

# **Multi-Paradigm Programming**

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Extend functional languages with features for

- ① logic (constraint) programming
- ② object-oriented programming
- ③ concurrent programming
- ④ distributed programming

# **DECLARATIVE PROGRAMMING**

#### **General idea:**

- no coding of algorithms
- description of logical relationships
- powerful abstractions
  - → domain specific languages
- higher programming level
- reliable and maintainable programs
  - $\rightarrow$  pointer structures  $\Rightarrow$  algebraic data types
  - → complex procedures ⇒ comprehensible parts (pattern matching, local definitions)

# DECLARATIVE PROGRAMMING: PARADIGMS

#### Functional programming:

- → functions,  $\lambda$ -calculus
- → equations
- → (lazy) deterministic reduction

#### Logic programming:

- ➔ predicates, predicate logic
- → logical formulas, Horn clauses
- → constraint solving (unification)
- → non-deterministic search for solutions

# FUNCTIONAL LOGIC LANGUAGES

- efficient execution principles of functional languages
- flexibility of logic languages
- avoid non-declarative features of Prolog (arithmetic, I/O, cut)
- combine best of both worlds in a single model
  - $\rightarrow$  higher-order functions  $\rightsquigarrow$  design patterns
  - → declarative I/O
  - → concurrent constraints

# IMPERATIVE VS. DECLARATIVE PROGRAMMING

#### **Readability, safety:**

```
function fac(n: nat): nat =
begin
    z := 1; p := 1;
    while z<n+1 do
    begin p := p*z; z := z+1 end;
    return(p)
end</pre>
```

```
fac 0 = 1
fac (n+1) = (n+1) * (fac n)
```

#### Quicksort: Classical imperative version:

```
procedure qsort(1,r: index);
var i,j: index; x,w: item
begin
   i := l; j := r;
   x := a[(l+r) div 2];
   repeat
      while a[i] < x do i := i+1;
      while x < a[j] do j := j-1;
      if i <= j then
      begin w := a[i]; a[i] := a[j]; a[j] := w;
         i := i+1; j := j-1
      end
   until i > j;
   if l < j then qsort(l,j);</pre>
   if i < r then qsort(i,r);</pre>
end
```

#### Quicksort: Classical imperative version:

```
Declarative version:
procedure qsort(1,r: index);
var i,j: index; x,w: item
                                  (qsort [] = []
begin
                                  qsort(x:1) =
   i := l; j := r;
                                      qsort (filter (<x) l)</pre>
   x := a[(l+r) div 2];
                                      ++ [x]
                                       ++ qsort (filter (>=x) 1)
   repeat
      while a[i] < x do i := i+1;
      while x < a[j] do j := j-1;
      if i <= j then
      begin w := a[i]; a[i] := a[j]; a[j] := w;
         i := i+1; j := j-1
      end
   until i > j;
   if l < j then qsort(l,j);</pre>
   if i < r then qsort(i,r);</pre>
end
```

# IMPERATIVE VS. DECLARATIVE PROGRAMMING

#### **Program development and maintenance:**

```
function f(n: nat): nat =
begin
    write('Hello');
    return(n*n)
end
.... z:=f(3)*f(3) ...
```

Optimization: ...  $x:=f(3); z:=x*x \dots (?)$ 

 $\sim$  side effects complicate program optimization and transformation

# CURRY

As a language for concrete examples, we use **Curry**: [Dagstuhl'96, POPL'97]

- multi-paradigm language
- extension of Haskell (non-strict functional language)
- developed by an international initiative
- provide a standard for functional logic languages (research, teaching, application)
- several implementations available

## BASIS OF DECLARATIVE PROGRAMMING: ALGEBRAIC DATA TYPES

Values in imperative languages: basic types + pointer structures

Declarative languages: algebraic data types (Haskell-like syntax)



#### Value $\approx$ data term, constructor term:

well-formed expression containing variables and data type constructors

(S Z) 1:(2:[]) [1,2] Node [Leaf 3, Node [Leaf 4, Leaf 5]]

# FUNCTIONAL PROGRAMS

**Functions**: operations on values defined by equations (or rules)



Reduce expressions to their values

Replace equals by equals

Apply reduction step to a subterm (redex, reducible expression):

variables in rule's left-hand side are universally quantified  $\rightarrow$  match lhs against subterm (instantiate these variables)

Z + y = y	$Z \leq y$	= True	
(S x) + y = S(x+y)	(S x) $\leq$ Z	= False	
	(S x) $\leq$ (S y	) = $x \le y$	

 $(S Z)+(S Z) \rightarrow S (Z+(S Z)) \rightarrow S (S Z)$ 

# **EVALUATION STRATEGIES**

Expressions with several redexes: which evaluate first?

Strict evaluation: select an innermost redex ( $\approx$  call-by-value)

Lazy evaluation: select an outermost redex

Strict evaluation:

 $Z \leq (S Z)+(S Z) \rightarrow Z \leq (S (Z+(S Z)) \rightarrow Z \leq (S (S Z)) \rightarrow True$ 

Lazy evaluation:

 $Z \leq (S Z)+(S Z) \rightarrow True$ 

Strict evaluation might need more steps, but it can be even worse...

