

Inverse covariation of spectral density and correlation dimension in cyclic EEG dynamics of the human brain

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Received: 15 December 1998 / Accepted in revised form: 18 May 1999

Abstract. The responsiveness or excitability of the central nervous system (CNS) to external or internal stimuli is systematically altered corresponding to transient changes of the EEG background activity, mainly in the alpha range. We hypothesise that a transient alpha power increase is due to an underlying increase in synchronisation or coupling strength between various neuronal elements or cortical networks. Consequently, the 'network' of the CNS may be more ordered and, hence, less complex in the case of high spectral density, and vice versa. The goals of the present paper are (1) to prove the inverse covariation between spectral density and correlation dimension for a set of human EEG data, (2) to falsify the null hypothesis that the observed relationship is a random one, and (3) to propose a neuronal approach which may explain the observed correlations. A sliding computation of the spectral density and correlation dimension [Grassberger P, Procaccia I (1983) *Physica D* 9:189–208] of mid-occipital EEG recordings derived from eight awake subjects with eyes closed was performed. The similarity between the two time courses was quantified by similarity measures and descriptive correlation coefficients. The temporal pattern of dimensional complexity showed an inverse relationship with simultaneously computed spectral power changes most pronounced in the alpha range. The group means of similarity measures and correlation coefficients were compared with the corresponding means of a sample set established by 20 Gaussian random signals. Statistically significant differences were obtained at the 0.1% level, rejecting the null hypothesis that the observed relationship is a random one. The results support the idea that the dynamics of the EEG signals investigated reflect a chaotic deterministic process with state transitions from 'high-dimensional' to 'low-dimensional' non-linear dynamics, and vice versa. Adequate neuronal models and approaches to interpret the disclosed transients and the inverse covariation

between spectral density and dimensional complexity are proposed, giving additional insight into the integrative functioning of the CNS with respect to the strategy of information processing.

1 Introduction

Since the first numerical analysis of the electroencephalogram (EEG) by Dietsch (1932), a number of procedures for the quantification of the EEG signal have been developed. Besides the conventional methods in the time or in the frequency domains, such as zero-crossing techniques, interval amplitude analysis and spectral analysis which are based on the linear system theory, non-linear approaches modelling the underlying dynamical system of the brain have been presented (Babloyantz and Destexhe 1986a; Mayer-Kress and Holzfuss 1987; Mayer-Kress and Layne 1987; Röschke and Basar 1989). Extensive surveys were given by Basar (1990a), and Pritchard and Duke (1992a). Most of the procedures quantify the information content of the EEG signal with respect to the distribution of amplitudes and frequencies, dimensional complexity and topography of the measured parameters. However, the temporal structure of long-term recordings is not quantified optimally when consecutive segments of the signal are analysed separately. Thus, the introduction of the sliding analysis with overlapping segments, previously applied to EEG signals (Keidel et al. 1987a,b, 1990a; Tirsch et al. 1988), has given insight into the temporal fine structure of long-term EEG recordings, taking the non-stationarity of the time series into account.

Figure 1 shows an example of an EEG recording with a duration of 4 min which was used in the present paper. When visually inspecting the temporal structure of the EEG curve, only the well-known alpha spindles are evident, but no information about long-lasting fluctuations of the signal's activity can be established. Underlying systematic, oscillatory changes of the neuronal activity in the 1-min range can be detected by appropriate

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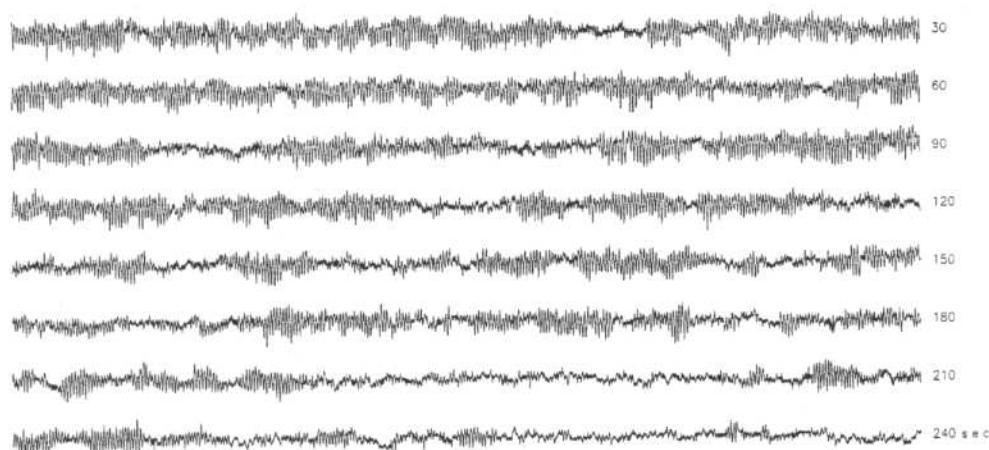


Fig. 1. 4-min record of EEG activity derived from the mid-occipital region (Oz) of subject 101

computer-assisted procedures such as the previously mentioned sliding analysis.

In various investigations (Keidel et al. 1987a,b, 1990a; Tirsch et al. 1988) which were based on spectral parameters, we demonstrated that the bioelectrical activity of the human brain as the output of the spontaneously active nervous 'network' is not maintained at a static level. Rather it shows an oscillating behaviour with a well-ordered change of activity and synergy over time. The described cyclic variations of the spontaneous awake EEG at rest were not caused randomly; they were related to changes of finger and muscle tremor as a mechanical correlate of the motoneuronal activity (Keidel et al. 1987b, 1989, 1990a; Mayer-Kress et al. 1988). The discerned period lengths of the oscillations in spectral power and coherence were in the range 20–70 s. These findings were in line with the results of Koepchen et al. (1980), Lips et al. (1988), and Galambos and Makeig (1988).

From these results, we have concluded that the central nervous system (CNS), with its responsiveness to external and internal stimuli, periodically alters the level of activity and degree of 'synergy' (Haken 1977) between different parallel processing structures and systems rather than maintaining a steady state. These conclusions also agree with Basar's (1983b) concept of continuously spontaneous changing states of the brain. Thus, we assume that a power increase in the main frequency range may be due to an increase in synchronisation or coupling strength between various neuronal elements within cortical networks generating the derived signals, and vice versa. This assumption is in line with the concept of synchronising oscillations at the cellular and neuronal network level from Lopes da Silva (1991) and the model of synchronisation and desynchronisation of coupled self-oscillatory networks from Basar (1983a). Consequently, the 'network' of the CNS may be more ordered and, hence, less complex in the case of high spectral density, and vice versa.

In recent years, methods of non-linear, dynamical system theory have been developed to estimate the complexity of EEG time series. The commonly used concept was to measure the dimension of the attractor in the phase space of the underlying dynamical system

which provides the widely used measure of the correlation dimension D_2 (Grassberger and Procaccia 1983). Following the reports of Farmer et al. (1983), the measure of dimensionality denotes the minimum number of essential variables needed to model the dynamics of a general dynamical system. Especially for a dynamical system, such as the brain, Mayer-Kress and Layne (1987), and Mayer-Kress et al. (1988) suggested that the dimensionality is related to the number of active coherent modes modulating the underlying process. An increase of dimensionality could correspond to an increase in the number of independent or incoherent oscillating subsystems or modes. Related to linear systems, this number may correspond to the number of independent frequencies in the power spectrum of the EEG signal which implies that, for example, a sharp peak and a fairly broad peak are correlated with a low and a high dimensionality, respectively.

Taking into consideration the temporal pattern of the dynamics of the ongoing spontaneous EEG activity, we hypothesise an inverse relationship between the cyclic dynamics of spectral density and correlation dimension during long-term analysis of EEG recordings, i.e. an increase of the spectral density may involve a decrease of the correlation dimension, and vice versa. The concept of the sliding calculation of the correlation dimension applied here is supported by the reports of Havstad and Ehlers (1989). Following the ideas presented by Keidel et al. (1987b), they suggested that for non-stationary dynamical systems small successive but overlapping data segments in which the system is approximately stationary should be preferred. They demonstrated the ability of such data sets to follow fluctuations in dimension with sufficient accuracy which enables studies of dimension as a function of time in non-stationary dynamical systems.

