERG, M., AND SOMMER, G. The multidimensional isotropic generalization of quadrature filters in geometric algebra. In *Proc. Int. Workshop on Algebraic Frames for the Perception-Action Cycle, Kiel* (2000), G. Sommer and Y. Zeevi, Eds., Lecture Notes in Computer Science, Springer-Verlag, Heidelberg. accepted.

[4] FELSBERG, M., AND SOMMER, G. Structure multivector for local analysis of images. Tech. Rep. 2001, Institute of Computer Science and Applied Mathematics, Christian-Albrechts-University of Kiel, Germany, February 2000.

[5] GRANLUND, G. H. Hierarchical computer vision. In *Proc. of EUSIPCO-90, Fifth European Signal Processing Conference, Barcelona* (1990), L. Torres, E. Masgrau, and M. A. Lagunas, Eds., pp. 73–84.

[6] GRANLUND, G. H., AND KNUTSSON, H. *Signal Processing for Computer Vision*. Kluwer Academic Publishers, Dordrecht, 1995.

[7] HAGLUND, L. *Adaptive Multidimensional Filtering*. PhD thesis, Linköping University, 1992.

[8] HAHN, S. L. *Hilbert Transforms in Signal Processing*. Artech House, Boston, London, 1996.

[9] JÄHNE, B. *Digitale Bildverarbeitung*. Springer, Berlin, 1997.

[10] KOVESI, P. *Invariant Measures of Image Features from Phase Information*. PhD thesis, University of Western Australia, 1996.

[11] KOVESI, P. Image features from phase information. *Videre: Journal of Computer Vision Research 1*, 3 (1999).

[12] KRIEGER, G., AND ZETZSCHE, C. Nonlinear image operators for the evaluation of local intrinsic dimensionality. *IEEE Transactions on Image Processing 5*, 6 (June 1996), 1026–1041.

[13] MERRON, J., AND BRADY, M. Isotropic gradient estimation. In *IEEE Computer Vision and Pattern Recognition* (1996), pp. 652–659.

[14] NABIGHIAN, M. N. Toward a three-dimensional automatic interpretation of potential field data via generalized Hilbert transforms: Fundamental relations. *Geophysics 49*, 6 (June 1984), 780–786.

[15] STEIN, E., AND WEISS, G. *Introduction to Fourier Analysis on Euclidean Spaces*. Princeton University Press, New Jersey, 1971.