Extended Kalman Filter Design for Motion Estimation by Point and Line Observations

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Abstract. The paper develops three extended Kalman filters (EKF) for 2D-3D pose estimation. The measurement models are based on three constraints which are constructed by geometric algebra. The dynamic measurements for these EKF are either points or lines. The real monocular vision experiments show that the results of EKFs perform more stable than that of LMS method.

1 Introduction and problem statement

The paper describes the design of EKFs which are used to estimate pose parameters of known objects in the framework of kinematics. Pose estimation in the framework of kinematics will be treated as nonlinear optimization with respect to geometric constraint equations expressing the relation between 2D image features and 3D model data.

The problem is described as follows. First, we make the following assumptions. The model of an object is given by points and lines in the 3D space. Further we extract line subspaces or points in an image of a calibrated camera and match them with the model of the object. The aim is to find the pose of the object from observations of points and lines in the images at different poses. Figure 1 shows the scenario with respect to observed line subspaces. The method of obtaining these is out of scope of this paper.

To be more detailed, in the scenario of figure 1 we describe the following situation: We assume 3D points $\{\boldsymbol{y_i}\}$ and lines $\{\boldsymbol{S_i}\}$, $i = 1, 2, ...$, belonging to an object model. Further we extract points $\{\boldsymbol{b_i}\}$ and lines $\{\boldsymbol{l_i}\}$ in an image of a calibrated camera and match them with the model.

Three constraints can be depicted:

1. Point-line constraint: A transformed point, e.g. $\boldsymbol{x_1}$, of the model point $\boldsymbol{y_1}$ must lie on the projection ray $\boldsymbol{L}_{b_1}$, given by the optical center $\boldsymbol{c}$ and the corresponding image point b_1.
2. Point-plane constraint: A transformed point, e.g. $\boldsymbol{x_1}$, must lie on the projection plane $\boldsymbol{P_{12}}$, given by $\boldsymbol{c}$ and the corresponding image line $\boldsymbol{l_1}$.
3. Line-plane constraint: A transformed line, e.g. $\boldsymbol{L_1}$, of the model line $\boldsymbol{S_1}$ must lie on the projection plane $\boldsymbol{P_{12}}$, given by $\boldsymbol{c}$ and the the corresponding image line $\boldsymbol{l_1}$.

We want to estimate optimal motion parameters based on these three constraints which formally are written [1, 2] in motor algebra [3, 4, 5] as

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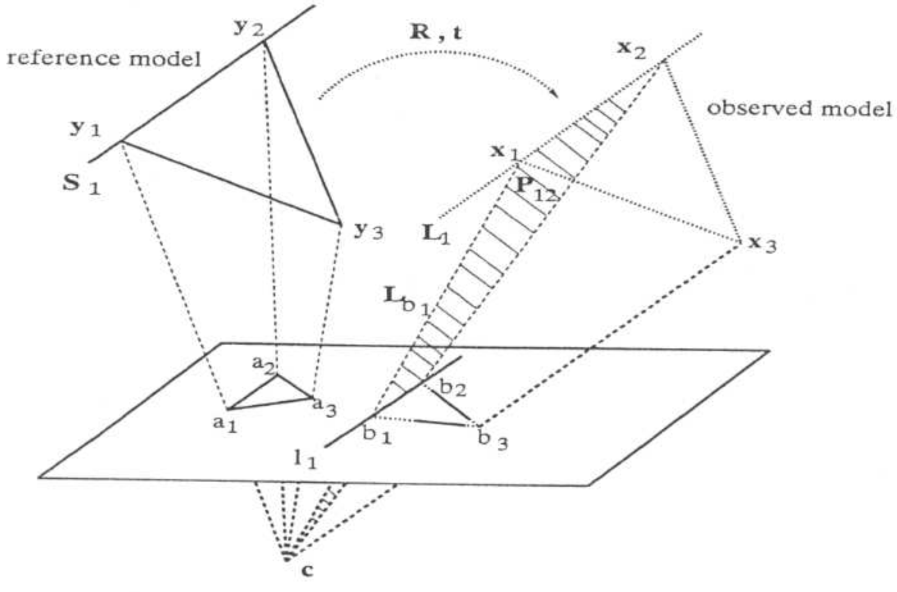

Fig. 1. The scenario. The solid lines at the left hand describe the assumptions: the camera model, the model of the object and the initially extracted lines on the image plane. The dashed lines at the right hand describe the actual pose of the model, which leads to the best fit of the object with the actual extracted lines.

Point-line constraint: $\boldsymbol{X_1 L_{b_1}} - \overline{\boldsymbol{L}}_{\boldsymbol{b_1}} \boldsymbol{X_1} = \boldsymbol{I}(\boldsymbol{m_1} - \boldsymbol{n_1} \times \boldsymbol{x_1}) = \boldsymbol{0}$,
Point-plane constraint: $\boldsymbol{P_{12} X_1} - \overline{\boldsymbol{X_1 P_{12}}} = \boldsymbol{I}(d_1 + \boldsymbol{p_1} \cdot \boldsymbol{x_1}) = \boldsymbol{0}$,
Line-plane constraint: $\boldsymbol{L_1 P_{12}} + \boldsymbol{P_{12}} \overline{\boldsymbol{L}}_{\boldsymbol{1}} = \boldsymbol{u_1} \cdot \boldsymbol{p_1} + \boldsymbol{I}(d_1 \boldsymbol{u_1} - \boldsymbol{p_1} \times \boldsymbol{v_1}) = \boldsymbol{0}$.