The goals of the present paper are (1) to prove the postulated covariation between spectral density and correlation dimension by means of real EEG data, (2) to falsify the null hypothesis that the observed relationship is a random one by means of pseudo-random signals and statistical test procedures, and (3) to propose a neuronal approach which may explain the observed correlations.

2 Materials and data acquisition

Eight healthy young volunteers (mean age: 23.5 ± 2.5 years) were examined. The data acquisition was performed in a reduced sound and electrically shielded room. The subjects were in a waking state and in a supine position with eyes closed. Unipolar scalp-EEG recordings were obtained from left and right motor cortices (C3/C4), from mid-frontal (Fz) and mid-occipital regions (Oz) corresponding to the 10/20 system and referred to linked earlobes. Only the occipital derivations were investigated in the present study. Signals were recorded for 4 min continuously and A/D converted using a sampling rate of 500 Hz. To suppress artefacts, the data were submitted to a band-pass filter with a frequency range of 1–40 Hz using a Butterworth filter of tenth order in a parallel configuration. Phase shifts were eliminated by forward and backward filtering.

3 Methods

3.1 Spectral analysis

As a classical method, spectral analysis was applied to EEG periods which were taken from the 4-min recording and divided into segments of 2.56 s, overlapping each other by 1.28 s. After elimination of linear trends and tapering, each segment was submitted to fast Fourier transform, resulting in spectral power values with a frequency resolution of 0.391 Hz. In order to increase the number of degrees of freedom, the segmental spectra were averaged within one period according to the segmentation procedure of Welch (1967). For data reduction, the mean of power values within the alpha range of 7.4–12.5 Hz was calculated. This is due to the fact that we have observed striking changes of amplitude in the alpha range of the ongoing EEG activity.

3.2 Non-linear dynamical analysis

3.2.1 Geometrical reconstruction of "strange attractors"

In contrast to the preceding analysis, which describes the spectral properties of the signal and which is based on the superposition of harmonic and linear periodic oscillators, procedures and measures were introduced characterising the local structure of the post-transient phase-space orbits, i.e. the so-called strange attractors. With respect to the brain dynamics, 'these attractors do not directly provide an adequate model, rather they may be interpreted as the result of self-organizing processes within the brain which produce coordinated states or pattern activity' (Gregson et al. 1990).

The dynamical approach of non-linear modelling described here is to interpret the measured single scalar time series from one electrode position on the scalp as the one-dimensional projection of a multi-dimensional phase-space trajectory of an underlying non-linear dynamical system, which is as yet unknown. Based on a geometrical view of the underlying process and follow-

ing the delay scheme of Packard et al. (1980), a technique was used which reconstructs the phase-space trajectory by 'embedding' the time series into a higher-dimensional space. This technique, by which time signals can be interpreted as multi-dimensional geometrical objects and reconstructed as phase-space vectors, is a basic tool for the characterisation of a dynamical system such as the human brain.

Let the variable $y(t_i)$ with $i = 1, \dots, N$ sample points represent the time series of the EEG signal derived from one lead on the scalp. The general form of the reconstructed phase-space vector of the 'embedding' dimension d at time t_i is given by:

$$\bar{y}_i = \{y(t_i), y(t_i + \tau), y(t_i + 2\tau), \dots, y(t_i + (d-1)\tau)\}.$$

Following this formula, the instantaneous state of the system is represented by a point in the multi-dimensional phase space. With increasing discrete time index from 1 up to N , the consecutive data points describe a geometrical object on which the trajectories of the system move. The reconstructed trajectories may fill out the entire phase space or converge to a limited subset, i.e. the attractor derived from the time series.

For the reconstruction of an attractor in a pseudo-state subspace with the 'embedding' dimension d , the following assumptions are made: (1) the pseudo-state subspace is a sufficient reconstruction of the original state space, as long as $d > 2D + 1$ (where D is the true fractal dimension of the attractor), following definite embedding theorems (Whitney 1936; Mañé 1981; Takens 1981). Then, the reconstructed attractor is diffeomorphic to the original attractor; both have the same metric properties; (2) as long as $d > D$ (Eckmann and Ruelle 1985), the dimensions of the reconstructed and the original attractors will be nearly the same; and (3) the dynamics of the signal may be described by a set of d variables which are derived from the time series by introducing a time lag τ (Packard et al. 1980).

The reconstructed trajectories can be plotted as phase portraits in different projections. Their shape corresponds to the dynamics of the system and depends markedly on the time lag τ . Various values of τ are possible in practical applications. In order to find the suitable time delay, the mutual information content of the time series is taken into account.

3.2.2 Mutual information content

To estimate time delays in non-linear systems, several procedures have been proposed. In addition to approaches using autocorrelation functions, the procedure of mutual information content has been widely applied. This measure describes the relation between input and output of non-linear systems, providing the amount of information about a random vector coupled with another vector. Based on Shannon's information theory (Shannon and Weaver 1948) and on Shaw's idea (1985) to predict a time series vector at time $t + \tau$ if the measurement at time t is known, a procedure was developed which computes the average amount of mutual information (Mars and Arragon 1982; Fraser

and Swinney 1986). This is a measure for the redundancy of a vector measurement at time $t + \tau$, which is generally expressed in bits when the used logarithm for calculation is taken to the base of two.

With respect to higher-dimensional attractor reconstructions, the available two-dimensional formulations were improved and extended taking four-dimensional time-delayed vectors into account and using a subspace of the embedding-dimensional phase space. Our investigations with EEG data had revealed that a dimension of four is sufficient related to the embedding dimension used and that this procedure provides a significantly higher resolution of information about the statistical dependencies within the time series investigated. In the four-dimensional discrete case, the stochastic variables X , Y , Z , and V with the finite sets of realisations $\{x_i\}$, $\{y_j\}$, $\{z_k\}$, and $\{v_l\}$, $i, j, k, l = 1, \dots, M$, respectively, are considered. The elements x_i , y_j , z_k , and v_l are derived from the corresponding time series: $x(t)$, $x(t + \tau)$, $x(t + 2\tau)$, and $x(t + 3\tau)$. Then, the average amount of mutual information is given by:

$$I = \sum_{i,j,k,l=1}^M P_{xyzu}(x_i, y_j, z_k, v_l) \cdot \ln \frac{P_{xyzu}(x_i, y_j, z_k, v_l)}{P_x(x_i) \cdot P_y(y_j) \cdot P_z(z_k) \cdot P_v(v_l)},$$

in which

- $P_x(x_i)$ probability of occurrence of $\{x_i\}$ derived from the time series vector $x(t)$,
- $P_y(y_j)$ probability of occurrence of $\{y_j\}$ derived from the time series vector $x(t + \tau)$,
- $P_z(z_k)$ probability of occurrence of $\{z_k\}$ derived from the time series vector $x(t + 2\tau)$,
- $P_v(v_l)$ probability of occurrence of $\{v_l\}$ derived from the time series vector $x(t + 3\tau)$,
- P_{xyzu} joint probability of occurrence of (x_i, y_j, z_k, v_l) ,
- M number of cells.

This method enables the optimal time delay to be determined for constructing an 'attractor' from a limited time-series. This value is based on the criterion that the mutual information content reaches the first significant minimum in the graph of I versus τ ; in this case the phase-space coordinates are approximately uncorrelated and independent. Compared to the decorrelation time derived from the autocorrelation function, the latter time delay is larger (Fraser and Swinney 1986).