Lazy evaluation:

 $Z+Z \leq f \rightarrow Z \leq f \rightarrow True$ 

#### Strict evaluation:

 $\mathsf{Z}\mathsf{+}\mathsf{Z} \ \leq \mathbf{f} \quad \rightarrow \quad \mathsf{Z}\mathsf{+}\mathsf{Z} \ \leq \ \mathbf{f} \quad \rightarrow \quad \mathsf{Z}\mathsf{+}\mathsf{Z} \ \leq \ \mathbf{f} \quad \rightarrow \quad \cdots$ 

Ideal strategy: evaluate only needed redexes

(i.e., redexes necessary to compute a value)

Determine needed redexes with definitional trees

# DEFINITIONAL TREES [ANTOY 92]

- → data structure to organize the rules of an operation
- → each node has a distinct pattern
- → *branch* nodes (case distinction), *rule* nodes



# **EVALUATION WITH DEFINITIONAL TREES**



Evaluating function call  $t_1 \leq t_2$ :

- ① Reduce  $t_1$  to head normal form (constructor-rooted expression)
- ② If  $t_1 = Z$ : apply rule
- ③ If  $t_1 = (S \dots)$ : reduce  $t_2$  to head normal form

# **PROPERTIES OF REDUCTION WITH DEFINITIONAL TREES**

#### Normalizing strategy

i.e., always computes value if it exists  $\approx$  sound and complete

- Independent on the order of rules
- Definitional trees can be automatically generated
   → pattern matching compiler
- Identical to lazy functional languages (e.g, Miranda, Haskell) for the subclass of uniform programs

   (i.e., programs with strong left-to-right pattern matching)
- Optimal strategy: each reduction step is needed
- Easily extensible to more general classes

# **HIGHER-ORDER FUNCTIONS**

#### Functions are first class citizens

- ➔ passing functions as parameters and results
- → combinator-oriented programming
- → expressing design patterns
- → code reuse

map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = (f x) : map f xs

map (1 +)  $[2,3,4] \rightarrow [3,4,5]$ 

Partial application: (1 +) is a function of type Int->Int

 $\lambda$ -abstraction:  $x \rightarrow 1+x$  (anonymous function)

# HIGHER-ORDER FUNCTIONS: EXAMPLES

#### Accumulate list elements with a binary operator:

foldr f z	[] =	Z	
foldr f z	(x:xs) =	f x (foldr f z xs)	)

Multiply all list elements: foldr (\*) 1 xs

Concatenate a list of lists: concat xs = foldr (++) [] xs

Tree example: computing list of all leaves in a tree:

frontier :: Tree a -> [a]
frontier (Leaf v) = [v]
frontier (Node ns) = concat (map frontier ns)

Filter all elements in a list satisfying a given predicate:

```
filter :: (a -> Bool) -> [a] -> [a]
filter p [] = []
filter p (x:xs) = if p x then x : filter p xs
else filter p xs
```

Now the code for quicksort becomes straightforward:

qsort [] = []
qsort (x:1) = qsort (filter (<x) 1)
++ [x] ++ qsort (filter (>=x) 1)

# **APPLICATION: HTML PROGRAMMING**

#### Data type for representing HTML expressions:

data HtmlExp =	HText String		
I	HStruct String	[(String,String)]	[HtmlExp]

HStruct "A" [("HREF","http://...")] [HText "click here"]

#### Get all hypertext links in an HTML document:

### NON-DETERMINISTIC EVALUATION

Previous functions: inductively defined on data structures

Sometimes overlapping rules more natural:

True  $\lor$  x = True x  $\lor$  True = True False  $\lor$  False = False

First two rules overlap on True  $\lor$  True

 $\rightsquigarrow$  Problem: no needed argument:  $e_1 \lor e_2$  evaluate  $e_1$  or  $e_2$ ?

Functional languages: backtracking: Evaluate  $e_1$ , if not successful:  $e_2$ 

**Disadvantage:** not normalizing ( $e_1$  may not terminate)

**NON-DETERMINISTIC EVALUATION** 

Tru	e ∨	х	=	True
	x V	True	=	True
<pre>Fals</pre>	e V	False	=	False

Evaluation of  $e_1 \vee e_2$ ?

- 1. Parallel reduction of  $e_1$  and  $e_2$  [Sekar/Ramakrishnan 93]
- 2. Non-deterministic reduction: try (don't know)  $e_1$  or  $e_2$

Extension to definitional trees / pattern matching: Introduce *or*-nodes to describe non-deterministic selection of redexes

 $\rightsquigarrow$  non-deterministic evaluation:  $e \rightarrow$ 



disjunctive expression

 $\rightsquigarrow$  non-deterministic functions

# **NON-DETERMINISTIC FUNCTIONS**

Functions can have more than one result value:

choose x y = x choose x y = y

choose 1 2  $\rightarrow$  1 | 2

Non-deterministic list insertion and permutations:

insert x [] = [x] insert x (y:ys) = choose (x:y:ys) (y:insert x ys) permute [] = [] permute (x:xs) = insert x (permute xs)

permute [1,2,3]  $\rightarrow$ [1,2,3] | [2,1,3] | [2,3,1] | [1,3,2] | [3,1,2] | [3,2,1]

# LOGIC PROGRAMMING

Distinguished features:

- → compute with partial information (constraints)
- → deal with free variables in expressions
- → compute solutions to free variables
- → built-in search
- → non-deterministic evaluation

Functional programming: values, no free variables

Logic programming: computed answers for free variables

Operational extension: instantiate free variables, if necessary

# FROM FUNCTIONAL PROGRAMMING TO LOGIC PROGRAMMING

Evaluate (f x): - bind x to 0 and reduce (f 0) to 2, or:

- bind x to 1 and reduce (f 1) to 3



Compute necessary bindings with needed strategy ~ needed narrowing [Antoy/Echahed/Hanus POPL'94/JACM'00]

# **EVALUATION WITH DEFINITIONAL TREES**



#### Evaluating function call $t_1 \leq t_2$ :

- 1 Reduce  $t_1$  to head normal form
- ② If  $t_1 = Z$ : apply rule
- ③ If  $t_1 = (S \dots)$ : reduce  $t_2$  to head normal form

# NEEDED NARROWING



#### Evaluating function call $t_1 \leq t_2$ :

- 1 Reduce  $t_1$  to head normal form
- ② If  $t_1 = Z$ : apply rule
- ③ If  $t_1 = (S \dots)$ : reduce  $t_2$  to head normal form
- ④ If  $t_1$  variable: bind  $t_1$  to Z or (S x)

# **PROPERTIES OF NEEDED NARROWING**

#### Sound and complete (w.r.t. strict equality, no termination requirement)

#### **Optimality:**

① No unnecessary steps:

Each narrowing step is needed, i.e., it cannot be avoided if a solution should be computed.