In above equations, we denote the point $\boldsymbol{X_1} = 1 + \boldsymbol{I x_1}$, the lines $\boldsymbol{L_{b_1}} = \boldsymbol{n_1} + \boldsymbol{I m_1}$ and $\boldsymbol{L_1} = \boldsymbol{u_1} + \boldsymbol{I v_1}$ and the plane $\boldsymbol{P_{12}} = \boldsymbol{p_1} + \boldsymbol{I} d_1$. More detailed derivation and interpretation of these constraints are described in [1, 2]. We use rotor algebra to describe points and their 3D kinematics and motor algebra to present lines and to model their kinematics. The reason we use rotor and motor algebra instead of matrix algebra is as follows. In EKF we define the state vector to be estimated is the parameter vector of rotation and translation. By rotor and motor algebra, there are 7 and 8 parameters, respectively. If we directly use matrix algebra, there will be up to 12 parameters (9 for rotation and 3 for translation). It is obviously that rotor or motor algebra will be more efficient. Moreover, using motor algebra we linearize the 3D Euclidean line motion model straightforwardly.

There are several approaches of optimal pose estimation based on least square methods [6]. Our preference is to use EKF for pose estimation because of their incremental, real-time potential and because of their robustness in case of noisy data. The robustness of the Kalman filter results from the fact that stronger modeling of the dynamic model is possible using additional priors compared to usual LMS estimators.

Because EKF means a general frame for handling nonlinear measurement models [7], the estimation of each considered constraint requires an individually designed EKF. The commonly known EKFs for pose estimation are related to 3D-3D point based measurements. The only EKF for line based measurements has been recently published by the authors [3]. But also that one has to estimate the motion of a line from 3D-3D measurements in motor algebra and not from 2D-3D measurements as in this paper. Zhang and Faugeras [8] used line segments in the frame of a point based standard EKF. Bar-Itzhack and Oshman [9] designed a quaternion EKF for point based rotation estimation.

The paper is organized as follows. After introduction and problem statement, in section two we will present three EKF approaches for motion estimation. In section three we compare the performance of different algorithms for constraint based pose estimation.

2 The Extended Kalman Filter for pose estimation

In this section we want to present the design of EKFs for estimating the pose based on three constraints. Because an EKF is defined in the frame of linear vector algebra, it will be necessary to map the estimation task from any chosen algebraic embedding to linear vector algebra (see e.g. [3]), at least so long no other solution exists. We present the design method in detail for constraint no.1 in subsection 2.1. The design results for constraints no.2 and 3 will be given in subsections 2.2 and 2.3, respectively.

2.1 EKF pose estimation based on point-line constraint

In case of point based measurements of the object at different poses, an algebraic embedding of the problem in the 4D linear space of the algebra of rotors $\mathcal{G}^+_{3,0,0}$, which is isomorphic to that one of quaternions $\mathbb{H}$, will be sufficient [4, 3]. Thus, rotation will be represented by a unit rotor $\boldsymbol{R}$ and translation will be a bivector $\boldsymbol{t}$. A point $\boldsymbol{y_1}$ transformed to $\boldsymbol{x_1}$ reads

$$\boldsymbol{x_1} = \boldsymbol{R}\boldsymbol{y_1}\tilde{\boldsymbol{R}} + \boldsymbol{t}.$$

We denote the four components of the rotor as

$$\boldsymbol{R} = r_0 + r_1\sigma_2\sigma_3 + r_2\sigma_3\sigma_1 + r_3\sigma_1\sigma_2.$$

To convert a rotor $\boldsymbol{R}$ into a rotation matrix $\boldsymbol{\mathcal{R}}$, simple conversion rules are at hand:

$$\boldsymbol{\mathcal{R}} = \begin{pmatrix} r_0^2 + r_1^2 - r_2^2 - r_3^2 & 2(r_1r_2 + r_0r_3) & 2(r_1r_3 - r_0r_2) \\ 2(r_1r_2 - r_0r_3) & r_0^2 - r_1^2 + r_2^2 - r_3^2 & 2(r_2r_3 + r_0r_1) \\ 2(r_1r_3 + r_0r_2) & 2(r_2r_3 - r_0r_1) & r_0^2 - r_1^2 - r_2^2 + r_3^2 \end{pmatrix}.$$

In vector algebra, the above point transformation model can be described as

$$\mathbf{x_1} = \boldsymbol{\mathcal{R}}\mathbf{y_1} + \mathbf{t}.$$

The projection ray $\mathbf{L}_{b_1}$ in the point-line equation is represented by Plücker coordinates $(\mathbf{n_1}, \mathbf{m_1})$, where $\mathbf{n_1}$ is its unit direction and $\mathbf{m_1}$ its moment. The point-line constraint equation in vector algebra of $\mathbb{R}^3$ reads

$$\mathbf{f_1} = \mathbf{m_1} - \mathbf{n_1} \times \mathbf{x_1} = \mathbf{m_1} - \mathbf{n_1} \times (\boldsymbol{\mathcal{R}}\mathbf{y_1} + \mathbf{t}) = \mathbf{0}.$$

Let the state vector $\mathbf{s}$ for the EKF be a 7D vector, composed in terms of the rotor coefficients for rotation and translation,

$$\mathbf{s} = (\mathbf{R}^T, \mathbf{t}^T)^T = (r_0, r_1, r_2, r_3, t_1, t_2, t_3)^T.$$

The rotation coefficients must satisfy the unit condition

$$\mathbf{f_2} = \mathbf{R}^T\mathbf{R} - 1 = r_0^2 + r_1^2 + r_2^2 + r_3^2 - 1 = 0.$$

The noise free measurement vector $\mathbf{a_i}$ is given by the actual line parameters $\mathbf{n_i}$ and $\mathbf{m_i}$, and the actual 3D point measurements $\mathbf{y_i}$,

$$\mathbf{a_i} = (\mathbf{n_i}^T, \mathbf{m_i}^T, \mathbf{y_i}^T)^T = (n_{i1}, n_{i2}, n_{i3}, m_{i1}, m_{i2}, m_{i3}, y_{i1}, y_{i2}, y_{i3})^T.$$