3.2.2 Dimensional analysis

The basic idea for the characterisation of chaotic dynamical systems is to calculate the dimension of their attractors measuring their self-similar structure. Initial reports on this subject were published by Farmer et al. (1983). This measure has been widely used in the field of non-linear dynamical analysis to estimate the number of independent variables needed to model the dynamical process and to discriminate between a deterministic and a random activity. The dimensionality is also a measure of the complexity related to the number of independent oscillators of modes modulating the process (Eckmann and Ruelle 1985) and the number of active degrees of

freedom of the system, which is reflected by its entropy (Theiler 1990). According to the entropy concept of Basar (1983a), the entropy is also a measure of the order of the system, i.e. an increased order involves a lower entropy, and vice versa. Thus, it can be deduced that the order of the system is associated with its dimensional complexity. To compute the dimensionality of an attractor, the procedure introduced by Grassberger and Procaccia (1983) usually has been applied, utilising the scaling structure of the attractor. It can be quantified by measuring the spatial correlation between pairs of randomly chosen points (y_i, y_j) on the attractor. This requires the introduction of an additional concept: the correlation integral providing a measure for the probability of pairs of phase-space vectors to be separated by a distance less or equal to a prior defined value.

The discrete form of such an integral is given as:

$$C(r) = \frac{2}{N^2 - N} \sum_{i=1}^N \sum_{j=i+W}^N \Theta(r - |\vec{y}_i - \vec{y}_j|),$$

in which N denotes the number of sample points. Θ denotes the Heaviside function which is defined as: $\Theta(x) = 0$ for $x \leq 0$ and $\Theta(x) = 1$ for $x > 0$, r denotes a defined distance, y_i the phase-space vector and $W > 1$ the correction variable introduced by Theiler (1986). For $W = 1$, a standard calculation of the correlation integral is performed.

For a sufficiently large number N of data points and for small distances r , this function has the following scaling property which is due to the exponential divergence of trajectories:

$$C(r) \sim r^{D_2} \quad \text{or} \quad \log[C(r)] = D_2 \log(r) + c$$

in which the exponent D_2 is the correlation dimension.

It has been shown that the correlation exponent is a useful measure of the local structure of a strange attractor (Grassberger and Procaccia 1983). Considering the latter relation with its power law, the exponent D_2 can consequently be obtained as the slope of the straight line figuring the logarithmic plot of C versus r . However, dealing with physiological data sets from living systems, this ideal function may not be apparent and graphs in which different regions are nearly linear are usually evident (Gregson et al. 1990). Moreover, Eckmann and Ruelle (1985), Theiler (1986), Pritchard and Duke (1992a) and Jeong et al. (1998) pointed out the occurrence of distortions or 'knees' in the correlation integral due to the effect of autocorrelations and noise contamination within the time series. Applied to the EEG data investigated here, Theiler's correction was successful only for $W=2$, but led to an unnecessarily overestimated value of the correlation dimension.

3.2.3 Localisation of the optimal fitting centre

The problem now is to determine the most linear scaling region which is essential for the condition that the reconstructed attractor is self-similar and fractal. For this purpose, a procedure was developed which automatically localises the corresponding optimal fitting

centre within the graph of the correlation integral (see Fig. 2). For the localisation of the fitting centre, consecutive sections in the curve with equal length (in our case 12% of the total length of the curve) are considered. Thus, walking along the graph, the starting point of the sections is shifted by increments of one. Each section corresponds to a window with different increment length in which a regression line based on perpendicular distances is fitted to the curve. From the resulting sequence of residual variances, the first, second, and third minima are determined, the increments of which represent the first, second, and third best-fitting centres, respectively.

Through the first fitting centre, which is generally the optimal one, several regression lines with variable lengths are fitted; the slope of the regression line with minimum residual variance is denoted as the correlation dimension D_2 . Application to EEG data revealed that this algorithm is rather robust against the distortions described above and that only very rarely the values from the second fit, but never from the third fit must be used. Compared to hitherto existing fitting procedures, the approach adopted involves a substantial improvement.

3.2.4 Saturation and limitations

The procedure is performed stepwise by considering successively higher values of the embedding dimension. For a sufficiently large value, the dimension of the attractor will be obtained as the saturation value of this procedure. This is guaranteed even if model attractors are considered. However, for a physiological time series such as the EEG, the slope values of the dimension do

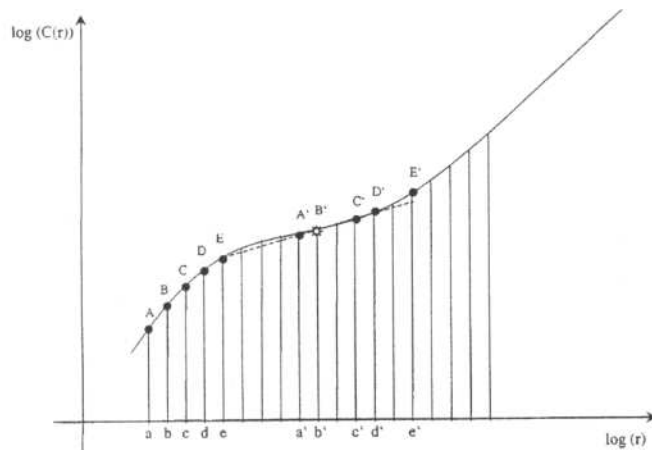


Fig. 2. Sliding fitting of various regression lines with different lengths through the graph of the correlation integral. The points $A, B, C, \dots$ denote the coordinates of n function values according to the equidistant abscissa values $a, b, c, \dots$ with index $1-n$. The distances $A-A', B-B', \dots$ denote equal curve sections on the graph (determined by length measuring) with a defined length. By means of this procedure a sequence of fitting windows $\{aa'\}, \{bb'\}, \dots$ with the fitting centres $(a'-a)/2, (b'-b)/2, \dots$ and variable increment length depending on the steepness of the graph is revealed. For each window, a regression line is fitted through the number of comprising data points. The point (*) represent the best fitting centre for window $\{ee'\}$ based on the minimum residual variance of the regression

not saturate in all cases, which is due to finite length of time series and superimposed noise. This problem was especially pointed out by Mayer-Kress and Layne (1987), Pritchard and Duke (1992a), and Ding et al. (1993). Since the correlation dimension was automatically determined for a large number of EEG epochs, no attention was paid to saturation and, subsequently, no epoch was discarded.

With respect to the theorem of Takens (1981), the embedding dimension should be greater than $|2D_2 + 1|$, where D_2 is the true correlation dimension of the attractor. Following the theoretical considerations of Eckmann and Ruelle (1990), D_2 is limited by:

$$D_{2\max} = \frac{2 \log_{10} N}{\log_{10}(1/p)} \quad \text{with} \quad p = \frac{r_{\min}}{D_2},$$

in which N denotes the number of sample points, $r_{\min}$ the minimum distance, and D the diameter of the reconstructed attractor. Using $N = 5000$ data points and $p = 0.1$, the upper limit of D_2 is 7.4 for the present study, resulting in a minimum embedding dimension of 16, which was used for further analysis.

3.3 Sliding analysis

In contrast to conventional analysis techniques consisting of the consecutive evaluation of defined periods, we used the sliding technique. This technique provides the detection of systematic, oscillatory changes in the signal's activity more clearly. A sliding shifting of a defined period is performed like a 'running' analysis window over the whole record length using a time shift Δt . Following the results of the sliding spectral analysis applied to human neurobiological signals (Tirsch et al. 1988) and of the sliding dimensional analysis applied to EEG signals (Keidel et al. 1990b), the period length was selected for 10 s and time shift for 1 s in the present study. These values had revealed the best results in disclosing temporal patterns from the signals. Using an EEG recording of 4 min, this procedure produces time series of 230 values for spectral power and correlation dimension. With respect to the computation of the mutual information content, a sliding analysis was also performed, providing time delays for each period separately.

