# Fast Local Estimation of Optical Flow Using Variational and Wavelet Methods

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**Abstract.** We present a framework for fast (linear time) local estimation of optical flow in image sequences. Starting from the commonly used brightness constancy assumption, a simple differential technique is derived in a first step. Afterwards, this approach will be extended by the application of a nonlinear diffusion process to the flow field in order to reduce smoothing at motion boundaries. Due to the ill-posedness of the determination of optical flow from the related differential equations, a Wavelet-GALERKIN projection method is applied to regularize and linearize the problem.

**Keywords:** optical flow estimation, wavelet methods, variational methods

## 1 Introduction

The main purpose of this article is the presentation of a fast algorithm for the determination of optical flow fields from given image sequences, which is a very important task in image processing. The question of a *reliable estimation* has been adressed by several authors and just as many different approaches were made (see, e.g. [9], [20], [7], [23], [1] and many others mentioned therein). Our approach falls into the large group of differential methods and the flow is computed locally, because here we are mostly interested in fast computations — nonlocal approaches lead to very large equation systems, that can only be solved iteratively, which is usually more expensive. On the other hand, nonlocal methods give commonly better results and are better suited to handle large displacements. Howerver, the presented methods are not limited to the local case and might also be applied to global flow calculations.

The paper is organized as follows: in Section 2, we will briefly recall the differential flow model and the most important invariant, which occurs as *brightness constancy assumption*. Additionally, we will show, how a Wavelet-GALERKIN

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procedure helps to linearize and regularize the associated flow equations in order to get stable estimations ([11]) and we argue, how certain properties of MRA-Wavelets may be utilized in this context. Since this simple model has some drawbacks, in particular blurring at motion edges and numerical instabilites at image points with very slow motion, we derive a certain extension in the following section; this extension is closely related to nonlinear diffusion processes, which are also briefly presented there. Finally, we will show some experimental results of the implementations in Section 4 and we discuss the advantages and disadvantages of this method in comparison to existing approaches of optical flow calculation.

## 2 A Simple Differential Model

As mentioned above, the starting point to calculate the optical flow will be the assumption, that the image brightness between distinct frames of the considered sequence is pointwise time-invariant under motion. This idealization has shown to be reliable for most real world image sequences unless no rapid illumination changes occur. A simple TAYLOR expansion of first order of the statement $I(\,x(t), y(t)\,) = \text{const}$ leads to the famous differential flow equation

$$I_x \cdot u + I_y \cdot v = -I_t, \tag{1}$$

where the vector field $(u, v)$ contains the optical flow information. Unfortunately, there are several problems concerned with this formulation, starting from the problem to determine two unknowns from one equation or from the difficulty to find good approximations of the partial derivatives of the image sequence in order to achieve credible solutions of (1). Many of these and other aspects of model problems are intensively discussed in [2] and [12].

We propose the following method to regularize the ill-posed optical flow equation: by projecting (1) into several subspaces $V_i$ of the signal space $\mathrm{L}_2(\mathbb{R})$, we obtain a number of equations, that contain the whole flow information and we may hope to get a solution to (1) by an optimization procedure of the received equation system. In more detail, we use a wavelet basis (or frame) to obtain a multiresolution analysis (MRA) of $\mathrm{L}_2(\mathbb{R})$ (many details may be found in [6]). Now, the projection can be easily done by taking the inner products of (1) with the building functions of the $V_i$ (sometimes called *test functions*), which are the scaling functions and wavelets the MRA stems from. Moreover, we may also embed the image representation into this multiscale framework, by approximating $I(x, y)$ as well as the temporal derivation $I_t(x, y)$ by series of the type

$$I(x,y) \approx \sum_i \sum_{k_x,k_y} c_{k_x,k_y} \cdot \varphi_i(x-k_x, y-k_y) \qquad \text{and}$$

$$I_t(x,y) \approx \sum_i \sum_{k_x,k_y} t_{k_x,k_y} \cdot \varphi_i(x-k_x, y-k_y).$$