**②** Shortest derivations:

If common subterms are shared, needed narrowing derivations have minimal length.

**③ Minimal set of computed solutions:** 

Two solutions  $\sigma$  and  $\sigma'$  computed by two distinct derivations are independent.

#### **Determinism:**

No non-deterministic step during the evaluation of ground expressions ( $\approx$  functional programming)

#### **Restriction: inductively sequential rules**

(i.e., no overlapping left-hand sides)

Extensible to

- → conditional rules [Hanus ICLP'95]
- → overlapping left-hand sides [Antoy/Echahed/Hanus ICLP'97]
- → multiple right-hand sides [Antoy ALP'97]
- → concurrent evaluation [Hanus POPL'97]

### STRICT EQUALITY

Problems with equality in the presence of non-terminating rules:

1. Equality on infinite objects undecidable:

$$f = 0:f$$
  $g = 0:g$ 

Is f = g valid?

2. Semantics of non-terminating functions:

f x = f (x+1) g x = g (x+1)

Is f 0 = g 0 valid?

Avoided by **strict equality**: identity on *finite* objects (both sides reducible to same ground data term)

# EQUATIONAL CONSTRAINTS

Logic programming: solve goals, compute solutions

Functional logic programming: solve equations

Strict equality: only reasonable notion of equality in the presence of non-terminating functions

Equational constraint  $e_1 = := e_2$ 

satisfied if both sides evaluable to unifiable data terms

$$\Rightarrow e_1 = := e_2$$
 does not hold if  $e_1$  or  $e_2$  undefined or infinite

$$\Rightarrow e_1 = := e_2$$
 and  $e_1, e_2$  data terms  $\approx$  unification in logic programming

# FUNCTIONAL LOGIC PROGRAMMING: EXAMPLES

### List concatenation:

append :: [a] -> [a] -> [a] append [] ys = ys append (x:xs) ys = x : append xs ys

#### Functional programming:

```
append [1,2] [3,4] \rightsquigarrow [1,2,3,4]
```

#### Logic programming:

append x y =:= [1,2]  $\rightarrow$ 

 ${x=[],y=[1,2]} | {x=[1],y=[2]} | {x=[1,2],y=[]}$ 

Last list element: (last xs | append ys [x] =:= xs = x

Infinite list of natural numbers:

from x = x : from (S x)
first Z ys = []
first (S x) (y:ys) = y : first x ys

Lazy functional programming:

first (S(SZ)) (from Z)  $\sim$  [Z,(SZ)]

Lazy functional logic programming:

first x (from y) =:= [Z]  $\rightarrow \{x=(S Z), y=Z\}$ 

FUNCTIONAL LOGIC PROGRAMMING: EXAMPLES

# **PROGRAMMING DEMAND-DRIVEN SEARCH**

Non-deterministic functions for generating permutations:



Sorting lists with test-of-generate principle:

sorted [] = []
sorted [x] = [x]
sorted (x:y:ys) | x<=y = x : sorted (y:ys)
psort xs = sorted (permute xs)</pre>

#### Advantages of non-deterministic functions as generators:

- → demand-driven generation of solutions (due to laziness)
- → modular program structure

psort [5,4,3,2,1]  $\rightsquigarrow$  sorted (permute [5,4,3,2,1])

$$\rightarrow^*$$
 sorted (5:4:permute [3,2,1]) |  $\cdots$ 

undefined: discard this alternative

#### Effect: Permutations of [3,2,1] are not enumerated!

Permutation sort for [n, n-1, ..., 2, 1]: #or-branches/disjunctions

Length of the list:	4	5	6	8	10
generate-and-test	24	120	720	40320	3628800
test-of-generate	19	59	180	1637	14758
# SEARCH STRATEGIES AND ENCAPSULATED SEARCH

### How to deal with non-deterministic computation steps?

- $\rightarrow$  explore alternatives in parallel  $\sim$  parallel architectures
- → explore alternatives by backtracking ~> Prolog
- → support flexible search strategies ~> encapsulate search

Disadvantages of fixed search (like backtracking):

- ➔ no application-dependent strategy or efficiency control
- ➔ global search: local search has global effects
- → I/O operations not backtrackable
- ➔ problems with concurrency and backtracking

Solution: provide primitives for user-definable search strategies (Oz [Schulte/Smolka 94], Curry [Hanus/Steiner 98])

# ENCAPSULATED SEARCH

#### Idea:

Compute until a non-deterministic step occurs, then give programmer control over this situation

### Search:

- → solve constraint
- → evaluate until failure, success, or non-determinism
- → return result in a list

First approach to primitive **search operator**:

try :: Constraint -> [Constraint]

SEARCH OPERATOR: FIRST APPROACH

try :: Constraint -> [Constraint]

f 0 = 2
f 1 = 3

try 
$$(1=:=2)$$
 $\sim \in$  []failuretry  $([x]=:=[0])$  $\sim \in$  [x=:=0]successtry  $(f x =:= 3)$  $\sim \in$  [x=:=0 & f 0 =:= 3,x=:=1 & f 1 =:= 3]

Problem: incompatible bindings for x in disjunctions!