3.4 Similarity between the shapes of two time series

When processing biosignals, in many cases there arises the question whether or not, or to what extent, the shapes of two time series are similar. One method for solving this problem is to calculate the correlation coefficient r between the two time series, which should be considered as a descriptive measure. Another method represents a more empirical approach which quantifies the similarity between the two time series following their cyclic alterations. Local similarity coefficients ranging between

-1 and +1 can be derived as the normalised quotient of the slopes of the two regression lines fitted in steps of one increment along the two curves which is denoted as:

$$q = \frac{\text{Min}\{|b_1|, |b_2|\}}{\text{Max}\{|b_1|, |b_2|\}},$$

in which b_1 and b_2 are the slopes of the two regression lines.

The global similarity coefficient is then defined as the measure:

$$\eta = \frac{n_+ - n_-}{n_+ + n_-},$$

in which n_+ and n_- are the numbers of positive and negative local similarity coefficients, respectively, using a cut-off level of 0-1.

A further possibility to investigate the similarity between two time courses is to apply a spectral analysis to the time series and to compare, for example, the corresponding peak frequencies and period lengths in the power spectrum. An alternative method, looking for the endogenous auto-rhythm within the time series, was also used. The principle of this procedure is to approximate the given time series by a sinusoid function with defined period T . Amplitude and phase were calculated by means of a least-square fitting - a technique which was taken from the so-called cosinor procedure of Halberg et al. (1967). Then, various sine functions with stepwise increasing period lengths ranging over 5-90 s were fitted in the search for an optimal fit. This was obtained by taking into account the minimum residual variance, i.e. the mean quadratic deviation between the time series

and the resulting fitted sine wave. The resulting optimal period lengths derived from the two time series were also compared with each other. In contrast to spectral analysis, in the case of relatively small data sets with only 230 points, the frequency resolution of this method is considerably higher. This is advantageous for further analysis in which solely period lengths were processed.

4 Results

4.1 Similarity between the shapes of the time series of spectral density and complexity

4.1.1 EEG data

Firstly, the sliding spectral analysis was applied to each of the 4-min EEG epochs derived from eight subjects. For each frequency step, the resulting time courses of 230 spectral densities were smoothed by an autoregressive low-pass filter and drawn as chronospectrograms. In Fig. 3, an example of such a chronospectrogram for subject 101 is illustrated. The dominant power in the alpha range and the pronounced rhythmic variations in power densities with periods of about 30-40 s are clearly evident.

Furthermore, the sliding dimensional analysis was applied to the same EEG data set. In the upper graph of Fig. 4, the smoothed time courses of 230 sliding computations of the correlation dimension for the same 4-min epoch as above are shown. The embedding dimension increases from 11 up to 20. The numeric range of the correlation dimension is comparable with specific findings in the literature (Babloyantz and Destexhe

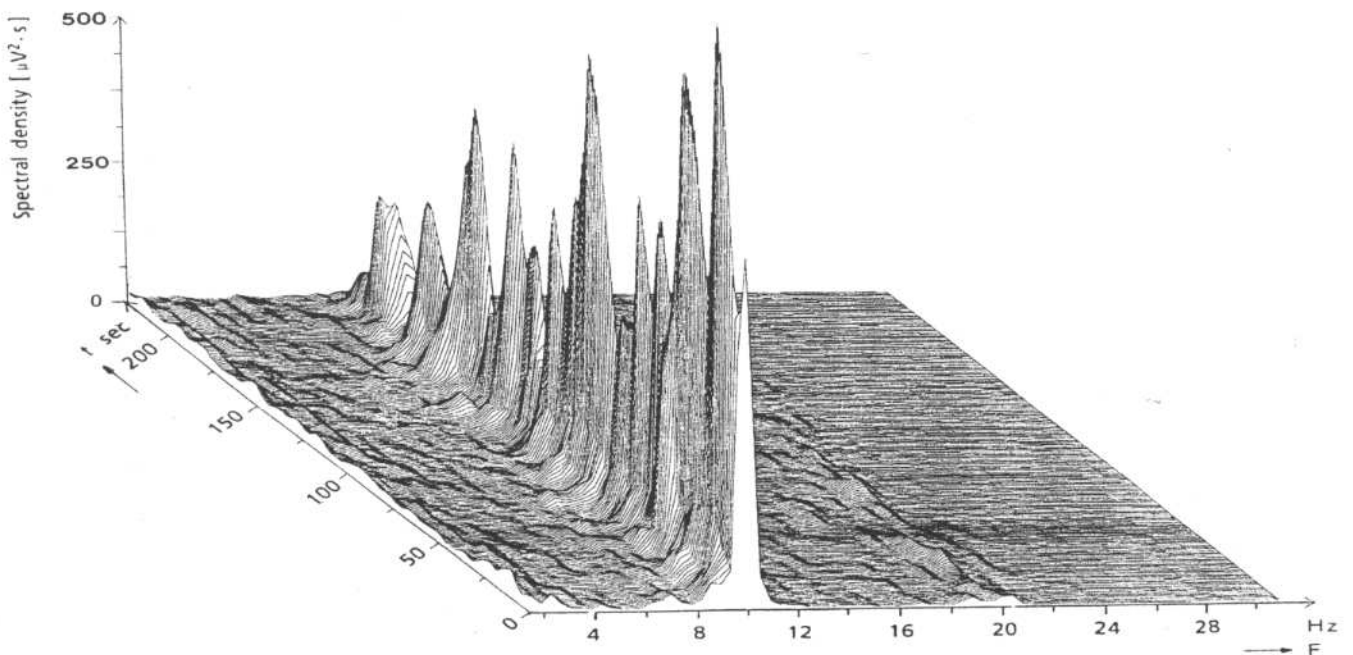


Fig. 3. Pseudo-three-dimensional chronospectrogram of sliding spectral density derived from the 4-min EEG epoch (Oz) of subject 101. The spectral frequency with a resolution of 0.391 Hz is marked on the *abscissa*. The time in seconds and the number of shifts ($\Delta t = 1$ s) are drawn on the *z*-axis