To complete our modelling, the optical flow is assumed to be locally polynomial, i.e. we assume

$$\begin{aligned} u(x,y)|_{B(l_x,l_y)} &= u_{00} + u_{10} \cdot (x - l_x) + u_{01} \cdot (y - l_y) + \dots \\ v(x,y)|_{B(l_x,l_y)} &= v_{00} + v_{10} \cdot (x - l_x) + v_{01} \cdot (y - l_y) + \dots \end{aligned}$$

for a capable surrounding $B(l_x, l_y)$ of a considered image point $I(l_x, l_y)$. After this procedure, we arrive at the system

$$\begin{aligned} \sum_{i,j,k,l,k_x,k_y,l_x,l_y} c_{k_x,k_y} \cdot \Big( u_{k,l} \cdot \Gamma^{k,l}_{i,j,x}(k_x - l_x - s_x, k_y - l_y - s_y) \, + \\ v_{k,l} \cdot \Gamma^{k,l}_{i,j,y}(k_x - l_x - s_x, k_y - l_y - s_y) \Big) \\ = - \sum_{i,j,k_x,k_y,l_x,l_y} t_{k_x,k_y} \cdot \Gamma^{0,0}_{i,j,1}(k_x - l_x - s_x, k_y - l_y - s_y) \end{aligned}$$

for $(l_x + s_x, l_y + s_y) \in B(l_x, l_y)$. Hereby, the $\Gamma^{(\cdot,\cdot)}_{(\cdot,\cdot,\cdot)}(\cdot,\cdot)$ denote the *generalized wavelet connection coefficients*, which are given by

$$\Gamma^{k,l}_{i,j,\zeta}(m_x, m_y) \;=\; \int_{\mathbb{R}^2} x^k \cdot y^l \cdot \varphi_i(x - m_x, y - m_y) \cdot \frac{\partial}{\partial \zeta} \varphi_j(x - m_x, y - m_y).$$

As was shown in [10] for 1D and in [15] for the general multidimensional case, these connection coefficients can be evaluated (which might be quite expensive) by a finite-dimensional linear system, if the filter mask of the scaling function is finite and the scaling matrix for the associated MRA satisfies a certain mild criterion. However, one obviously sees, that the connection coefficients are independent of the data to be processed and can thus be computed offline and stored in look-up-tables afterwards. Therefore, the linear equation system determining the optical flow can be built up by simple discrete convolutions of the image data with the connection coefficient matrices. Moreover, since we use compactly supported wavelets (see e.g. [14] for design principles of compactly supported multidimensional wavelets), only a rather small number of nontrivial linear equations has to be solved for each image point; the whole processing can be done in linear time and is in addition massively parallelisable.

One might see this approach as a kind of advanced finite difference method, which might be enlightened before the background, that connection coefficients are closely related to discrete differentiation schemes [18]. This is very much in the spirit of [20] and [3], where somewhat similar approaches were made. However, in [20] a scale-space-embedding using Gaussians and Gaussian derivatives on several scales were used to obtain a linear equation system from (1) and in [3], partial integration and analytical wavelets were employed instead of the connection coefficient framework.

## 3 Application of Nonlinear Diffusion

The algorithm derived in the previous section has some nice features, in particular, it is very simple to implement and it is also very fast. But on the other hand there are also some deficiencies. The linear filtering, which is implicitly performed by using the MRA framework leads to blurring that especially detoriorates the flow estimation at motion boundaries and moreover, the equations become numerically instable for small displacement areas; this is a general problem in optimization of overdetermined linear systems with entries of small magnitude. Here, the consequence are estimation outliers.

To overcome these problems, several authors ([13], [23]) proposed the usage of some additional nonlinear flow-based functional, that shall be minimized in order to reduce smoothing at motion boundaries and to stabilize the numerical robustness of the estimations [4]. One very well-known approach is the minimization of the functional

$$\int_{\Omega} \lambda \cdot W_\sigma(|\nabla u|^2 + |\nabla v|^2) + (I_x \cdot u + I_y \cdot v + I_t)^2 \, dx\, dy, \tag{2}$$

where $W_\sigma(|\nabla u|^2 + |\nabla v|^2)$ is some potential function, that shall guarantee piecewise smooth flow fields as solutions. Here, $\lambda$ is just a weighting factor and $\sigma$ is a steering parameter, that thresholds motion boundaries. A necessary condition for $(u, v)$ to be a minimizing solution to (2) is the satisfaction of the related EULER differential equations

$$\begin{aligned} \lambda \cdot \operatorname{div}\left(W'_\sigma(|\nabla u|^2 + |\nabla v|^2) \cdot \nabla u\right) &= I_x \cdot (I_x \cdot u + I_y \cdot v + I_t)\,, \\ \lambda \cdot \operatorname{div}\left(W'_\sigma(|\nabla u|^2 + |\nabla v|^2) \cdot \nabla v\right) &= I_y \cdot (I_x \cdot u + I_y \cdot v + I_t)\,, \end{aligned} \tag{3}$$