Solution: abstract search variable in constraints: x - c

SEARCH OPERATOR: FINAL APPROACH

Search goal: constraint with abstracted search variable

Search operator try: maps search goal into list of search goals

try :: (a->Constraint) -> [a->Constraint]

	f 0 = 2
l	f 1 = 3

## **ENCAPSULATED SEARCH: SEARCH STRATEGIES**

try  $x \rightarrow c$ : evaluate c, stop after non-deterministic step

**Depth-first search:** collect all solutions in a list

all :: (a->Constraint) -> [a->Constraint]
all g = collect (try g)
where collect [] = []
 collect [g] = [g]
 collect (g1:g2:gs) = concat (map all (g1:g2:gs))

all (\xs -> append xs [1] =:= [0,1])  $\sim$  [\xs -> xs =:= [0]]

# **ENCAPSULATED SEARCH: FURTHER SEARCH STRATEGIES**

• compute only the first solution:

once g = head (all g) where head (x:xs) = x

Note: lazy evaluation is important here!

(strict languages, like Oz, must define new search operator)

 $\sim$  lazy evaluation supports better reuse

- findall, best solution search, parallel search, ...
- negation as failure:

naf c = (all  $\_->c$ ) =:= []

 $\rightsquigarrow$  control failures

## HANDLING SOLUTIONS

Extract value of the search variable by application of search goal:

**Prolog's findall:** 

unpack :: (a->Constraint) -> a
unpack g | g x = x where x free
findall g = map unpack (all g)

Compute all splittings of a list:

```
findall (\(x,y) -> append x y =:= [1,2])

\stackrel{*}{\Rightarrow} [([],[1,2]), ([1],[2]), ([1,2],[])]
```

## EXPLOITING LAZINESS

Show a list of search goals, as requested by the user:

printloop [] = putStr "no\n"
printloop (a:as) = browse a >> putStr "? " >>
 getChar >>= evalAnswer as
evalAnswer as ';' = newline >> printloop as
evalAnswer as '\n' = newline >> putStr "yes\n"

Laziness easily supports demand-driven encapsulated search

- ⇒ Separation of Logic and Control
- $\Rightarrow$  Modularity:
  - Prolog's top-level with breadth-first search: prolog\_bfs g = printloop (bfs g)
  - Prolog's top-level with depth-bounded search:
     prolog\_bound g bd = printloop (bound g bd)

MONADIC INPUT/OUTPUT

Problem: Handling input/output in a declarative manner?

Solution: Consider the external world as a parameter to all I/O operations (Haskell, Mercury)

I/O actions: transformations on the external world

Interactive program: sequence(!) of actions applied to the external world

Type of I/O actions:  $(IO a \approx World \rightarrow (a, World))$ 

But: the "world" is implicit parameter, not explicitly accessible!

#### Some primitive I/O actions:.

getChar	•••	IO Char	 read character from stdin
putChar	::	Char -> IO ()	 write argument to stdout
return	•••	a -> IO a	 do nothing and return argument

getChar applied to a world  $\rightsquigarrow$  character + new (transformed) world

**Compose actions:** (>>=) :: IO a -> (a -> IO b) -> IO b getChar >>= putChar: copy character from input to output

Specialized composition: ignore result of first action:

(>>) :: IO a -> IO b -> IO b x >> y = x >>=  $\_->y$  **Example:** output action for strings (String  $\approx$  [Char])

```
putStr :: String -> IO ()
putStr [] = return ()
putStr (c:cs) = putChar c >> putStr cs
```

#### Example: read a line

#### Monadic composition not well readable

→ syntactic sugar: Haskell's do notation

do  $p <-a_1 \approx a_1 \gg = p \rightarrow a_2$  $a_2$ 

Example: read a line (with do notation)

Note: no I/O in disjunctions ("cannot copy the world")

 $\sim$  encapsulate search between I/O actions

# **CONSTRAINT PROGRAMMING**

## Logic Programming:

- → compute with partial information (constraints)
- → data structures (constraint domain): constructor terms
- → basic constraint: (strict) equality
- → constraint solver: unification

Constraint Programming: generalizes logic programming by

- → new specific constraint domains (e.g., reals, finite sets)
- → new basic constraints over these domains
- → sophisticated constraint solvers for these constraints

## CONSTRAINT PROGRAMMING OVER REALS

Constraint domain: real numbers

Basic constraints: equations / inequations over real arithmetic expressions

Constraint solvers: Gaussian elimination, simplex method

**Examples:** 

5.1 =:= x + 3.5  $\rightsquigarrow \{x=1.6\}$ 

 $x \le 1.5 \& x+1.3 \ge 2.8 \quad \rightsquigarrow \quad {x=1.5}$ 

EXAMPLE: CIRCUIT ANALYSIS

Define relation cvi between electrical circuit, voltage, and current

Circuits are defined by the data type

data Circuit = Resistor Float | Series Circuit Circuit | Parallel Circuit Circuit :

Rules for relation cvi:

cvi (Resistor r) v i = v =:= i \* r -- Ohm's law cvi (Series c1 c2) v i = -- Kirchhoff's law v=:=v1+v2 & cvi c1 v1 i & cvi c2 v2 i cvi (Parallel c1 c2) v i = -- Kirchhoff's law i=:=i1+i2 & cvi c1 v i1 & cvi c2 v i2 Querying the circuit specification:

### Current in a sequence of resistors:

### Relation between resistance and voltage in a circuit:

cvi (Series (Series (Resistor r) (Resistor r)) (Resistor r)) v 5.0  $\rightsquigarrow$  {v=15.0\*r}

Also synthesis of circuits possible

# **CONSTRAINT PROGRAMMING WITH FINITE DOMAINS**

Constraint domain: finite set of values

Basic constraints: equality / disequality / membership / ...

Constraint solvers: OR methods (e.g., arc consistency)

Application areas: combinatorial problems (job scheduling, timetabling, routing,...)