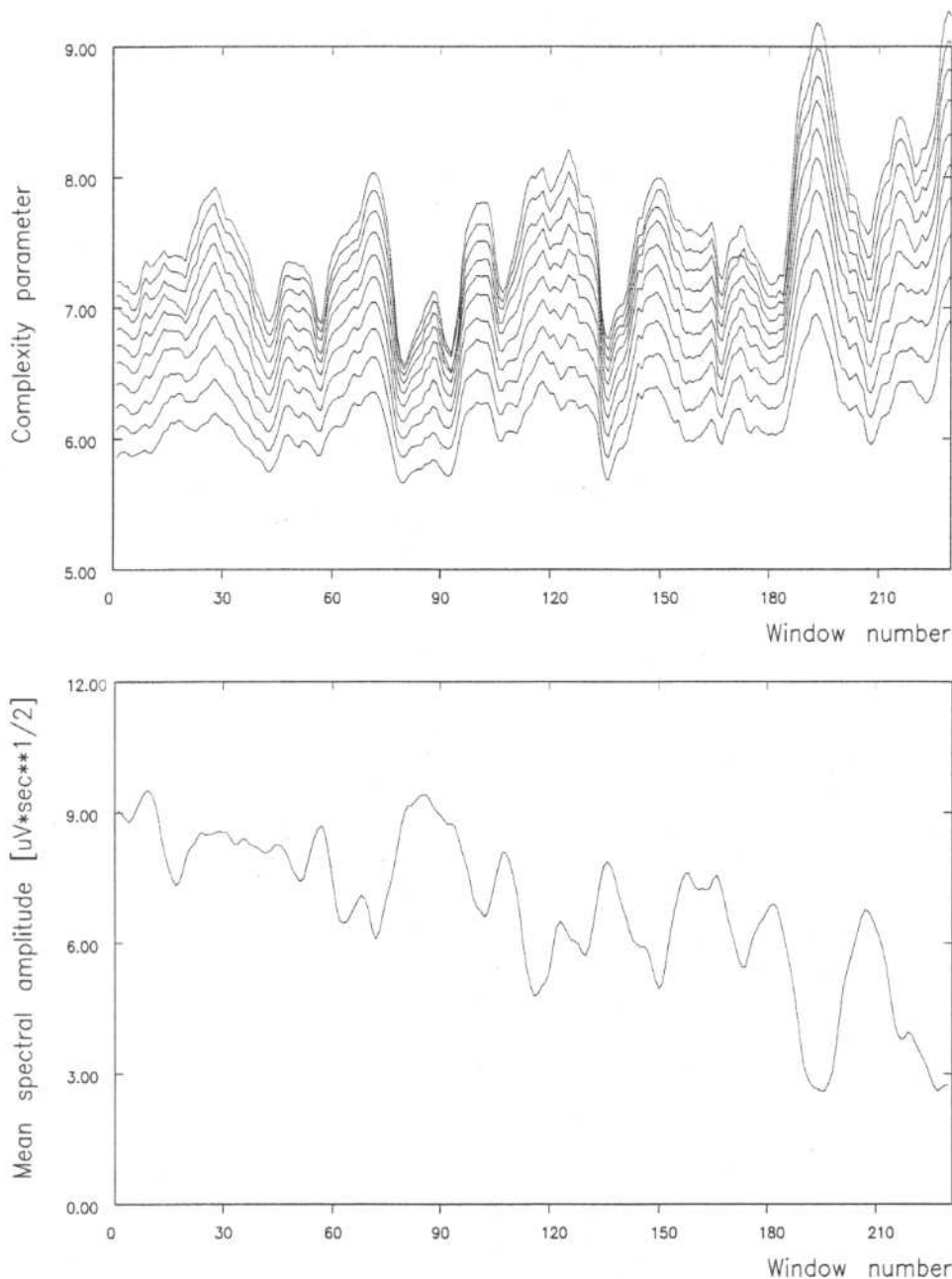


Fig. 4. Synopsis of sliding dimensional analysis and spectral analysis derived from the same EEG epoch as shown in Fig. 1. *Upper graph* Smoothed time courses of dimensional complexity over 230 windows using time delays of 24–40 ms. Embedding dimension ranging over 11–20. The length of analysis window is 10 s (5000 data points) which is shifted by 1 s. *Lower graph* Time series of spectral amplitude (as the square root of spectral density) in the alpha frequency band at 7.4–12.5 Hz. Note the reciprocal behaviour of the two graphs indicating that an increase of correlation dimension involves a decrease of spectral density, and vice versa

1986a; Dvorak and Siska 1986; Basar et al. 1989b; Pritchard and Duke 1992b; Pritchard and Duke 1997). In contrast to consecutive analyses, the rhythmic variation of dimensional complexity becomes evident just by the sliding analysis.

The time course of the spectral density in the total frequency band of 1–40 Hz derived from the same EEG data is shown below. Comparing the cyclic structure of the two graphs, an inverse covariation between spectral density and complexity is evident. This correlation is demonstrated more clearly in Fig. 5. For this purpose, the time courses of the complexity according to the embedding dimension of 16 and of the spectral density were normalised between -1 and $+1$ and superimposed (Fig. 5, upper graph). For clear evidence of the inverse covariation

between the two graphs, the time series of the spectral density was inverted.

The inverted spectral density, represented by the dotted line, clearly follows the cyclic alterations of the complexity. The similarity of the two graphs was quantified by a simple procedure described previously resulting in 79 positive and 24 negative similarity coefficients with a cut-off level of ± 0.5 and a similarity measure of $\eta = 0.53$ (Fig. 5, lower graph). The great majority of positive similarities and the high correlation coefficient of $r = 0.84$ indicate a high degree of congruence between the two graphs, and a high negative correlation between the dimensional complexity and non-inverted spectral density of the signal. This finding is due to the improved spectral properties of the EEG signal, resulting in a pronounced cyclic activity with sharp peaks in the alpha

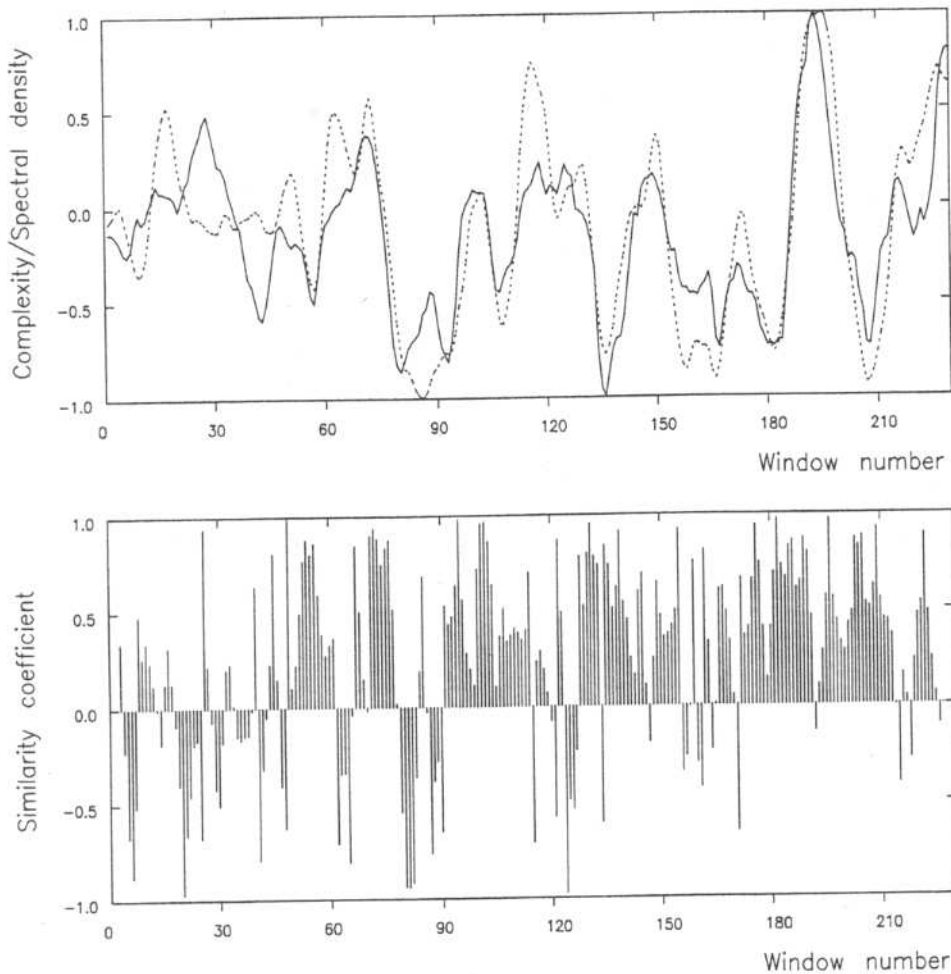


Fig. 5. Relationship between sliding dimensional and spectral analysis derived from the same EEG epoch as in Fig. 1. *Upper graph* Normalised time series of dimensional complexity (embedding dimension = 16) and spectral density in the alpha frequency band at 7.4–12.5 Hz (dashed line). For better evidence of the relationship, the time series of spectral density is inverted. *Lower graph* Time series of 79 positive and 24 negative coefficients describing the similarity between the two graphs at each increment. The high degree of congruence between the shapes of the two time series is expressed by the great majority of positive similarities and the high correlation coefficient ($r = 0.84$)

band as shown in the chronospectrogram of Fig. 3. The covariation between the time series of complexity and spectral density was also investigated for the remaining seven cases, resulting in different similarity measures with $\bar{\eta} = -0.33 \pm 0.20$ and correlation coefficients with $\bar{r} = -0.50 \pm 0.28$ (see Table 1).

4.1.2 Random data

With respect to a statistical substantiation of the observed inverse covariation between the spectral density and correlation dimension in the sense that this relationship is not a random one, the following null hypothesis (which is based on a rather heuristic approach) was formulated: "the observed covariation is not EEG-specific and can also be generated by a random process".