For a sequence of measurements $\mathbf{a_i}$ and states $\mathbf{s_i}$, the constraint equations

$$\mathbf{f_i}(\mathbf{a_i}, \mathbf{s_i}) = \begin{pmatrix} \mathbf{f_{1i}} \\ \mathbf{f_{2i}} \end{pmatrix} = \begin{pmatrix} \mathbf{m_i} - \mathbf{n_i} \times (\boldsymbol{\mathcal{R}}_\mathbf{i}\mathbf{y_i} + \mathbf{t_i}) \\ \mathbf{R_i}^T\mathbf{R_i} - 1 \end{pmatrix} = \mathbf{0}$$

relate measurements and states in a nonlinear manner. The system model in this static case should be

$$\mathbf{s_{i+1}} = \mathbf{s_i} + \boldsymbol{\zeta}_\mathbf{i},$$

where $\boldsymbol{\zeta}_\mathbf{i}$ is a vector random sequence with known statistics,

$$E[\boldsymbol{\zeta}_\mathbf{i}] = 0,$$
$$E[\boldsymbol{\zeta}_\mathbf{i}^T \boldsymbol{\zeta}_\mathbf{k}] = \boldsymbol{\mathcal{Q}}_\mathbf{i}\delta_\mathbf{ik},$$

where $\delta_\mathbf{ik}$ is the Kronecker delta and the matrix $\boldsymbol{\mathcal{Q}}_\mathbf{i}$ is assumed to be nonnegative definite.

We assume that the measurement system is disturbed by additive white noise, i.e., the real observed measurement $\mathbf{a}'_\mathbf{i}$ is expressed as

$$\mathbf{a}'_\mathbf{i} = \mathbf{a_i} + \boldsymbol{\eta}_\mathbf{i}.$$

The vector $\boldsymbol{\eta}_\mathbf{i}$ is an additive, random sequence with known statistics,

$$E[\boldsymbol{\eta}_\mathbf{i}] = 0,$$
$$E[\boldsymbol{\eta}_\mathbf{i}^T \boldsymbol{\eta}_\mathbf{k}] = \boldsymbol{\mathcal{W}}_\mathbf{i}\delta_\mathbf{ik},$$

where the matrix $\boldsymbol{\mathcal{W}}_\mathbf{i}$ is assumed to be nonnegative definite.

Since the observation equation is nonlinear (that means, the relationship between the measurement $\mathbf{a}'_\mathbf{i}$ and state $\mathbf{s_i}$ is nonlinear), we expand $\mathbf{f_i}(\mathbf{a_i}, \mathbf{s_i})$ into a Taylor series about the $(\mathbf{a}'_\mathbf{i}, \hat{\mathbf{s}}_\mathbf{i/i-1})$, where $\mathbf{a}'_\mathbf{i}$ is the real measurement and $\hat{\mathbf{s}}_\mathbf{i/i-1}$ is the predicted state at situation i. By ignoring the second order terms, we get the linearized measurement equation

$$\mathbf{z_i} = \boldsymbol{\mathcal{H}}_\mathbf{i}\mathbf{s_i} + \boldsymbol{\xi}_\mathbf{i},$$

where

$$\begin{aligned}\mathbf{z_i} &= \mathbf{f_i}(\mathbf{a}'_\mathbf{i}, \hat{\mathbf{s}}_\mathbf{i/i-1}) - \frac{\partial \mathbf{f_i}(\mathbf{a}'_\mathbf{i}, \hat{\mathbf{s}}_\mathbf{i/i-1})}{\partial \mathbf{s_i}}\hat{\mathbf{s}}_\mathbf{i/i-1} \\ &= \begin{pmatrix} \mathbf{m}'_\mathbf{i} - \mathbf{n}'_\mathbf{i} \times (\hat{\boldsymbol{\mathcal{R}}}_\mathbf{i/i\,-\,1}\mathbf{y}'_\mathbf{i} + \hat{\mathbf{t}}_\mathbf{i/i-1}) \\ \hat{\mathbf{R}}^T_\mathbf{i/i\,-\,1}\hat{\mathbf{R}}_\mathbf{i/i-1} - 1 \end{pmatrix} + \boldsymbol{\mathcal{H}}_\mathbf{i}\hat{\mathbf{s}}_\mathbf{i/i-1}.\end{aligned}$$

The measurement matrix $\boldsymbol{\mathcal{H}}_\mathbf{i}$ of the linearized measurement $\mathbf{z_i}$ reads

$$\boldsymbol{\mathcal{H}}_\mathbf{i} = -\frac{\partial \mathbf{f_i}(\mathbf{a}'_\mathbf{i}, \hat{\mathbf{s}}_\mathbf{i/i-1})}{\partial \mathbf{s_i}} = \begin{pmatrix} \boldsymbol{\mathcal{C}}_{\mathbf{n}'_\mathbf{i}}\boldsymbol{\mathcal{D}}_{\hat{\boldsymbol{\mathcal{R}}}\mathbf{y}'} & \boldsymbol{\mathcal{C}}_{\mathbf{n}'_\mathbf{i}} \\ \boldsymbol{\mathcal{D}}_\mathbf{R} & \mathbf{0}_{1\times 3} \end{pmatrix},$$