which are closely related to nonlinear diffusion processes ([16], [21]). In this context, the solutions are obtained by an evolutionary iteration of the PDEs using e.g. GAUSS-SEIDEL iterations or some advanced semi-implicit numerical schemes like additive operator splitting [22]. Here, we want to go a different way, by solving the EULER differential equations not by temporal evolution, but directly with the connection coefficient framework presented in Section 2. Since we want to use non-polynomial potentials $W_\sigma$ (e.g. like in [17]), the PDEs (3) cannot by linearized directly by the usage of connection coefficients — aiming to receive a system of linear equations, we have to do some modification first. The technique, we use here is called *half-quadratic regularization* [5], its starting point is the fact, that $W_\sigma$ may be rewritten as

$$W_\sigma(x^2) = \inf_\gamma (\gamma \cdot x^2 + \rho(\gamma))$$

with some well-chosen convex function $\rho$ depending on $W_\sigma$. Rewriting (2) leads to the minimization of

$$\int_{\Omega} \lambda \cdot \left(\gamma(x,y) \cdot (|\nabla u|^2 + |\nabla v|^2) + \rho(\gamma(x,y))\right) + (I_x \cdot u + I_y \cdot v + I_t)^2 \, dx\, dy,$$

which is done in a two-step-way. First, the functional is minimized with respect to $\gamma$, while $(u, v)$ are fixed. Under certain conditions, which are fulfilled by the used potential, the solution is given by $\gamma = W'_\sigma$. In the second step, we keep this $\gamma$ fixed and minimize with respect to $(u, v)$ which leads to the linear EULER differential equations

$$\begin{aligned} \lambda \cdot \operatorname{div}(\gamma \cdot \nabla u) &= I_x \cdot (I_x \cdot u + I_y \cdot v + I_t), \\ \lambda \cdot \operatorname{div}(\gamma \cdot \nabla v) &= I_y \cdot (I_x \cdot u + I_y \cdot v + I_t). \end{aligned} \tag{4}$$

These linear PDEs are again solved by the Wavelet-GALERKIN projection method under application of the connection coefficient framework as described before. The processing is still feasible in linear time, but requires about six times more operations than the *simple* method. We close this section with one final remark about the half-quadratic regularization utilized here: obviously, in our case this method is equivalent to a BANACH-type iteration of the system (3) with $W'_\sigma(|\nabla u_{k-1}|^2 + |\nabla v_{k-1}|^2)$ as linearizer from the previous step. Thus, a good preprocessing for the initial guess of $(u, v)$ is very desirable and in our implementation realized by the simple differential approach described before. With this initializing step, very few iterations of the system (4) are sufficient to achieve a stable solution.

## 4 Implementation and Experimental Results

First, we will give a brief overview of our implementation. We used symmetric, nonorthogonal, twodimensional, nonseparable wavelets with at least two vanishing moments as generators of an MRA. The same wavelets (scaling functions) and their connection coefficients were used to build the linearized flow equations, which are solved by QR-algorithm. In order to handle large displacements, a multiscale coarse-to-fine strategy with subsequent image warping is utilized, motion outliers are identified via thresholding the temporal derivatives.

**Table 1.** Various results for the *Yosemite fly-through.*

| Method | Av. ang. err. | Density | Type |
|---|---|---|---|
| FLEET & JEPSON | 4.36° | 34.1% | phase-based |
| WEBER & MALIK | 4.31° | 64.2% | local / differential |
| SINGH | 12.90° | 97.8% | region-based |
| URAS | 10.44° | 100% | local / differential |
| ALVAREZ ET AL. | 5.53° | 100% | global / differential |
| NAGEL | 11.71° | 100% | global / differential |
| Section 2 | 6.94° | 96.1% | local / differential |
| Section 3 | 6.88° | 100% | local / differential |

We applied our methods to the famous *Yosemite* sequence (synthetic), see Figure 1. Additionally, in Table 1, we present the results that were achieved by the described methods regarding the average angular error from the true flow field.

One sees, that both described methods perform competitively well in comparison with other methods, especially to those with high estimation densities.

