Synchronous Languages—Lecture 16

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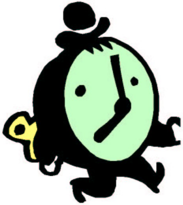

Lustre

The 5-Minute Review Session

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3. What is a characteristic of the causality handling and compilation in the Blech language?
4. In addition to event-triggered execution, which other execution models do you know?

The 5-Minute Review Session

1. In sequential constructiveness, what is the *iur-protocol*?
2. When are threads *statically concurrent*?
3. What is a characteristic of the causality handling and compilation in the Blech language?
4. In addition to event-triggered execution, which other execution models do you know?
5. What is the idea of *dynamic ticks*?

Overview

Lustre

- A synchronous data flow language
- Developed since 1984 at IMAG, Grenoble [HCRP91]
- Also graphical design entry available (SAGA)
- Moreover, the basis for SCADE, a tool used in software development for avionics and automotive industries

$\rightsquigarrow$ Translatable to FSMs with finitely many control states

- Same advantages as Esterel for hardware and software design

Lustre Modules

General form:

```
node f(x_1:α_1, ..., x_n:α_n) returns (y_1:β_1,...,y_m : β_m)
var z_1:γ_1,...,z_k:γ_k;
let
  z_1 = τ_1; ...; z_k = τ_k;
  y_1 = π_1; ...; y_m = π_k;
  assert φ_1; ...; assert φ_ℓ;
tel
```

where

- f is the name of the module
- Inputs x_i, outputs y_i, and local variables z_j
- Assertions φ_i (boolean expressions)

Lustre Programs

- Lustre programs are a list of modules that are called nodes
- All nodes work synchronously, *i. e.* at the same speed
- Nodes communicate only via inputs and outputs
- No broadcasting of signals, no side effects
- **Equations** $z_i = \tau_i$ **and** $y_i = \pi_i$ **are not assignments**
- Equations must have solutions in the mathematical sense

Lustre Programs

- As $z_i = \tau_i$ and $y_i = \pi_i$ are equations, we have the Substitution Principle:
  The definitions $z_i = \tau_i$ and $y_i = \pi_i$ of a Lustre node allow one to replace z_i by τ_i and y_i by π_i.
- Behavior of z_i and y_i completely given by equations $z_i = \tau_i$ and $y_i = \pi_i$

Assertions

- Assertions `assert` φ do not influence the behavior of the system
- `assert` φ means that during execution, φ must invariantly hold
- Equation `X = E` equivalent to assertion `assert(X = E)`
- Assertions can be used to optimize the code generation
- Assertions can be used for simulation and verification

Data Streams

- All variables, constants, and all expressions are streams, *i. e.*, sequences of values of a certain type
- Streams can be composed to new streams
- Example: given $x = (0, 1, 2, 3, 4, \ldots)$ and $y = (0, 2, 4, 6, 8, \ldots)$, then $x + y$ is the stream $(0, 3, 6, 9, 12, \ldots)$
- However, **streams may refer to different clocks**

$\leadsto$ Each stream has a corresponding clock, which filters out elements whenever the clock is false

- Per default, streams run on the base clock, which is always true

Data Types

- Primitive data types: `bool`, `int`, `real`
  - Semantics is clear?
- Imported data types: `type` α
  - Similar to Esterel
  - Data type is implemented in host language
- Tuples of types: $\alpha_1 \times \ldots \times \alpha_n$ is a type
  - Semantics is Cartesian product

Expressions (Streams)

- Every declared variable x is an expression
- Boolean expressions:
  - τ_1 `and` τ_2, τ_1 `or` τ_2, `not` τ_1
- Numeric expressions:
  - $\tau_1 + \tau_2$ and $\tau_1 - \tau_2$, $\tau_1 * \tau_2$ and τ_1 / τ_2, τ_1 `div` τ_2 and τ_1 `mod` τ_2
- Relational expressions:
  - $\tau_1 = \tau_2$, $\tau_1 < \tau_2$, $\tau_1 \leq \tau_2$, $\tau_1 > \tau_2$, $\tau_1 \geq \tau_2$
- Conditional expressions:
  - `if b then` τ_1 `else` τ_2 for all types