where

$$\begin{aligned}\boldsymbol{\mathcal{D}}_\mathbf{R} &= \frac{\partial(\hat{\mathbf{R}}^T_\mathbf{i/i\,-\,1}\hat{\mathbf{R}}_\mathbf{i/i-1} - 1)}{\partial \mathbf{R_i}} \\ &= \begin{pmatrix} -2\hat{r}_{(i/i-1)0} & -2\hat{r}_{(i/i-1)1} & -2\hat{r}_{(i/i-1)2} & -2\hat{r}_{(i/i-1)3} \end{pmatrix},\end{aligned}$$

$$\mathcal{D}_{\hat{\mathcal{R}}\mathbf{y}'} = \frac{\partial(\hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{y}'_{\mathbf{i}})}{\partial \mathbf{R}_{\mathbf{i}}} = \begin{pmatrix} d_1 & d_2 & d_3 & d_4 \\ d_4 & -d_3 & d_2 & -d_1 \\ -d_3 & -d_4 & d_1 & d_2 \end{pmatrix},$$

$$\begin{aligned} d_1 &= 2(\hat{r}_{(i/i-1)0}y'_{i1} + \hat{r}_{(i/i-1)3}y'_{i2} - \hat{r}_{(i/i-1)2}y'_{i3}), \\ d_2 &= 2(\hat{r}_{(i/i-1)1}y'_{i1} + \hat{r}_{(i/i-1)2}y'_{i2} + \hat{r}_{(i/i-1)3}y'_{i3}), \\ d_3 &= 2(-\hat{r}_{(i/i-1)2}y'_{i1} + \hat{r}_{(i/i-1)1}y'_{i2} - \hat{r}_{(i/i-1)0}y'_{i3}), \\ d_4 &= 2(-\hat{r}_{(i/i-1)3}y'_{i1} + \hat{r}_{(i/i-1)0}y'_{i2} + \hat{r}_{(i/i-1)1}y'_{i3}). \end{aligned}$$

The 3×3 matrix $\mathcal{C}_{\mathbf{n}'_{\mathbf{i}}}$ is the skew-symmetric matrix of $\mathbf{n}'_{\mathbf{i}}$. For any vector $\mathbf{y}$, we have $\mathcal{C}_{\mathbf{n}'_{\mathbf{i}}}\mathbf{y} = \mathbf{n}'_{\mathbf{i}} \times \mathbf{y}$ with

$$\mathcal{C}_{\mathbf{n}'_{\mathbf{i}}} = \begin{pmatrix} 0 & -n'_{i3} & n'_{i2} \\ n'_{i3} & 0 & -n'_{i1} \\ -n'_{i2} & n'_{i1} & 0 \end{pmatrix}.$$

The measurement noise is given by

$$\begin{aligned} \boldsymbol{\xi}_{\mathbf{i}} &= -\frac{\partial \mathbf{f}_{\mathbf{i}}(\mathbf{a}'_{\mathbf{i}}, \hat{\mathbf{s}}_{\mathbf{i/i-1}})}{\partial \mathbf{a}_{\mathbf{i}}}(\mathbf{a}_{\mathbf{i}} - \mathbf{a}'_{\mathbf{i}}) = \frac{\partial \mathbf{f}_{\mathbf{i}}(\mathbf{a}'_{\mathbf{i}}, \hat{\mathbf{s}}_{\mathbf{i/i-1}})}{\partial \mathbf{a}_{\mathbf{i}}}\boldsymbol{\eta}_{\mathbf{i}} \\ &= \begin{pmatrix} \mathcal{C}_{\hat{\mathbf{x}}_{\mathbf{i/i-1}}} & \mathbf{I}_{3\times 3} & -\mathcal{C}_{\mathbf{n}'_{\mathbf{i}}}\hat{\mathcal{R}}_{\mathbf{i/i-1}} \\ \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} \end{pmatrix}_{4\times 9} \boldsymbol{\eta}_{\mathbf{i}}, \end{aligned}$$

where $\mathbf{I}_{3\times 3}$ is a unit matrix and $\mathcal{C}_{\hat{\mathbf{x}}_{\mathbf{i/i-1}}}$ is the skew-symmetric matrix of $\hat{\mathbf{x}}_{\mathbf{i/i-1}}$ with

$$\hat{\mathbf{x}}_{\mathbf{i/i-1}} = \hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{y}'_{\mathbf{i}} + \hat{\mathbf{t}}_{\mathbf{i/i-1}}.$$

The expectation and the covariance of the new measurement noise $\boldsymbol{\xi}_{\mathbf{i}}$ are easily derived from that of $\mathbf{a}'_{\mathbf{i}}$ as

$$\begin{aligned} &E[\boldsymbol{\xi}_{\mathbf{i}}] = \mathbf{0}, \\ &E[\boldsymbol{\xi}_{\mathbf{i}}^T\boldsymbol{\xi}_{\mathbf{i}}] = \mathcal{V}_{\mathbf{i}} = \left(\frac{\partial \mathbf{f}_{\mathbf{i}}(\mathbf{a}'_{\mathbf{i}}, \hat{\mathbf{s}}_{\mathbf{i/i-1}})}{\partial \mathbf{a}_{\mathbf{i}}}\right)\mathcal{W}_{\mathbf{i}}\left(\frac{\partial \mathbf{f}_{\mathbf{i}}(\mathbf{a}'_{\mathbf{i}}, \hat{\mathbf{s}}_{\mathbf{i/i-1}})}{\partial \mathbf{a}_{\mathbf{i}}}\right)^T. \end{aligned}$$

The EKF motion estimation algorithms based on point-plane and line-plane constraints can be derived in a similar way. We list the results below.