For this purpose, a sample set of 20 random signals of 4-minutes duration taken from a normal Gaussian distribution was generated by means of a standard pseudo-random number generator with a mean equal zero and a defined standard deviation using different starting points. In Fig. 6, an example of such a random signal is given. In analogy to the investigated EEG signals, the random data were filtered by the same band-pass filter.

In Table 1, the similarity measures η and correlation coefficients r between the two graphs of spectral density

and correlation dimension calculated for the ensemble of 20 random signals are summarised. The resulting distribution with a mean value of about zero approximately follows a normal Gaussian distribution which was confirmed by an adequate statistical test (Kolmogoroff-Smirnow). The mean values of η (-0.05 ± 0.15) and r (-0.07 ± 0.14) were compared with the corresponding means (-0.33 ± 0.20 and -0.50 ± 0.28) of the data set established by eight EEG signals. The equality of the means was tested by two-sided t -tests, resulting in significances $P < 0.001$, i.e. the null hypothesis that the means of η and r of the EEG sample set are identical with the corresponding means of the random sample set was rejected. This implies that the observed EEG covariation between the spectral density and correlation dimension is not a random one.

In addition, the η and r values of each EEG data set were tested to determine whether they are not random but rather EEG-specific by means of two-sided z -tests. For this purpose, a measure of significance was calculated as the difference between a value derived from the EEG data of one subject and the mean value derived from the random data divided by the standard deviation s of the random distribution. This significance measure is dimensionless and expressed in units of s . In most of the cases the significances for η and r were beyond the

Table 1. Similarity measures η and correlation coefficients r derived from the time series of complexity (embedding dimension = 16) and spectral density in the alpha band. Comparison between the results derived from 8 EEG signals and 20 pseudo-random signals taken from a normal Gaussian distribution. The differences of means were tested by two-sided t-tests resulting in significances of $P < 0.001$. The significances P_η and P_r indicate the error probability when rejecting the null hypothesis that one EEG result originate from the approximately normal distribution of the pseudo-random sample (two sided z-test)

Case	EEG signal					Pseudo-random signal (normal Gaussian distribution)	
	ID	η	P_η	r	P_r	η	r
1	101	-0.53	< 0.01	-0.84	< 0.001	-0.18	-0.45
2	201	-0.19	n.s.	-0.11	n.s.	-0.15	-0.04
3	301	-0.59	< 0.001	-0.65	< 0.001	-0.10	-0.13
4	401	0.03	n.s.	-0.19	n.s.	-0.19	-0.02
5	501	-0.31	n.s.	-0.71	< 0.001	-0.09	-0.06
6	601	-0.35	< 0.05	-0.60	< 0.001	-0.09	0.01
7	701	-0.22	n.s.	-0.22	n.s.	-0.24	-0.05
8	801	-0.46	< 0.01	-0.68	< 0.001	-0.04	0.15
9						-0.15	-0.15
10						0.11	0.13
11						0.16	-0.25
12						-0.03	0.01
13						-0.19	-0.26
14						-0.03	-0.07
15						0.19	-0.18
16						-0.18	0.01
17						-0.01	0.13
18						0.13	0.03
19						0.02	-0.07
20						-0.39	-0.05
Mean		-0.33		-0.50		-0.05	-0.07
Std. dev		0.20		0.28		0.15	0.14
P-value						<0.001	<0.01

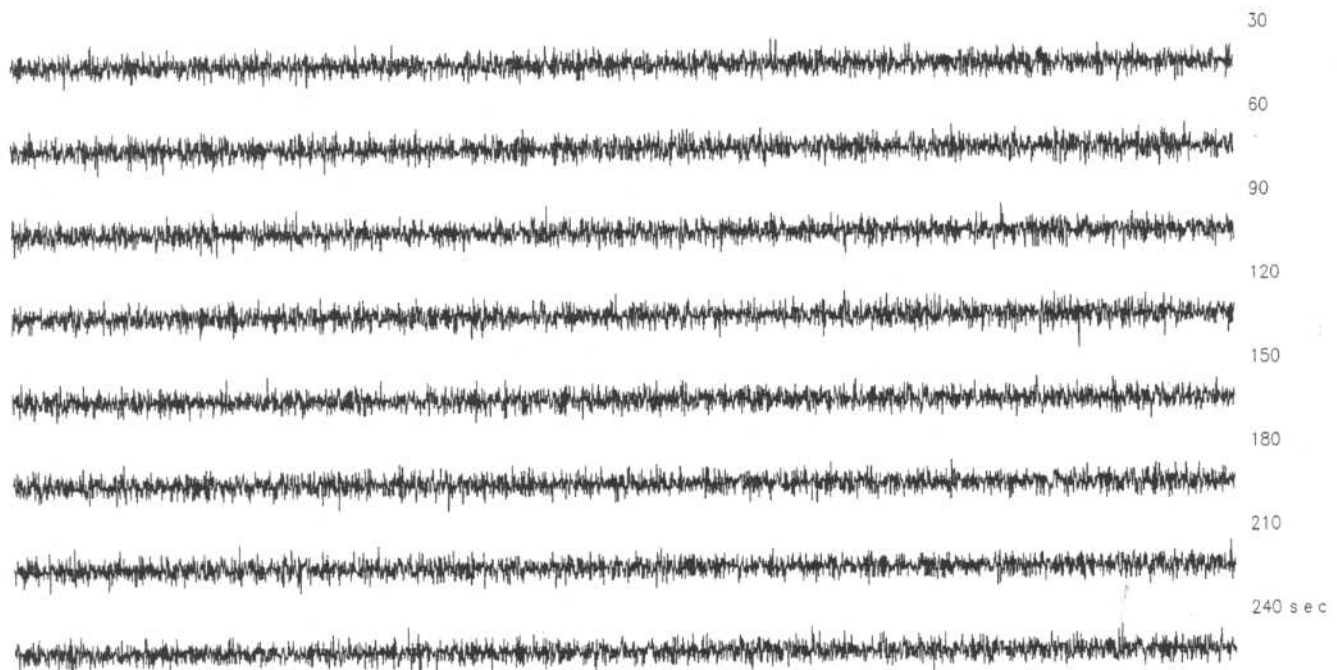


Fig. 6. Example of a Gaussian pseudo-random signal with a duration of 4 min. The random signal was smoothed by the same band-pass filter as used for the EEG data (1-40 Hz)

5% level coming down to 0.1% and rejecting the null hypothesis that the EEG results are random, i.e. they originate from the normal distribution of the pseudo-random sample set. It is pointed out that the significances based on the correlation coefficients are much better than those based on similarity measures. Nevertheless, the correlations should be considered descriptive.

Furthermore, it is stressed that the statistical comparison of the EEG data with Gaussian random data, as described in the above procedure, does not present any test of the dynamical model underlying the EEG signals.

4.2 Similarity between the cyclic alterations of the time series of the spectral density and complexity

The observed closed relationship between the two time series can also be confirmed by the application of the previously described sine-wave-fitting technique (Sect. 3.4) that considers the temporal order of systematic changes in complexity and spectral density. In Fig. 7 (left part), the time series of dimensional complexity (derived from the EEG of subject 101) according to an embedding dimension of 16 is plotted together with the optimal sine waves corresponding to the fits of the first to third order; in the same way, the time series of spectral density is presented (see Fig. 7, right part). Comparing the results of sine wave fitting derived from the two methods of analysis, the period lengths of either the first or second fit are identical, indicating both a 41-s periodicity and an identical temporal order of the two time series.