Node Expansion

- Assume implementation of a node f with inputs $x_1 : \alpha_1, \ldots,$ $x_n : \alpha_n$ and outputs $y_1 : \beta_1, \ldots, y_m : \beta_m$
- Then, f can be used to create new stream expressions, *e. g.*, $f(\tau_1, \ldots, \tau_n)$ is an expression
  - Of type $\beta_1 \times \ldots \times \beta_m$
  - If $(\tau_1, \ldots, \tau_n)$ has type $\alpha_1 \times \ldots \times \alpha_n$

Vector Notation of Nodes

By using tuple types for inputs, outputs, and local streams, we may consider just nodes like

```
node f(x:α) returns (y:β)
var  z:γ;
let
  z = τ;
  y = π;
  assert φ;
tel
```

Clock-Operators

- All expressions are streams
- Clock-operators modify the temporal arrangement of streams
- Again, their results are streams
- The following clock operators are available:
  - `pre` τ for every stream τ
  - τ_1 `->` τ_2, (initialization) where τ_1 and τ_2 have the same type
  - τ_1 `when` τ_2 where τ_2 has boolean type (downsampling)
  - `current` τ (upsampling)

Clock-Hierarchy

- As already mentioned, streams may refer to different clocks
- We associate with every expression a list of clocks
- A clock is thereby a stream φ of boolean type

Clock-Hierarchy

- clocks(τ) $:=$ [] for expressions without clock operators

Clock-Hierarchy

- $\text{clocks}(\tau) := []$ for expressions without clock operators
- $\text{clocks}(\texttt{pre}(\tau)) := \text{clocks}(\tau)$

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- clocks(τ) := [] for expressions without clock operators
- clocks(`pre`(τ)) := clocks(τ)
- clocks(τ_1 `->` τ_2) := clocks(τ_1),
  where clocks(τ_1) = clocks(τ_2) is required

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- $\text{clocks}(\texttt{pre}(\tau)) := \text{clocks}(\tau)$
- $\text{clocks}(\tau_1 \texttt{ -> } \tau_2) := \text{clocks}(\tau_1)$,
  where $\text{clocks}(\tau_1) = \text{clocks}(\tau_2)$ is required
- $\text{clocks}(\tau \texttt{ when } \varphi) := [\varphi, c_1, \ldots, c_n]$,
  where $\text{clocks}(\varphi) = \text{clocks}(\tau) = [c_1, \ldots, c_n]$

Clock-Hierarchy

- clocks(τ) $:=$ [] for expressions without clock operators
- clocks($\texttt{pre}(\tau)$) $:=$ clocks(τ)
- clocks(τ_1 `->` τ_2) $:=$ clocks(τ_1),
  where clocks(τ_1) $=$ clocks(τ_2) is required
- clocks(τ `when` φ) $:= [\varphi, c_1, \ldots, c_n]$,
  where clocks(φ) $=$ clocks(τ) $= [c_1, \ldots, c_n]$
- clocks($\texttt{current}(\tau)$) $:= [c_2, \ldots, c_n]$,
  where clocks(τ) $= [c_1, \ldots, c_n]$

Semantics of Clock-Operators

- $[\![\texttt{pre}(\tau)]\!] := (\perp, \tau_0, \tau_1, \ldots)$, provided that $[\![\tau]\!] = (\tau_0, \tau_1, \ldots)$

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  provided that $[\![\tau]\!] = (\tau_0, \tau_1, \ldots)$ and $[\![\pi]\!] = (\pi_0, \pi_1, \ldots)$
- $[\![\tau \texttt{ when } \varphi]\!] = (\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, \ldots)$, provided that
  - $[\![\tau]\!] = (\tau_0, \tau_1, \ldots)$
  - $\{t_0, t_1, \ldots\}$ is the set of points in time where $[\![\varphi]\!]$ holds