2.2 EKF pose estimation based on point-plane constraint

The projection plane $\mathbf{P_{12}}$ in the point-plane constraint equation is represented by $(d_1, \mathbf{p_1})$, where d_1 is its Hesse distance and $\mathbf{p_1}$ its unit direction. The point-plane constraint equation in vector algebra of $\mathbb{R}^3$ reads

$$d_1 - \mathbf{p_1}^T(\mathcal{R}\mathbf{x_1} + \mathbf{t}) = 0.$$

With the measurement vector $\mathbf{a_i} = (d_\mathbf{i}, \mathbf{p_i}^T, \mathbf{y_i}^T)^T$ and the same state vector $\mathbf{s}$ as above, the measurement $\mathbf{z_i}$ of linearized measurement equation reads

$$\mathbf{z_i} = \begin{pmatrix} d'_{\mathbf{i}} - \mathbf{p}'^T_{\mathbf{i}}(\hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{y}'_{\mathbf{i}} + \hat{\mathbf{t}}_{\mathbf{i/i-1}}) \\ \hat{\mathbf{R}}^T_{\mathbf{i/i-1}}\hat{\mathbf{R}}_{\mathbf{i/i-1}} - 1 \end{pmatrix} + \mathcal{H}_{\mathbf{i}}\hat{\mathbf{s}}_{\mathbf{i/i-1}}.$$

The measurement matrix $\mathcal{H}_\mathbf{i}$ of the linearized measurement $\mathbf{z_i}$ now reads

$$\mathcal{H}_{\mathbf{i}} = \begin{pmatrix} \mathbf{p_i}^T \mathcal{D}_{\hat{\mathcal{R}}\mathbf{y}'} & \mathbf{p_i}^T \\ \mathcal{D}_{\mathbf{R}} & \mathbf{0}_{1\times 3} \end{pmatrix}.$$

The measurement noise is given by

$$\xi_{\mathbf{i}} = \begin{pmatrix} 1 & -(\hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{y}'_{\mathbf{i}} + \hat{\mathbf{t}}_{\mathbf{i/i-1}})^T & -(\mathbf{p}'^T_{\mathbf{i}} \hat{\mathcal{R}}_{\mathbf{i/i-1}}) \\ 0 & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} \end{pmatrix}_{2\times 7} \eta_{\mathbf{i}}.$$

2.3 EKF pose estimation based on line-plane constraint

Using the line-plane constraint, the reference model entity in $\mathcal{G}^+_{3,0,1}$ [3, 4] is the Plücker line $\boldsymbol{S_1} = \boldsymbol{n_1} + \boldsymbol{I}\boldsymbol{m_1}$. This line transformed by a motor $\boldsymbol{M} = \boldsymbol{R} + \boldsymbol{I}\boldsymbol{R}'$ reads

$$\boldsymbol{L_1} = \boldsymbol{M}\boldsymbol{S_1}\widetilde{\boldsymbol{M}} = \boldsymbol{R}\boldsymbol{n_1}\widetilde{\boldsymbol{R}} + \boldsymbol{I}(\boldsymbol{R}\boldsymbol{n_1}\widetilde{\boldsymbol{R}'} + \boldsymbol{R}'\boldsymbol{n_1}\widetilde{\boldsymbol{R}} + \boldsymbol{R}\boldsymbol{m_1}\widetilde{\boldsymbol{R}}) = \boldsymbol{u_1} + \boldsymbol{I}\boldsymbol{v_1}.$$

We denote the 8 components of the motor as

$$\begin{aligned} \boldsymbol{M} &= \boldsymbol{R} + \boldsymbol{I}\boldsymbol{R}' \\ &= r_0 + r_1\gamma_2\gamma_3 + r_2\gamma_3\gamma_1 + r_3\gamma_1\gamma_2 + \boldsymbol{I}(r'_0 + r'_1\gamma_2\gamma_3 + r'_2\gamma_3\gamma_1 + r'_3\gamma_1\gamma_2). \end{aligned}$$

The line motion equation can be equivalently expressed by vector form,

$$\mathbf{u_1} = \mathcal{R}\mathbf{n_1},$$
$$\mathbf{v_1} = \mathcal{A}\mathbf{n_1} + \mathcal{R}\mathbf{m_1},$$

with

$$\mathcal{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

$$\begin{aligned} a_{11} &= 2(r'_0r_0 + r'_1r_1 - r'_2r_2 - r'_3r_3), \quad a_{12} = 2(r'_3r_0 + r'_2r_1 + r'_1r_2 + r'_0r_3), \\ a_{13} &= 2(-r'_2r_0 + r'_3r_1 - r'_0r_2 + r'_1r_3), \ a_{21} = 2(-r'_3r_0 + r'_2r_1 + r'_1r_2 - r'_0r_3), \\ a_{22} &= 2(r'_0r_0 - r'_1r_1 + r'_2r_2 - r'_3r_3), \quad a_{23} = 2(r'_1r_0 + r'_0r_1 + r'_3r_2 + r'_2r_3), \\ a_{31} &= 2(r'_2r_0 + r'_3r_1 + r'_0r_2 + r'_1r_3), \quad a_{32} = 2(-r'_1r_0 - r'_0r_1 + r'_3r_2 + r'_2r_3), \\ a_{33} &= 2(r'_0r_0 - r'_1r_1 - r'_2r_2 + r'_3r_3). \end{aligned}$$

The line-plane constraint equation in vector algebra of $\mathbb{R}^3$ reads

$$\begin{pmatrix} \mathbf{f_1} \\ \mathbf{f_2} \end{pmatrix} = \begin{pmatrix} \mathbf{p_1}^T\mathbf{u_1} \\ d_1\mathbf{u_1} + \mathbf{v_1} \times \mathbf{p_1} \end{pmatrix} = \begin{pmatrix} \mathbf{p_1}^T(\mathcal{R}\mathbf{n_1}) \\ d_1\mathcal{R}\mathbf{n_1} + (\mathcal{A}\mathbf{n_1} + \mathcal{R}\mathbf{m_1}) \times \mathbf{p_1} \end{pmatrix} = \mathbf{0}.$$