### General method:

- ① define the domain of the variables (possible values)
- ② define the constraints between all variables
- ③ "labeling", i.e., non-deterministic instantiation of the variables

constraint solver reduces the domain of the variables by sophisticated pruning techniques using the given constraints

Usually: finite domain  $\approx$  finite subset of integers

# EXAMPLE: A CRYPTO-ARITHMETIC PUZZLE

Assign a different digit to each different letter		S	е	e n	d	
such that the following calculation is valid:	+	m		o r	е	
	m	0	n	n e	У	
puzzle s e n d m o r y =						
domain [s,e,n,d,m,o,r,y] 0 9 & -		defi	ne	doma	in	
s > 0 & m > 0 & -		defi	ne	cons	train	ts
all_different [s,e,n,d,m,o,r,y] &						
1000 * s + 100 * e + 1	10	* n	+ d			
+ $1000 * m + 100 * o + 1$	10	* r	+ e			
= 10000 * m + 1000 * o + 100 * n + 100	10	* e	+ y	&		
labeling [s,e,n,d,m,o,r,y] -		inst	ant	iate	vari	ables
puzzle s e n d m o r y $\rightsquigarrow$ {s=9,e=5,n=	=6,0	d=7,	m=1.	,o=0	,r=8,	y=2}

### Disadvantage of narrowing:

- $\rightarrow$  functions on recursive data structures  $\sim$  narrowing may not terminate
- $\rightarrow$  all rules must be explicitly known  $\sim$  combination with external functions?

Solution: Delay function calls if a needed argument is free

→ residuation principle [Aït-Kaci et al. 87] (used in Escher, Le Fun, Life, NUE-Prolog, Oz,...)

Distinguish: rigid (consumer) and flexible (generator) functions

Necessary: Concurrent conjunction of constraints:  $c_1 \& c_2$ 

Meaning: evaluate  $c_1$  and  $c_2$  concurrently, if possible

FLEXIBLE VS. RIGID FUNCTIONS

$$f 0 = 2$$
  
 $f 1 = 3$ 

rigid/flexible status not relevant for ground calls:

f 1  $\rightarrow$  3

f flexible:

f x =:= y  $\rightsquigarrow$  {x=0,y=2} | {x=1,y=3}

f rigid:

Default in Curry: constraints are flexible, all others are rigid

# PARALLEL FUNCTIONAL PROGRAMMING

## Parallel evaluation of arguments:

f t1 t2 = letpar 
$$x = g t1$$
  
y = h t2 in k x y

with concurrent conjunction of equations:

f t1 t2 | x =:= g t1 & y = h t2 = k x y where x,y free

Skeleton-based parallel programming:

farm: parallel version of map

## **EXTERNAL FUNCTIONS**

External functions: implemented in another language (e.g., C, Java,...)

Conceptually definable by an infinite set of equations, e.g.,

0+0 = 0 1+0 = 1 2+0 = 20+1 = 1 1+1 = 2 ... 0+2 = 2 ...

Definition not accessible, infinite disjunctions

- suspend external function calls until arguments are fully known, i.e., ground [Bonnier/Maluszynski 88, Boye 91]
- no extension to presented computation model (external functions are rigid), but not possible in narrowing-based languages!
- → reuse of existing libraries

# STANDARD ARITHMETIC

Implementation of standard arithmetic (+, -, \*,...) as external functions:

- $0, 1, 2, \ldots$ : constructors
- +, -, \*,...: external functions

 $x = := 2 + 3 + 4 \quad \rightsquigarrow \quad \{x = 14\}$  $x = := 2 + 3 + y \quad \rightsquigarrow \quad \{\} \quad x = := 6 + y \quad (suspend)$ 

 $\begin{array}{ll} x + x = := y & x = := 2 \\ & & \\ &$ 

 $\Rightarrow$  Rigid functions as passive constraints (Life)

External functions as passive constraints:

digit 0 = success ... digit 9 = success

The constraint digit acts as a generator:

x+x=:=y & x\*x=:=y & digit x $\rightsquigarrow \{x=0, y=0\} \mid \{x=2, y=4\}$ 

# HIGHER-ORDER FUNCTIONAL LOGIC PROGRAMMING

Functional programming: map (1 +) [2,3,4]  $\rightarrow$  [3,4,5] Logic programming: map f [2,3,4] =:= [3,4,5]  $\rightarrow$  ???

- $\rightarrow$  consider application function f x = (f x) as external
- → consider partial applications as data terms
- → first-order definition of application function (\$) (as in [Warren 82]):

(+) \$ x = (+ x) -- right-hand side is data term
(+ x) \$ y = x+y -- evaluate right-hand side

HIGHER-ORDER FUNCTIONAL LOGIC PROGRAMMING

Reasonable: application function (\$) is rigid

```
\rightsquigarrow delay applications of unknown functions
```

```
\rightsquigarrow map f [2,3,4] suspends
```

### Other solutions possible but more expensive:

- ightarrow (\$) is flexible ightarrow guess unknown functions
- solver for higher-order equations
   (higher-order unification, higher-order needed narrowing)

# UNIFICATION OF DECLARATIVE COMPUTATION MODELS

Computation model	Restrictions on programs
Needed narrowing	inductively sequential rules; optimal strategy
Weakly needed narrowing (~Babel)	only flexible functions
Resolution (~Prolog)	only (flexible) predicates ( $\sim$ constraints)
Lazy functional languages (~Haskell)	no free variables in expressions
Parallel functional langs. (~Goffin, Eden)	only rigid functions, concurrent conjunction
Residuation (~Life, Oz)	constraints are flexible; all others are rigid

# **CONCURRENT OBJECTS WITH STATE**

## Modeling objects with state as a (rigid!) constraint function:

- ➔ first parameter: current state
- → second parameter: message stream (rigid  $\approx$  wait for input)

## Example: Counter object

```
data CounterMessage = Set Int | Inc | Get Int
counter :: Int -> [CounterMessage] -> Constraint
counter eval rigid -- declare as rigid
counter _ (Set v : ms) = counter v ms
counter n (Inc : ms) = counter (n+1) ms
counter n (Get v : ms) = v=:=n & counter n ms
counter _ [] = success
```