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- $[\![\tau \texttt{ when } \varphi]\!] = (\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, \ldots)$, provided that
  - $[\![\tau]\!] = (\tau_0, \tau_1, \ldots)$
  - $\{t_0, t_1, \ldots\}$ is the set of points in time where $[\![\varphi]\!]$ holds
- $[\![\texttt{current}(\tau)]\!] = (\perp, \ldots, \perp, \tau_0, \ldots, \tau_0, \tau_1, \ldots, \tau_1, \tau_2, \ldots)$,
  provided that
  - $[\![\tau]\!] = (\tau_0, \tau_1, \ldots)$
  - Stream holds value of τ from last tick of *clock of* clock of τ

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
$\texttt{pre}(\tau)$							

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φ	0	1	0	1	0	0	1
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$\mathtt{pre}(\tau)$	$\perp$	τ_0					

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φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
`pre`(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `->` `pre`(τ)							

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τ `->` `pre`(τ)	τ_0	τ_0					

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τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
`pre`(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `-> pre`(τ)	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `when` φ							

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
pre(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ -> pre(τ)	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ when φ		τ_1					

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
`pre`(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `->` `pre`(τ)	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `when` φ		τ_1		τ_3			τ_6
`current`$(\tau$ `when` $\varphi)$							

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
`pre`(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `->` `pre`(τ)	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `when` φ		τ_1		τ_3			τ_6
`current`$(\tau$ `when` $\varphi)$	$\perp$						

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
`pre`(τ)	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `->` `pre`(τ)	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `when` φ		τ_1		τ_3			τ_6
`current`(τ `when` φ)	$\perp$	τ_1					

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
$\texttt{pre}(\tau)$	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
$\tau\ \texttt{->}\ \texttt{pre}(\tau)$	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
$\tau\ \texttt{when}\ \varphi$		τ_1		τ_3			τ_6
$\texttt{current}(\tau\ \texttt{when}\ \varphi)$	$\perp$	τ_1	τ_1				

Example for Semantics of Clock-Operators

φ	0	1	0	1	0	0	1
τ	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
$\mathtt{pre}(\tau)$	$\perp$	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `->` $\mathtt{pre}(\tau)$	τ_0	τ_0	τ_1	τ_2	τ_3	τ_4	τ_5
τ `when` φ		τ_1		τ_3			τ_6
$\mathtt{current}(\tau\ \mathtt{when}\ \varphi)$	$\perp$	τ_1	τ_1	τ_3	τ_3	τ_3	τ_6

- Note: $[\![\tau\ \mathtt{when}\ \varphi]\!] = (\tau_1, \tau_3, \tau_6, \ldots)$, *i. e.*, **gaps are not filled!**
- This is done by $\mathtt{current}(\tau\ \mathtt{when}\ \varphi)$

Example for Semantics of Clock-Operators

0	

Example for Semantics of Clock-Operators

	0	0	0	0	0	0	0	...
	1							

Example for Semantics of Clock-Operators

0	0	0	0	0	0	0	...
1	1	1	1	1	1	1	...
`n = (0 -> pre(n)+1)`							

Example for Semantics of Clock-Operators

0	0	0	0	0	0	0	...
1	1	1	1	1	1	1	...
`n = (0 -> pre(n)+1)`	0	1	2	3	4	5	...
`e = (1 -> not pre(e))`							

Example for Semantics of Clock-Operators

`0`	0	0	0	0	0	0	...
`1`	1	1	1	1	1	1	...
`n = (0 -> pre(n)+1)`	0	1	2	3	4	5	...
`e = (1 -> not pre(e))`	1	0	1	0	1	0	...
`n when e`							

Example for Semantics of Clock-Operators

0	0	0	0	0	0	0	...
1	1	1	1	1	1	1	...
`n = (0 -> pre(n)+1)`	0	1	2	3	4	5	...
`e = (1 -> not pre(e))`	1	0	1	0	1	0	...
`n when e`	0		2		4		...
`current(n when e)`							