We use the 8 components of the motor as the state vector for the EKF,

$$\mathbf{s} = (r_0, r_1, r_2, r_3, r'_0, r'_1, r'_2, r'_3)^T$$

and these 8 components must satisfy both the unit and orthogonal conditions:

$$f_3 = r_0^2 + r_1^2 + r_2^2 + r_3^2 - 1 = 0,$$
$$f_4 = r_0r'_0 + r_1r'_1 + r_2r'_2 + r_3r'_3 = 0.$$

The 10D noise free measurement vector $\mathbf{a_i}$ is given by the true plane parameters d_i and $\mathbf{p_i}$, and the true 6D line parameters $(\mathbf{n_i}, \mathbf{m_i})$,

$$\mathbf{a_i} = (d_\mathrm{i}, \mathbf{p_i}^T, \mathbf{n_i}^T, \mathbf{m_i}^T)^T = (d_\mathrm{i}, p_{i1}, p_{i2}, p_{i3}, n_{i1}, n_{i2}, n_{i3}, m_{i1}, m_{i2}, m_{i3})^T.$$

The new measurement in linearized equation reads

$$\mathbf{z_i} = \begin{pmatrix} \mathbf{p}'^T_{\mathbf{i}}(\hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{n}'_{\mathbf{i}}) \\ d'_\mathrm{i}\hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{n}'_{\mathbf{i}} + (\hat{\mathcal{A}}_{\mathbf{i/i-1}}\mathbf{n}'_{\mathbf{i}} + \hat{\mathcal{R}}_{\mathbf{i/i-1}}\mathbf{m}'_{\mathbf{i}}) \times \mathbf{p}'_{\mathbf{i}} \\ \hat{\mathbf{R}}^T_{\mathbf{i/i-1}}\hat{\mathbf{R}}_{\mathbf{i/i-1}} - 1 \\ \hat{\mathbf{R}}^T_{\mathbf{i/i-1}}\hat{\mathbf{R}}'_{\mathbf{i/i-1}} \end{pmatrix} + \mathcal{H}_{\mathbf{i}}\hat{\mathbf{s}}_{\mathbf{i/i-1}}.$$

The measurement matrix $\mathcal{H}_i$ of the linearized measurement $\mathbf{z}_i$ reads

$$\mathcal{H}_i = \begin{pmatrix} -\mathbf{p}_i'^T \mathcal{D}_{\hat{\mathcal{R}}\mathbf{n}'} & \mathbf{0}_{1\times 4} \\ -d_i' \mathcal{D}_{\hat{\mathcal{R}}\mathbf{n}'} + \mathcal{C}_{\mathbf{p}_i'}(\mathcal{D}_{\hat{\mathcal{A}}\mathbf{n}'} + \mathcal{D}_{\hat{\mathcal{R}}\mathbf{m}'}) & \mathcal{C}_{\mathbf{p}_i'} \mathcal{D}_{\hat{\mathcal{R}}\mathbf{n}'} \\ \mathcal{D}_{\mathbf{R}} & \mathbf{0}_{1\times 4} \\ \mathcal{D}_{R'} & \frac{1}{2}\mathcal{D}_{\mathbf{R}} \end{pmatrix},$$

where $\mathcal{D}_{\hat{\mathcal{R}}\mathbf{n}'} = \frac{\partial(\hat{\mathcal{R}}_{i/i-1}\mathbf{n}_i')}{\partial \mathbf{R}_i}$, $\mathcal{D}_{\hat{\mathcal{R}}\mathbf{m}'} = \frac{\partial(\hat{\mathcal{R}}_{i/i-1}\mathbf{m}_i')}{\partial \mathbf{R}_i}$, $\mathcal{D}_{\hat{\mathcal{A}}\mathbf{n}'} = \frac{\partial(\hat{\mathcal{A}}_{i/i-1}\mathbf{n}_i')}{\partial \mathbf{R}_i}$, $\mathcal{D}_{R'} = \frac{\partial(\hat{\mathbf{R}}_{i/i-1}^T \hat{\mathbf{R}}'_{i/i-1})}{\partial \mathbf{R}_i}$ and $\frac{1}{2}\mathcal{D}_{\mathbf{R}} = \frac{\partial(\hat{\mathbf{R}}_{i/i-1}^T \hat{\mathbf{R}}'_{i/i-1})}{\partial \mathbf{R}_i'}$

The 3×3 matrix $\mathcal{C}_{\mathbf{p}_i'}$ is the skew-symmetric matrix of $\mathbf{p}_i'$.

The measurement noise is given by

$$\boldsymbol{\xi}_i = \begin{pmatrix} 0 & \mathbf{n}_i'^T \hat{\mathcal{R}}_{i/i-1} & \mathbf{p}_i'^T \hat{\mathcal{R}}_{i/i-1} & \mathbf{0}_{1\times 3} \\ \hat{\mathcal{R}}_{i/i-1}\mathbf{n}_i' & \mathcal{C}_{\hat{\mathbf{v}}_i} & d_i' \hat{\mathcal{R}}_{i/i-1} - \mathcal{C}_{\mathbf{p}_i'} \hat{\mathcal{A}}_{i/i-1} & -\mathcal{C}_{\mathbf{p}_i'} \hat{\mathcal{R}}_{i/i-1} \\ \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 3} \end{pmatrix} \eta_i$$

where $\mathcal{C}_{\hat{\mathbf{v}}_i}$ is skew-symmetric matrix of $\hat{\mathbf{v}}_i$, and $\hat{\mathbf{v}}_i$ is defined as

$$\hat{\mathbf{v}}_i = \hat{\mathcal{A}}_{i/i-1}\mathbf{n}_i' + \hat{\mathcal{R}}_{i/i-1}\mathbf{m}_i'.$$

Having linearized the measurement models, the EKF implementation is straightforward and standard. Further implementation details will not be repeated here [2, 7, 3, 8]. In next section, We will denote the EKF as $\mathbf{R}$tEKF, if the state explicitly uses the rotor components of rotation $\mathbf{R}$ and of translation $\mathbf{t}$, or MEKF, if motor components of motion $\mathbf{M}$ is used.