# **CONCURRENT OBJECTS WITH STATE: A COUNTER**

```
counter _ (Set v : ms) = counter v ms
counter n (Inc : ms) = counter (n+1) ms
counter n (Get v : ms) = v=:=n & counter n ms
counter _ [] = success
```

Also: incremental instantiation of s (message sending)

Several sending processes  $\rightsquigarrow$  merge message streams

# PORTS FOR DISTRIBUTED SYSTEMS

Distributed systems:  $n \rightarrow 1$ -communication with dynamic connections

**Port** [Janson et al. 93, AKL]: constraint between multiset p and stream s satisfied if elements in p and s are identical



Two constraints on ports:

openPort p s	open port $p$ with stream s
1	constrain to hold massage

send m p constrain p to hold message m

Previous counter with two clients:

```
openPort p s &> counter 0 s & client1 p & client2 p
```

# PORTS FOR DISTRIBUTED SYSTEMS

- communication based on logic (constraint solving)
- simple extension of base semantics
- send instantiates end of stream s (in constant time)

s\_tail =:= (m:new\_s\_tail)

 $\rightsquigarrow$  strict communication

- provides efficient implementation (senders have no access to old messages)
- free variables in messages  $\approx$  reply channels
- dynamic extension of senders (pass port variable)

# EXTERNAL PORTS

I/O actions for external communication

(between different programs running on different machines):

```
openNamedPort :: String -> IO [a]
```

```
connectPort :: String -> IO (Port a)
```

openNamedPort pn: open new external port with global name pn and return stream of incoming messages

connectPort pn: return port with global name pn

(similar concepts: external objects in Oz, registered processes in Erlang)

### A simple example: a global counter server

The server side: (started on medoc.cs.uni-kiel.de)

```
main = openNamedPort "counter" >>= c_server
```

```
c_server s | counter 0 s = done
```

The client side:

client pn m = connectPort pn >>= sendPort m

sendPort msg p | send msg p = done

### Increment the global counter:

client "counter@medoc.cs.uni-kiel.de" Inc

#### Ask the counters current value:

client "counter@medoc.cs.uni-kiel.de" (Get v)  $\sim$  {v=...}

# A NAME SERVER

Messages: "PutName n i" (assign i to name n) "GetName n i"

#### The client side:

client "nameserver@..." (PutName "talk" 42) client "nameserver@..." (GetName "talk" x)  $\rightsquigarrow \{x=42\}$ 

# A HIERARCHICAL NAME SERVER

Internet domain name server: ask master server if name locally unknown Implementation by slight modification of previous name server:
# A COMPUTATION SERVER

Strict communication, no RPCs ~> no direct way to distribute work

Computation server: accepts messages (f, x, y)

start\_cserver = openNamedPort "compserver" >>= compserver
compserver ((f,x,y) : ms) | y=:=(f x) = compserver ms

Client side: client "compserver@cs" (prime,1000,p)  $\rightsquigarrow$  {p=7919}

- → consider partially applied function calls as data terms
- → asynchronous RPCs (free result variable ≈ "promise" [Liskov/Shrira 88])
- → concurrent server:

compserver eval rigid compserver ((f,x,y) : ms) = y=:=(f x) & compserver ms

# A MODEL FOR MULTI-PARADIGM PROGRAMMING

## Integration of different programming paradigms is possible

Functional programming is a good starting point:

- $\rightarrow$  lazy evaluation  $\sim$  modularity, optimal evaluation
- → higher-order functions ~> code reuse, design patterns
- → polymorphism ~> type safety, static checking

Stepwise extensible in a conservative manner to cover

- → logic programming: non-determinism, free variables
- → constraint programming: specific constraint structures
- concurrent programming: suspending function calls, synchronization on logical variables
- → object-oriented programming: constraint functions, ports
- → imperative programming: monadic I/O, sequential composition
- → distributed programming: external ports

# WHY INTEGRATION OF DECLARATIVE PARADIGMS?

- more expressive than pure functional languages (compute with partial information/constraints)
- more structural information than in pure logic programs (functional dependencies)
- more efficient than logic programs (determinism, laziness)
- functions: declarative notion to improve control in logic programming
- avoid impure features of Prolog (arithmetic, I/O)
- combine research efforts in FP and LP
- do not teach two paradigms, but one: declarative programming [Hanus PLILP'97]
- choose the most appropriate features for application programming

# APPLICATION OF MULTI-PARADIGM LANGUAGES

## So far: high-level approach to

- → search problems
- → constraint solving
- → distributed systems

#### In the following: appropriate to develop domain-specific languages for

- ➔ graphical user interfaces
- → parsing
- → HTML/CGI programming

FUNCTIONAL LOGIC GUI PROGRAMMING

#### [Hanus PADL'00]

Graphical User Interfaces (GUIs) have a

- $\rightarrow$  layout structure  $\sim$  hierarchical structure, algebraic data type
- ightarrow logical structure  $\sim 
  ightarrow$  dependencies in the layout structure

Tcl/Tk: assign strings to layout elements  $\rightsquigarrow$  run-time errors Here: use logical variables as references  $\rightsquigarrow$  compiler errors

A simple "Hello world" GUI:



runWidget "Hello"

(TkCol [TkLabel [TkText "Hello world!"], TkButton tkExit [TkText "Stop"]])

# LAYOUT STRUCTURE OF GUIS

Specify hiearchical GUI layout as a "TkWidget" term:

```
data TkWidget a =
   TkButton (GUIRef -> a) [TkConfItem a]
   IKCheckButton [TkConfItem a]
   IKEntry [TkConfItem a]
   IKLabel [TkConfItem a]
   IKScale Int Int [TkConfItem a]
   IKTextEdit [TkConfItem a]
   IKTextEdit [TkConfItem a]
   IKRow [TkWidget a]
   IKCol [TkWidget a]
```