To determine whether the observed conformity in temporal order is also valid for the EEG data from other subjects, the sine-wave-fitting technique as well as the spectral analysis were applied to the two time series derived from seven subjects corresponding to the same mid-occipital lead as in the previous case. The period duration according to spectral analysis was calculated as the reciprocal value of the main peak frequency in the smoothed power spectrum. Figure 8 shows an example for such a spectrum derived from subject 101. In Table 2, the period lengths corresponding to the time courses of complexity and spectral density are summarised for the eight subjects. A matched pairs *t*-test was applied to the sample sets of period lengths derived from the sine wave fits and the power spectra. The high *P* values (*P*1) indicate that there are no significant differences between the two period lengths and suggest that an equivalence test should be applied. The corresponding alternative hypothesis is that the difference of the means is outside a given interval. In the case of sine-wave-fitting data, the revealed *P* value (*P*2) suggests that the alternative hypothesis (on the 5% level) that the means differ more than $\pm 15\%$ should be rejected. In contrast, for the period lengths derived from power spectra, the alternative hypothesis is supported due to the lower temporal resolution of the power spectral procedure.

5 Discussion

The approach of sliding spectral and dimensional analysis revealed a systematic relationship between

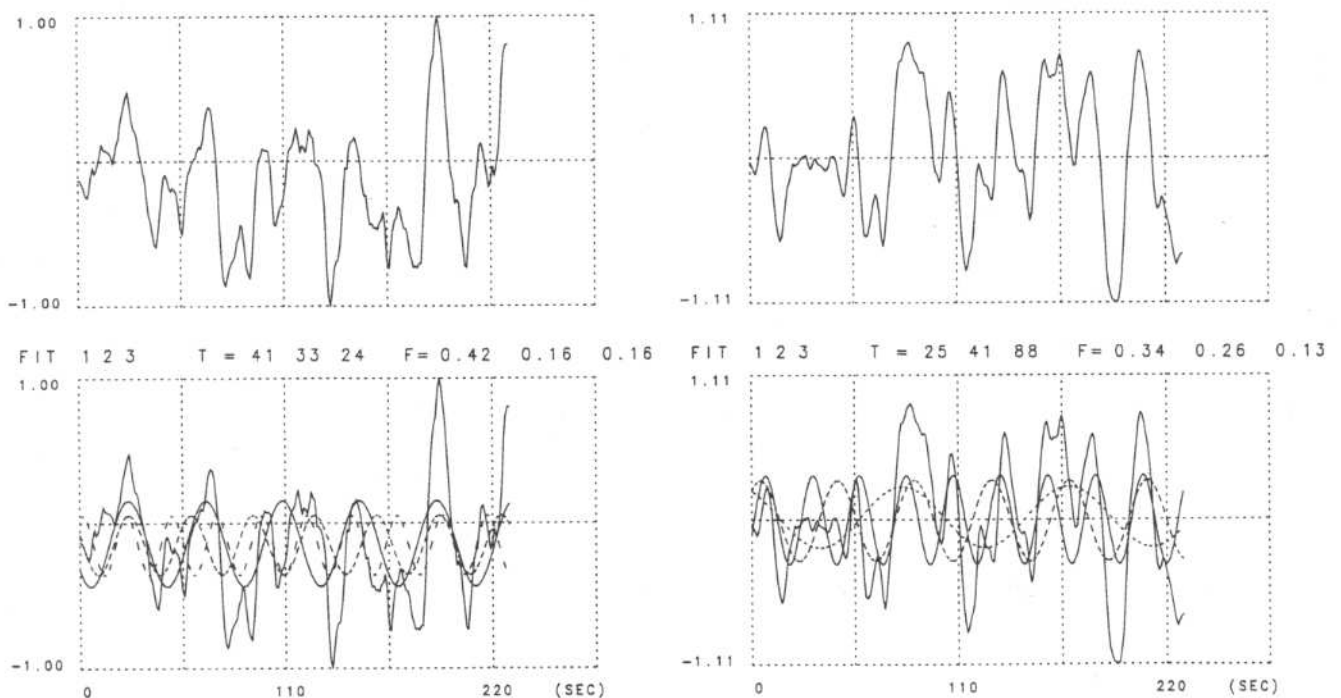


Fig. 7. Results of sine wave fitting technique. Left part upper graph Time series of complexity ($d=16$) derived from the same EEG recording as in Fig. 1. Lower graph Optimal fits of sine waves of first ($T=41$ s, solid line), second ($T=33$ s, dashed line), and third order ($T=24$ s, dashed line). Right part upper graph Time series of spectral density in the alpha band. Lower graph Optimal fits of first ($T=25$ s, solid line) and second order ($T=41$ s, dashed line). Note that the period lengths derived from complexity are nearly identical with those from the spectral density

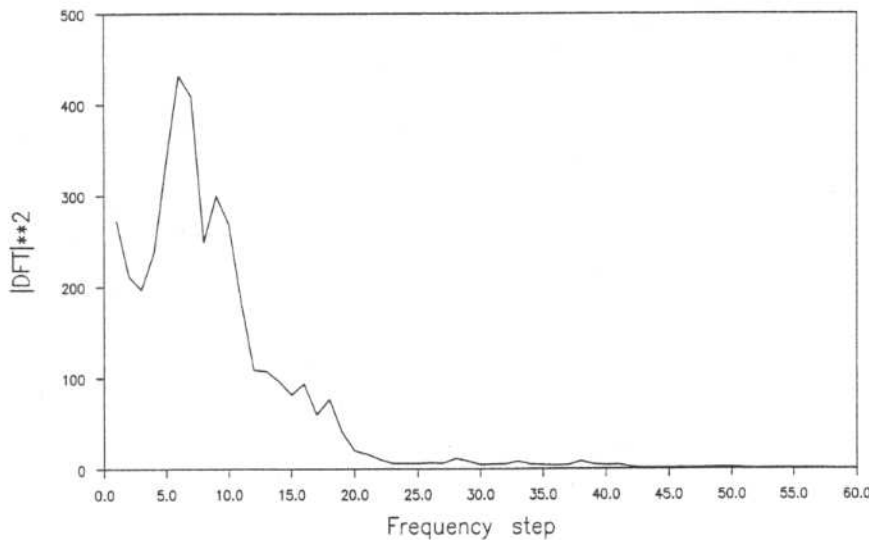


Fig. 8. Smoothed power spectrum derived from the time series of dimensional complexity as used in Fig. 7. Frequency resolution is 0.00434 Hz. The main peak at frequency step = 6 is corresponding to a period length of 38.3 s; the additional lower peak is corresponding to a period length of 25.5 s. The two peaks nearly agree with the fits of first and third order resulting from the sine-wave-fitting technique

Table 2. Comparison of period lengths calculated by sine-wave-fitting technique and spectral analysis applied to the time series of complexity with embedding dimension = 16 (T1) and alpha power (T2) derived from eight EEG signals. P_1 represents the P-value calculated by a matched pairs t-test between the period lengths derived from the two time series. The high value indicate that there are no significant differences. P_2 represents the P-value calculated by an equivalence-test rejecting the alternative hypothesis that the means differ more than 15%

Case	Sine wave fitting			Power spectrum		
	T1 (sec)	T2 (sec)	ΔT	T1 (sec)	T2 (sec)	ΔT
101	41	41	0	38	38	0
201	65	38	27	57	38	19
301	33	34	-1	32	32	0
401	46	51	-5	46	46	0
501	78	80	-2	76	76	0
601	25	22	3	46	23	+23
701	33	46	-13	38	46	-8
801	72	76	-4	76	76	0
Mean	49.13	48.50	0.62	51.12	46.87	4.25
Std. dev	19.95	20.14	11.64	17.04	19.44	10.75
P_1	0.88			0.30		
P_2	0.02			0.20		

rhythmic variations in the spectral density and correlation dimension for long-term EEG recordings exhibiting that the CNS is not maintained at a static level but operates in a dynamic oscillatory mode. The temporal pattern of dimensional complexity was found to display an inverse covariation with simultaneously computed spectral power changes most pronounced in the alpha activity of the occipital lead. The results are in line with other reports describing similar phenomena. Rösche and Basar (1989) studied the dynamics of human brain activity in different states and considered the power spectra to be a feature with complementary properties to the correlation dimension. In addition, based on the evaluation of consecutive EEG epochs from sleep data, Rösche and Aldenhoff (1991) concluded that the correlation dimension provides an additional tool with results being comparable to those obtained by spectral analysis. Pritchard and Duke (1992b) found a similar but not identical pattern of effects between the results of

spectral analysis and dimensional analysis within a multi-subject, factorial study investigating selected no-task human EEG epochs. Across the entire data set, he observed a high negative correlation between alpha power and the correlation dimension expressed in a correlation coefficient of $r = -0.622$. A step further, in our novel approach, we investigated the temporal pattern of covariation between the spectral density and dimensional complexity in the EEG dynamics of individual subjects, mainly in the alpha range. Using a special software developed by our group, we found a temporal ordered cyclic non-stationarity of alpha power and related complexity with periods of approximately 50 s during the resting (no-task) EEG state. Comparable results have not been described before, because appropriate algorithms as well as powerful computer systems have not been available. During these cycles, the relationship between power and complexity was maintained at a stable level showing high correlations.