Example for Semantics of Clock-Operators

`0`	0	0	0	0	0	0	...
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`n = (0 -> pre(n)+1)`	0	1	2	3	4	5	...
`e = (1 -> not pre(e))`	1	0	1	0	1	0	...
`n when e`	0		2		4		...
`current(n when e)`	0	0	2	2	4	4	...
`current (n when e) div 2`							

Example for Semantics of Clock-Operators

`0`	0	0	0	0	0	0	...
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`n = (0 -> pre(n)+1)`	0	1	2	3	4	5	...
`e = (1 -> not pre(e))`	1	0	1	0	1	0	...
`n when e`	0		2		4		...
`current(n when e)`	0	0	2	2	4	4	...
`current (n when e) div 2`	0	0	1	1	2	2	...

Example for Semantics of Clock-Operators

`n = 0 -> pre(n)+1`	

Example for Semantics of Clock-Operators

`n = 0 -> pre(n)+1`	0	1	2	3	4	5	6	7	8	9	10	11
`d2 = (n div 2)*2 = n`												

Example for Semantics of Clock-Operators

`n = 0 -> pre(n)+1`	0	1	2	3	4	5	6	7	8	9	10	11
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`n2 = n when d2`												

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`n2 = n when d2`	0		2		4		6		8		10	
`d3 = (n div 3)*3 = n`												

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`n2 = n when d2`	0		2		4		6		8		10	
`d3 = (n div 3)*3 = n`	1	0	0	1	0	0	1	0	0	1	0	0
`n3 = n when d3`												

Example for Semantics of Clock-Operators

`n = 0 -> pre(n)+1`	0	1	2	3	4	5	6	7	8	9	10	11
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`n2 = n when d2`	0		2		4		6		8		10	
`d3 = (n div 3)*3 = n`	1	0	0	1	0	0	1	0	0	1	0	0
`n3 = n when d3`	0			3			6			9		
`d3' = d3 when d2`												

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`n = 0 -> pre(n)+1`	0	1	2	3	4	5	6	7	8	9	10	11
`d2 = (n div 2)*2 = n`	1	0	1	0	1	0	1	0	1	0	1	0
`n2 = n when d2`	0		2		4		6		8		10	
`d3 = (n div 3)*3 = n`	1	0	0	1	0	0	1	0	0	1	0	0
`n3 = n when d3`	0			3			6			9		
`d3' = d3 when d2`	1		0		0		1		0		0	
`n6 = n2 when d3'`												

Example for Semantics of Clock-Operators

n = 0 -> pre(n)+1	0	1	2	3	4	5	6	7	8	9	10	11
d2 = (n div 2)*2 = n	1	0	1	0	1	0	1	0	1	0	1	0
n2 = n when d2	0		2		4		6		8		10	
d3 = (n div 3)*3 = n	1	0	0	1	0	0	1	0	0	1	0	0
n3 = n when d3	0			3			6			9		
d3' = d3 when d2	1		0		0		1		0		0	
n6 = n2 when d3'	0						6					
c3 = current(n2 when d3')												

Example for Semantics of Clock-Operators

`n = 0 -> pre(n)+1`	0	1	2	3	4	5	6	7	8	9	10	11
`d2 = (n div 2)*2 = n`	1	0	1	0	1	0	1	0	1	0	1	0
`n2 = n when d2`	0		2		4		6		8		10	
`d3 = (n div 3)*3 = n`	1	0	0	1	0	0	1	0	0	1	0	0
`n3 = n when d3`	0			3			6			9		
`d3' = d3 when d2`	1		0		0		1		0		0	
`n6 = n2 when d3'`	0						6					
`c3 = current(n2 when d3')`	0		0		0		6		6		6	

Example: Counter

```
node Counter(x0, d:int; r:bool) returns (n:int)
let
  n = x0 → if r then x0 else pre(n) + d
tel
```