2.4 Some notes about the algorithms

Here we will give some specific notes on EKF algorithms.

The EKF algorithm requires an initial guess of motion not very far from the true one. One reasonable hypothesis is that the motion is "small". So we can set the initial guess as "no motion": $\mathbf{s}_{1/0} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$ and $\mathbf{s}_{1/0} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$ for $\mathbf{R}$tEKF and MEKF, respectively. In experiments, the estimate converges rapidly from the initial guess to near the true one within 4 or 5 runs, but for a qualified estimation, more than 15 runs are required.

In our experiments, we find, if the translation $||\mathbf{t}|| \gg 1$, the algorithms based on constraints no. 1 and no. 3 will frequently diverge. The reason is that these constraints contain cross product terms. Such situation can be analyzed by the equation of measurement noise $\boldsymbol{\xi}_i$. In constraint no. 1, suppose we set the origin of the coordinate system at somewhere on the reference model. If the estimated translator $||\hat{\mathbf{t}}_i|| \gg 1$, then, (usually) $||\hat{\mathbf{y}}_i|| \gg 1$. That will directly cause the components of the covariance matrix $\mathcal{V}_i$ to be far greater than that of the original covariance matrix $\mathcal{W}_i$. Such enlarged noise will easy make the EKF diverging. To solve this problem, we simply multiply the measurement function by a scalar as follows. At the beginning of the algorithm, we check the distance, $||\mathbf{m}_1||$, of the projection line $\mathbf{L}_{b_1}$. If $||\mathbf{m}_i|| > 1$, we can use a modified measurement equation

$$\mathbf{f}_i/||\mathbf{m}_i|| = \mathbf{m}_i/||\mathbf{m}_i|| - \mathbf{n}_i \times (\mathcal{R}_i \mathbf{y}_i/||\mathbf{m}_i|| + \mathbf{t}_i/||\mathbf{m}_i||) = 0.$$

At the end of the algorithm we multiply the estimated translation $\hat{\mathbf{t}}_\mathbf{i}^* = \hat{\mathbf{t}}_\mathbf{i}/||\mathbf{m_i}||$ by $||\mathbf{m_i}||$ to recover the true estimation $\hat{\mathbf{t}}_\mathbf{i}$. Similar analysis can be done for constraint no. 3. In case of constraint no. 3, we use the distance , d_i, of projection plane $\mathbf{P_{ij}}$, that means, divide $\mathbf{f_{2i}}$ by d_i. At the end of the algorithm, we multiply the estimated dual part $\hat{\mathbf{R}}_\mathbf{i}'^*$, $\hat{\mathbf{R}}_\mathbf{i}'^* = \hat{\mathbf{R}}_\mathbf{i}'/d_i$, by d_i to recover the true estimation $\hat{\mathbf{R}}_\mathbf{i}'$.

3 Experiments

In this section we present some experiments by real images. The aim of the experiments is to study the performance of the EKF algorithms for pose estimation based on geometric constraints. We expect that both the special constraint and the algorithmic approach of using it may influence the results. This behavior should be shown with respect to different qualities of data.

In our experimental scenario we took a B21 mobile robot equipped with a stereo camera head and positioned it two meters in front of a calibration cube. We focused one camera on the calibration cube and took an image. Then we moved the robot, focused the camera again on the cube and took another image. The edge size of the calibration cube is 46 cm and the image size is 384×288 pixel. Furthermore we defined on the calibration cube a 3D object model.

In these experiments we actually selected certain points by hand and from these the depicted lines are derived and, by knowing the camera calibration, the actual projection ray and projection plane parameters are computed.

The results of different algorithms for pose estimation are shown in table 1 In the second column of table 1 **Rt**EKF and **M**EKF denote the use of the EKF, MAT denotes matrix algebra, SVD denotes the singular value decomposition of a matrix to ensure a rotation matrix as a result. In the third column the used constraints, point-line (XL), point-plane (XP) and line-plane (LP) are indicated. The fourth column shows the results of the estimated rotation matrix $\mathcal{R}$ and the translation vector $\mathbf{t}$, respectively. The fifth column shows the error of the equation system. Since the error of the equation system describes the Hesse distance of the entities [1], the value of the error is an approximation of the squared average distance of the entities.

In a second experiment we compare the noise sensitivity of the various approaches for pose estimation. Matrix based estimations result in both higher errors and larger fluctuations in dependence of the noise level compared to EKF estimates. This is in agreement with the well known behavior of error propagation in case of matrix based rotation estimation. The EKF performs more stable. This is a consequence of the estimator themselves and of the fact that in our approach rotation is represented as rotors. The concatenation of rotors is more robust than that of rotation matrices.