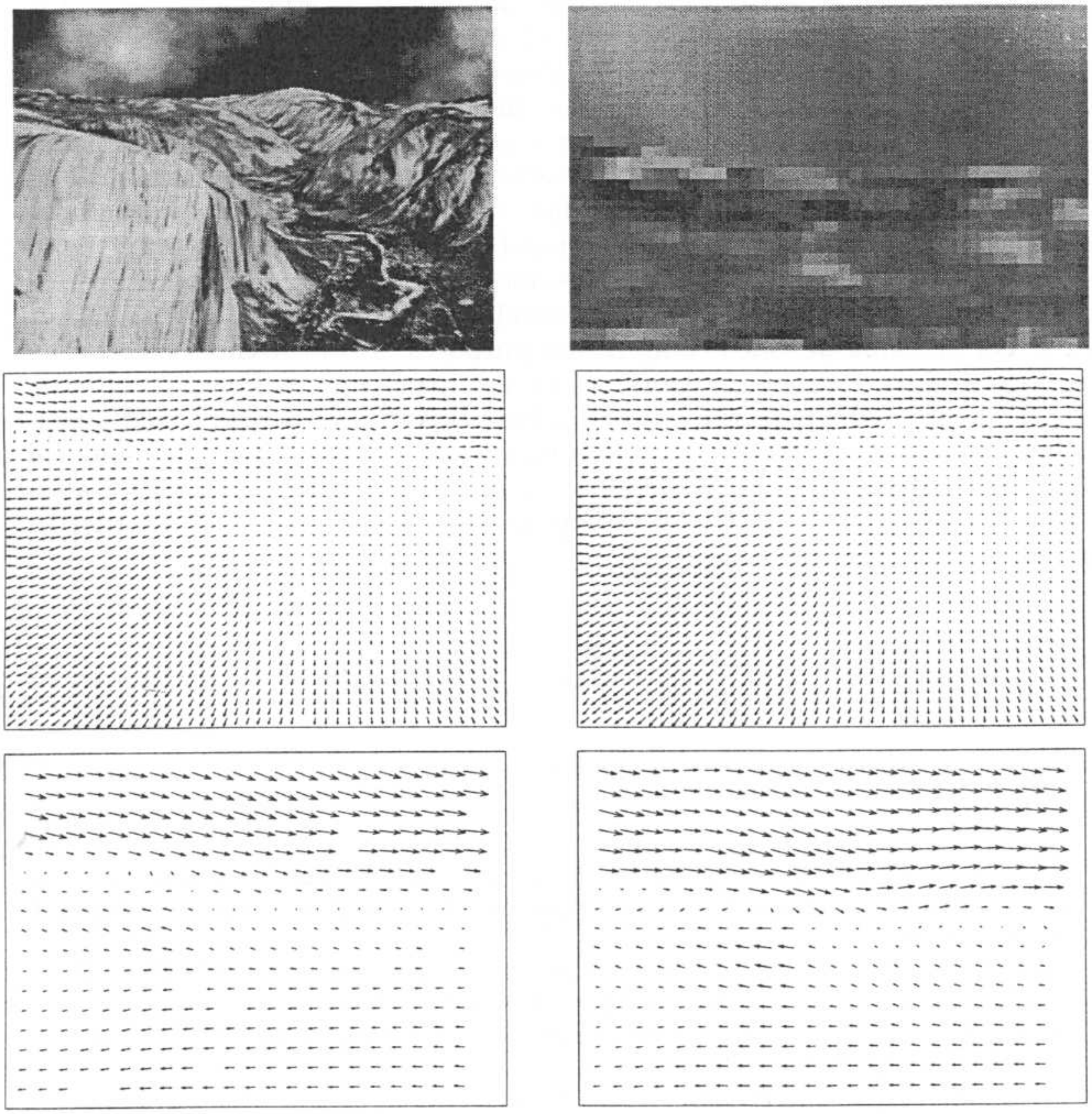

**Fig. 1. Top:** The data. **Left:** One frame of the *Yosemite* sequence. **Right:** A little section of the ridge in focus. **Middle left:** Calculated flow field for the method described in Section 2. **Middle Right:** Results belonging to the extended nonlinear method. **Bottom:** Improvement in the recovery of motion boundaries and supression of motion outliers — a little section of the ridge in the upper left from the *Yosemite* sequence is focussed. **Left:** Calculated flow by the simple differential method. **Right:** Results of nonlinear diffusion method.

## 5 Discussion

We presented a new very fast (i.e. linear time) algorithm for the estimation of optical flow, that mainly bases on the usage of wavelet connection coefficients. By adding a nonlinear diffusion term, flow blurring at motion boundaries could be reduced. However, as the experiments show, there is still some smoothing at such boundaries. This is mainly due to the fact, that the discrete filters, that are used to approximate the partial derivatives, cause some overlapping of the spatial information and thus, different motion information are mixed near singular curves (motion boudaries). This is a general problem with differential techniques and cannot be completely overcome. Therefore, if one is interested in exact detection of motion boundaries, treatments that try to minimize some kind of displacement energy functional are superior. On the other hand such approaches are nonlocal and require more computational amount. Which method one should finally use, depends surely on the application, that stands behind. For tracking or video compression purposes, a fast method is certainly preferrable, while for object detection or segmentation, nonlocal but more exact algorithms might be the better choice. However, as already mentioned, the wavelet connection coefficient framework could also be applied to nonlocal approaches, but the price of a higher computational cost has to be paid also. Nevertheless, this is one of the authors purposes for the next time in order to improve his flow calculations further.

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