Moreover, adequate statistical test procedures with use of pseudo-random data were carried out to reject the null hypothesis that the observed relationship is a random one revealing maximum significances at the 0.1% level. Furthermore, the period lengths of the endogenous rhythm derived from the time series of the spectral density and complexity were compared with each other, indicating that there are no significant differences and that there exists an identical temporal order of the two time series.

Our results confirm the previously defined hypothesis that cyclic alterations of spectral density are associated with corresponding variations of dimensional complexity during long-term analysis of EEG recordings in the minute range. The observed inverse covariation indicates that the spectral density of the EEG signal investigated here increases with decreasing complexity or increasing order of the underlying dynamical system. This may be due to an enhanced neuronal synchronisation with an increase of subsystem coherence, i.e. in-phase coherent oscillators causing a lower complexity of the CNS. The effects of an increase or decrease of spectral density are summarised in Fig. 9. The possible existence of 'synchronised phase-locked groups of neuronal oscillators and statistically unrelated phasors' being responsible for changes in EEG activity was the subject of a fundamental question raised by Nicolis (1987).

In addition, we propose a simple, more heuristic approach, i.e. a noise-based model which may explain the inverse relationship between the changes in spectral density and complexity observed for an EEG signal with distinct alpha activity. In this approach, an internal constant level of random-like, irregular activity, such as the thermic noise of electrical resistances, is superimposed by a deterministic, regular and oscillatory activity with varying amplitude like the alpha rhythm. An increase of regular activity (and spectral power) may reflect an increase of neuronal synchronisation and involves a decrease in the complexity of the entire signal. In the case of a decrease of regular activity which reflects

a decrease of neuronal synchronisation, the random-like activity will play a more dominant role and, consequently, the entire signal may become more complex. Finally, the complexity of the system's output activity is a function of the signal-to-noise ratio. It is pointed out that this model is only valid for EEG signals with distinct alpha activity and not for Gaussian random signals, which is confirmed by the statistics in Sect. 4.1.2. Besides this simple heuristic model, there are further competitive models, e.g. non-linear deterministic ones which we have not considered in the present paper.

Another model which may explain the described changes in complexity is reported by Basar (1983a). In the model of coupled oscillators, he suggested that the dimension of a neuronal dynamical system depends on the number of uncoupled independent oscillators within a large ensemble of coupled oscillators representing the neuronal populations of the brain. Accordingly, Röschke and Aldenhoff (1991) deduced for EEG sleep data that an increased dimensionality during sleep stages I, II and rapid eye movement (REM) may be caused by a dominance of weakly coupled oscillations of neuronal networks with independent frequencies. In contrast, a decreased dimensionality observed in slow wave sleep stages such as stage IV may be due to an increased number of strongly coupled oscillators.

Moreover, the reduced dimensionality could reflect an enhanced 'synergetic self-organisation' with fewer degrees of freedom and lower entropy (Haken 1977, 1988; Theiler 1990) which synchronises some of the sub-systems (oscillators) and cortical networks, and leads to state transitions from 'high-dimensional' to 'low-dimensional' non-linear dynamics, reflecting states with high and low entropies (Basar 1983b; Babloyantz and Destexhe 1986b; Keidel et al. 1987a; Röschke and Aldenhoff 1991). The transients in complexity are in line with Basar's (1990b) 'operative states', which were interpreted as functional operators of the EEG activity describing the degree of synchronisation and desynchronisation in various EEG frequencies in the brain. This concept also agrees with Accardo's assumptions

Increase of spectral density involves:

- Increase of order of the underlying system
- Increased neuronal synchronisation
- Increase of in-phase coherent oscillators (subsystems)
- Decrease of dimensional complexity

Decrease of spectral density involves:

- Decrease of order of the underlying system
- Increased neuronal desynchronisation
- Increase of out-of-phase incoherent oscillators (subsystems)
- Increase of dimensional complexity

Fig. 9. Model of interpretation according to the observed inverse covariation between spectral density and dimensional complexity for long-term EEG signals

about brain dynamics (Accardo et al. 1997). Besides the well-known findings of the latter authors mainly related to external stimuli, changes of sleep stages and to eyes open/closed conditions, our results claim an additional autonomous cyclic alteration of the degree of synchronisation which may be driven by a central pattern generator with an assumed location in the brain stem or diencephalic system (Keidel et al. 1987a, 1990a,b). Our findings are supported by the reports of Basar et al. (1989b), who observed large fluctuations in the correlation dimension ranging over 5–8 during 30-min EEG recordings with distinct alpha activity derived from relaxed but awake subjects with eyes closed. In contrast to our study, he used consecutive EEG epochs of 3 min duration which could not disclose the well-ordered temporal pattern of the transients in complexity.

The existence of such transients confirms the conclusions of Lopes da Silva (1991) in an experimental manner. He suggested that changing states in a neuronal network of the brain may be related to transitions from a random type of activity to an oscillatory mode which corresponds to the previously proposed noise-based model. Furthermore, he concluded that 'brain processes information in a parallel fashion' which makes sense in the state of a random-like, incoherent activity in which each neuron or neuronal population is firing or acting independently (Basar 1983b).

Accordingly, it may be deduced that transient periods of high complexity of the brain with independent active different areas may allow a fast parallel information processing mode, because numerous processes from sensory and cognitive 'channels' would be executed simultaneously, which may be related to active information processing (Accardo et al. 1997). This idea discussed by Keidel et al. (1987a), is in line with the computer metaphor of Nelson and Bower (1990) including parallel distributed processing and with the models of rapid information processing of Townsend (1990) involving parallel operations. In contrast, interposed periods of a central information processing mode with a high coupling strength between different brain structures (including hemispheres and subcortical-cortical connections) resembling a lower complexity of the brain may facilitate the data transfer to (and retrieval from) 'higher' association areas or between the hemispheres. In this case, the brain may be maintained in a 'resting' state with reduced responsiveness corresponding to Basar's (1983b) considerations and identified previously by Berger (1938) as a rather 'passive' state contrasting with an 'active' state of the brain. It could be argued that the brain shows endogenous cyclic changes between these two processing modes; this is reflected in the changes of the chaotic dynamics of the spontaneous EEG activity and fits within the processing-load hypothesis of Pritchard and Duke (1992b) and the information processing paradigm of Pritchard and Duke (1992a).

In conclusion, the occurrence of the formerly mentioned cyclic transients expressed by the inverse covariation between the spectral density and dimensional complexity seems to be an intrinsic property of the CNS with respect to the strategy of information processing

which might be responsible for higher mental functions, information storage and retrieval (Adey 1966; Keidel et al. 1987a; Llinas 1988; Basar et al. 1989a). The non-linear modelling of the EEG as used in this paper and the interpretation of the inverse covariation between the spectral density and dimensional complexity as well as the conclusions about the resulting cyclic transients give additional insight into the integrative functioning of the brain.

Acknowledgements. We are grateful to Dipl.-Math. H. Tritschler for useful comments and fruitful discussions on this paper.

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