4 Conclusions

In this paper we present three EKF algorithms for 2D-3D pose estimation. The aim of the paper is to design EKFs based on three geometric constraints. The model data are either points or lines. The observation frame is constituted by projection lines or projection planes. Any deviations from the constraint correspond the Hesse distance of the involved geometric entities. The representation

no.	$\mathcal{R}$ — t	Constraint	Experiment 1	Error
1	RtEKF — RtEKF	XL-XL	$\mathcal{R} = \begin{pmatrix} 0.986 & 0.099 & -0.137 \\ -0.127 & 0.969 & -0.214 \\ 0.111 & 0.228 & 0.967 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -35.66 \\ -203.09 \\ 156.25 \end{pmatrix}$	4.5
2	RtEKF — RtEKF	XP-XP	$\mathcal{R} = \begin{pmatrix} 0.986 & 0.115 & -0.118 \\ -0.141 & 0.958 & -0.247 \\ 0.085 & 0.260 & 0.962 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -37.01 \\ -198.55 \\ 154.86 \end{pmatrix}$	5.5
3	MEKF — MEKF	LP-LP	$\mathcal{R} = \begin{pmatrix} 0.985 & 0.106 & -0.134 \\ -0.133 & 0.968 & -0.213 \\ 0.108 & 0.228 & 0.968 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -53.30 \\ -214.37 \\ 138.53 \end{pmatrix}$	2.6
4	MEKF — MAT	LP-LP	$\mathcal{R} = \begin{pmatrix} 0.985 & 0.106 & -0.134 \\ -0.133 & 0.968 & -0.213 \\ 0.108 & 0.228 & 0.968 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -67.78 \\ -227.73 \\ 123.90 \end{pmatrix}$	2.7
5	SVD — MAT	LP-XP	$\mathcal{R} = \begin{pmatrix} 0.976 & 0.109 & -0.187 \\ -0.158 & 0.950 & -0.266 \\ 0.149 & 0.289 & 0.945 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -66.57 \\ -216.18 \\ 100.53 \end{pmatrix}$	7.1

Table 1. The results of experiment 1, depending on the used constraints and algorithms to evaluate their validity.

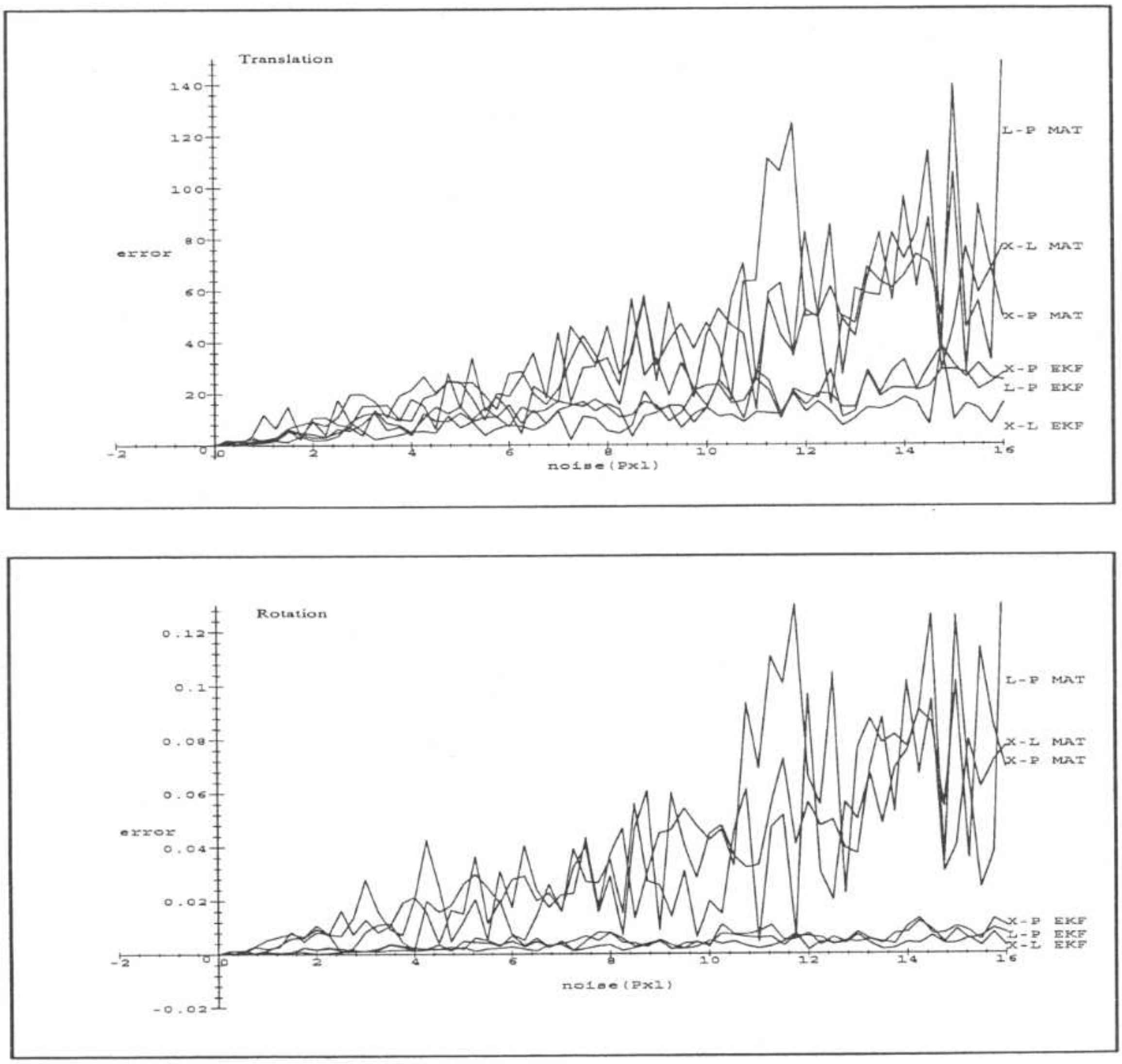

Fig. 2. Performance comparison with increasing noise. The EKFs perform with more accurate and more stable estimates than the matrix based method.