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node Counter(x0, d:int; r:bool) returns (n:int)
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```

- Initial value of n is $x0$
- If no reset r then increment by d
- If reset by r, then initialize with x_0
- *Counter* can be used in other equations, e.g.
  - $ex1 = Counter(0, 2, 0)$ yields

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- *Counter* can be used in other equations, e.g.
  - $ex1 = Counter(0, 2, 0)$ yields the even numbers
  - $ex2 = Counter(0, 1, \texttt{pre}(ex2) = 4)$ yields

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- Initial value of n is $x0$
- If no reset r then increment by d
- If reset by r, then initialize with x_0
- *Counter* can be used in other equations, e.g.
  - $ex1 = Counter(0, 2, 0)$ yields the even numbers
  - $ex2 = Counter(0, 1, \mathtt{pre}(ex2) = 4)$ yields numbers mod 5

ABRO in Lustre

```
node EDGE(X:bool) returns (Y:bool);
let
  Y = false -> X and not pre(X);
tel


node ABRO (A,B,R:bool) returns (O: bool);
  var seenA, seenB : bool;
let
  O = EDGE(seenA and seenB);
  seenA = false -> not R and (A or pre(seenA));
  seenB = false -> not R and (B or pre(seenB));
tel
```

Causality Problems in Lustre

- Synchronous languages have causality problems
- They arise if preconditions of actions are influenced by the actions
- Therefore they require to solve fixpoint equations
- Such equations may have none, one, or more than one solutions

$\rightsquigarrow$ Analogous to Esterel, one may consider reactive, deterministic, logically correct, and constructive programs

Causality Problems in Lustre

- $x = \tau$ is acyclic, if x does not occur in τ or does only occur as subterm $\texttt{pre}(x)$ in τ
- Examples:
  - `a = a and pre(a)` is

Causality Problems in Lustre

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Causality Problems in Lustre

- $x = \tau$ is acyclic, if x does not occur in τ or does only occur as subterm $\texttt{pre}(x)$ in τ
- Examples:
  - `a = a and pre(a)` is cyclic
  - `a = b and pre(a)` is acyclic
- Acyclic equations have a unique solution!
- Analyze cyclic equations to determine causality?
- But: **Lustre only allows acyclic equation systems**
- Sufficient for signal processing

Malik's Example

- However, some interesting examples are cyclic

```
y   = if c then y_f else y_g;
y_f = f(x_f);
y_g = g(x_g);
x_f = if c then y_g else x;
x_g = if c then x else y_f;
```

- Implements `if c then f(g(x)) else g(f(x))` with only one instance of `f` and `g`
- **Impossible without cycles**

Sharad Malik.
Analysis of cyclic combinatorial circuits.
in IEEE Transactions on Computer-Aided Design, 1994

Clock Consistency

Consider the following equations:

```
b = 0→not pre(b);
y = x + (x when b)
```

- We obtain the following:

x	x_0	x_1	x_2	x_3	x_4	...
b						

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x when b		x_1		x_3		...
$x + (x$ when $b)$	$x_0 + x_1$	$x_1 + x_3$	$x_2 + x_5$	$x_3 + x_7$	$x_4 + x_9$	...

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- To compute $y_i := x_i + x_{2i+1}$, we have to store $x_i, \ldots, x_{2i+1}$
- **Problem: not possible with finite memory**

Clock Consistency

- Expressions like $x + (x \texttt{ when } b)$ are not allowed
- **Only streams at the same clock can be combined**
- What is the 'same' clock?
- Undecidable to prove this semantically
- Check syntactically

Clock Consistency

- Two streams have the same clock if their clock can be syntactically unified
- Example:

$$
\begin{aligned}
x &= a \ \texttt{when} \ (y > z);\\
y &= b + c;\\
u &= d \ \texttt{when} \ (b + c > z);\\
v &= e \ \texttt{when} \ (z < y);
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$$

- x and u have the same clock
- x and v do not have the same clock

Arrays

- Given type α, α^n defines an array with n entries of type α
- Example: `x: bool`^{n}
- The bounds of an array must be known at compile time, the compiler simply transforms an array of n values into n different variables.
- The i-th element of an array X is accessed by $X[i]$.
- $X[i..j]$ with $i \leq j$ denotes the array made of elements i to j of X.
- Beside being syntactical sugar, arrays allow to combine variables for better hardware implementation.

Example for Arrays

```
node DELAY (const d: int; X: bool) returns (Y: bool);
  var A: bool^(d+1);
let
  A[0] = X;
  A[1..d] = (false^(d))→ pre(A[0..d--1]);
  Y = A[d];
tel
```

- `false`$^{(d)}$ denotes the boolean array of length d, which entries are all `false`
- Observe that `pre` and `->` can take arrays as parameters
- Since d must be known at compile time, this node cannot be compiled in isolation

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- `false`$^{(d)}$ denotes the boolean array of length d, which entries are all `false`
- Observe that `pre` and `->` can take arrays as parameters
- Since d must be known at compile time, this node cannot be compiled in isolation
- The node outputs each input delayed by d steps.
- So $Y_n = X_{n-d}$ with $Y_n = \mathit{false}$ for $n < d$

Static Recursion

- Functional languages usually make use of recursively defined functions
- Problem: termination of recursion in general undecidable

$\rightsquigarrow$ Primitive recursive functions guarantee termination

- Problem: still with primitive recursive functions, the reaction time depends heavily on the input data

$\rightsquigarrow$ Static recursion: recursion only at compile time

- Observe: If the recursion is not bounded, the compilation will not stop.

Example for Static Recursion

- Disjunction of boolean array

```
node BigOr(const n:int; x: bool^n) returns (y:bool)
let
y = with n=1 then x[0]
    else x[0] or BigOr(n--1,x[1..n--1]);
tel
```

- Constant *n* must be known at compile time
- Node is unrolled before further compilation

Example for Maximum Computation

Static recursion allows logarithmic circuits:

```
node Max(const n:int; x:int^n) returns (y:int)
  var y_1,y_2: int;
let
  y_1 = with n=1 then x[0]
        else Max(n div 2,x[0..(n div 2)--1]);
  y_2 = with n=1 then x[0]
        else Max((n+1) div 2, x[(n div 2)..n--1]);
  y = if y_1 >= y_2 then y_1 else y_2;
tel
```

Delay node with recursion

```
node REC_DELAY (const d: int; X: bool) returns (Y: bool);
let
  Y = with d=0 then X
  else false → pre(REC_DELAY(d--1, X));
tel
```

A call `REC_DELAY(3, X)` is compiled into something like:

Delay node with recursion

```
node REC_DELAY (const d: int; X: bool) returns (Y: bool);
let
  Y = with d=0 then X
  else false → pre(REC_DELAY(d--1, X));
tel
```

A call `REC_DELAY(3, X)` is compiled into something like:

```
  Y  = false → pre(Y2)
  Y2 = false → pre(Y1)
  Y1 = false → pre(Y0)
  Y0 = X;
```

Summary

- Lustre is a synchronous dataflow language.
- The core Lustre language are boolean equations and clock operators `pre`, `->`, `when`, and `current`.
- Additional datatypes for real and integer numbers are also implemented.
- User types can be defined as in Esterel.
- Lustre only allows acyclic programs.
- Clock consistency is checked syntactically.
- Lustre offers arrays and recursion, but both array-size and number of recursive calls must be known at compile time.

To Go Further

- Nicolas Halbwachs and Pascal Raymond, A Tutorial of Lustre, 2002 `http://www-verimag.imag.fr/~halbwach/lustre-tutorial.html`
- Nicolas Halbwachs, Paul Caspi, Pascal Raymond, and Daniel Pilaud,The Synchronous Data-Flow Programming Language Lustre, In Proceedings of the IEEE, 79:9, September 1991, `http://www-verimag.imag.fr/~halbwach/lustre:ieee.html`