EXAMPLE: A COUNTER GUI

## A specification of a counter GUI:



TkCol

```
[TkEntry [TkRef val, TkText "0"],
TkRow [TkButton (tkUpdate incr val) [TkText "Increment"],
TkButton (tkSetValue val "0") [TkText "Reset"],
TkButton tkExit [TkText "Stop"]]]
```

- → the free variable val is a reference to the entry widget
- → val is used in the event handlers of other widgets
- → val is part of the logical structure of the GUI

# LOGICAL STRUCTURE OF GUIS

## Configuration options for GUIs:

```
data TkConfItem a =
   TkText String -- initial text
   ITkBackground String -- background color
   ITkRef TkRefType -- widget reference
   ITkCmd (GUIRef -> a) -- event handler
   :
```

TkRef: reference to a widget, used in event handlers
(TkRefType is abstract ~> argument is a logical variable)
tkExit :: GUIRef -> IO ()
tkGetValue :: TkRefType -> GUIRef -> IO String
tkSetValue :: TkRefType -> String -> GUIRef -> IO ()
tkUpdate :: (String->String) -> TkRefType -> GUIRef -> IO ()

Remark: event handlers also available as constraints

# EXAMPLE: TEMPERATURE CONVERTER

## Convert a temperature from Celsius into Fahrenheit:

Temperature Conversion
Temperature in Celsius:
30
Temperature in Fahrenheit: 86

TkCol [TkLabel [TkText "Temperature in Celsius:"], TkScale 0 100 [TkRef cels, TkCmd convert], TkRow [TkLabel [TkText "Temperature in Fahrenheit: "], TkMessage [TkRef fahr, TkBackground "white"]]]

```
where cels,fahr free
```

```
convert gr =
  tkGetValue cels gr >>= \cs ->
  tkSetValue fahr (show ((parseIntcs) *9'div'5+32)) gr
```

# GUIS WITH STATE: A DESK CALCULATOR

Calculator			
1	2	3	•
4	5	6	-
7	8	9	•
С	0	-	1

Implementation consists of two parts:

- Object for storing the state state: (operand, accumulator function) messages: Display s, Button b
- 2. GUI for showing the state

#### **Object for storing the state:**

Message Button b: the user has pressed button b calcMgr (d,f) (Button b : ms) | isDigit b = calcMgr (10\*d+ordb-ord'0', f) ms | b=='+' = calcMgr (0, ((f d) +)) ms | b=='-' = calcMgr (0, ((f d) -)) ms | b=='\*' = calcMgr (0, ((f d) \*)) ms | b=='/' = calcMgr (0, ((f d) 'div')) ms | b=='=' = calcMgr (f d, id) ms | b=='C' = calcMgr (0, id) ms

#### **GUI for showing the state** with a reference cm to calculator object:

calc\_GUI cm = TkCol [TkEntry [TkRef display, TkText "0"],

💽 Ca	lculate	or 👘	2
31875			
1	2	3	•
4	5	6	-
7	8	9	•
С	0	-	1

TkRow (map cbutton ['1', '2', '3', '+']), TkRow (map cbutton ['4', '5', '6', '-']), TkRow (map cbutton ['7', '8', '9', '\*']), TkRow (map cbutton ['C', '0', '=', '/'])]

where display free

→ model-view-controller paradigm à la Smalltalk-80

→ different (distributed) views on one application

# FUNCTIONAL LOGIC GUI PROGRAMMING: SUMMARY

# Functional features useful for

- → layout specification
- → event handlers (data structures with functional components)
- → application-oriented extensions

## Logic programming features useful for

- → dealing with dependencies inside a structure (free variables)
- → handling state (concurrent objects)

Distributed features  $\rightsquigarrow$  GUIs for distributed applications

Specification (rather than imperative programming) of GUIs

Domain-specific language for GUIs, but:

no extension to base language necessary

# FUNCTIONAL LOGIC PROGRAMMING OF PARSERS

## [Caballero/Lopez-Fraguas FLOPS'99]

## Logic programming of parsers:

- → nonterminals consume corresponding tokens (difference lists)
- → definite clause grammars for nice notation
- → non-deterministic grammars/parsing
- → resulting representations as arguments

## Functional programming of parsers:

- ➔ parsers consume corresponding tokens
- ➔ powerful parser combinators
- ➔ more complex handling of alternatives and representations

## **Functional logic programming of parsers:**

simpler handling of representations and alternatives due to

- → non-deterministic functions
- ➔ free variables as arguments

Parser ≈ function of type [token] -> [token] Argument: list of tokens to be parsed Result: list of remaining unparsed tokens

```
A parser recognizing token 'a':
```

```
parse_a ('a':ts) = ts
```

```
A parser recognizing a given token:
terminal sym (t:ts) | sym=:=t = ts
```

Parser recognizing the empty word:

```
empty sentence = sentence
```

# PARSER COMBINATORS

#### Parser combinators: higher-order functions to combine parsers

```
Alternative of two parsers p and q: combinator p <|> q
(p <|> q) sentence = p sentence
(p <|> q) sentence = q sentence
```

```
Sequence of two parsers p and q: combinator p <*> q
(p1 <*> p2) s0 | p1 s0 =:= s1 = p2 s1 where s1 free
```

```
Repetition of a parser: (zero or more times)
star p = (p <*> star p) <|> empty
```

```
Parser for a(a|b)*:
terminal 'a' <*> star (terminal 'a' <|> terminal 'b')
```

## A parser for palindromes over the alphabet $\{a, b\}$

```
pali = empty <|> a <|> b <|> a<*>pali<*>a <|> b<*>pali<*>b
a = terminal 'a'
b = terminal 'b'
```

Checking a sentence for a palindrome:

pali "abaaba" =:= []

Using logic programming features, we can also generate palindromes:

# PARSERS WITH REPRESENTATIONS

Parsers should not only check a list of tokens but also return a representation (e.g., abstract syntax tree)

- $\rightarrow$  Functional programming: parsers have result (*rep*, *tokens*)
- → Logic programming: parsers have rep argument  $\sim$  simpler definitions

Parser with representation  $\approx$  rep -> [token] -> [token]

Representation argument:

- → usually free variable
- → will be instantiated during parsing

# PARSER COMBINATORS WITH REPRESENTATIONS

Alternative of two parsers p and q: combinator p <||> q (p <||> q) rep = p rep <|> q rep

(reuse combinator for parsers without representation)

Attach representation exp to a parser p: combinator p >>> exp
(p >>> exp) rep s\_in | p s\_in =:= s\_out & exp =:= rep = s\_out
where s\_out free

```
At least one repetition of a parser:
some p = pr <*> star prs >>> (r:rs) where r,rs free
```

EXAMPLE: PARSER FOR ARITHMETIC EXPRESSIONS

- expr = term t <\*> plus\_minus op <\*> expr e >>> (op t e)
   <||> term
- term = factor f <\*> prod\_div op <\*> term t >>> (op f t)
   <||> factor

```
num = some digit l >>> numeric_value l
```

```
Example: expr val "(10+5*2)/4" =:= [] \rightsquigarrow {val=5}
```

# FUNCTIONAL LOGIC PARSING: SUMMARY

## Higher-order features useful for

- → combining parsers (parsers are functions)
- → computing representations

#### Logic programming features useful for

- → dealing with alternatives (non-deterministic functions)
- → managing representations (free variables in arguments)
- → parsing with constraints

Domain-specific language for parsing, but:

no extension to base language necessary

# APPLICATION: HTML/CGI PROGRAMMING

Early days of the World Wide Web: web pages with static contents Common Gateway Interface (CGI): web pages with dynamic contents

Retrieval of a dynamic page:

- → server executes a program
- → program computes an HTML string, writes it to stdout
- → server sends result back to client

HTML with input elements (forms):

- → client fills out input elements
- ➔ input values are sent to server
- → server program decodes input values for computing its answer

# TRADITIONAL CGI PROGRAMMING

## CGI programs on the server can be written in any programming language

- → access to environment variables (for input values)
- → writes a string to stdout

## Scripting languages: (Perl, Tk,...)

- → simple programming of single pages
- ➔ error-prone: correctness of HTML result not ensured
- ➔ difficult programming of interaction sequences

Specialized languages: (MAWL, DynDoc,...)

- → HTML support (structure checking)
- → interaction support (partially)
- → restricted or connection to existing languages

# CGI PROGRAMMING IN A MULTI-PARADIGM LANGUAGE

## Library in multi-paradigm language

## Exploit functional and logic features for

- → HTML support (data type for HTML structures)
- → simple access to input values (free variables and environments)
- → simple programming of interactions (event handlers)
- → wrapper for hiding details

## Exploit imperative features for

→ environment access (files, data bases,...)

## Domain-specific language for HTML/CGI programming

# MODELING HTML

## Data type for representing HTML expressions:

data HtmlExp =	HText String		
I	HStruct String	[(String,String)]	[HtmlExp]

## Some useful abbreviations:

htxt	S	=	HText	(h	tmlQu	lote	e s)
bold	hexps	=	HStruc <sup>-</sup>	t	"B"	[]	hexps
italic	hexps	=	HStruc	t	"I"	[]	hexps
h1	hexps	=	HStruc <sup>-</sup>	t	"H1"	[]	hexps

- -- plain string
- -- bold font
- -- italic font
- -- main header

Example: [h1 [htxt "1. Hello World"],

italic [htxt "Hello"], bold [htxt "world!"]]

# → **1. Hello World**

Hello world!

. . .

#### Advantages:

- → static checking of HTML structure (well-balanced parentheses)
- ➔ flexible dynamic documents
- ➔ functions for computing HTML documents

Converting tree structure (leaves contain strings) into nested HTML lists:

```
data Tree a = Leaf a | Node [Tree a]
htmlTree :: Tree String -> [HtmlExp]
htmlTree (Leaf s) = [htxt s]
htmlTree (Node trees) = [ulist (map htmlTree trees)]
ulist :: [[HtmlExp]] -> HtmlExp
ulist items = HStruct "UL" [] (map litem items)
litem hexps = HStruct "LI" [] hexps
```

# HTML INPUT FORMS

# Specific HTML elements for dealing with user input

<INPUT TYPE="TEXT" NAME="INPTEXT" VALUE="fill out!">

Form is submitted  $\rightsquigarrow$ 

clients sends the current value of this field (identified by "INPTEXT")

## Expressible as HTML term:

## Problems:

- → server program must decode input values
- → server program must know right names of field identifiers ("INPTEXT")
- → error-prone

# **ABSTRACT INPUT FORMS**

## **Solution:**

- → use free variables as references to input fields (CGI references)
- → collect input values in CGI environments: mapping from CGI references to strings
- → associate event handlers to submit buttons
- → event handlers take a CGI environment and produces an HTML form

## Implementation:

straightforward in a functional logic language!

# CGI references:

data CgiRef = CgiRef String -- data constructor not exported

- ➔ no construction of wrong references
- → only free variables of type CgiRef
- ➔ global wrapper function instantiates with the right strings

## HTML elements with CGI references:

data HtmlExp = ... | HtmlCRef HtmlExp CgiRef

HTML form: title + list of HTML expressions data HtmlForm = Form String [HtmlExp]

CGI environments: map CGI references to strings type CgiEnv = CgiRef -> String

Event handlers have type CgiEnv -> IO Form

Event handlers are associated to submit buttons:

user presses a submit button

 $\rightsquigarrow$  execute associated event handler with current environment

# EXAMPLE: FORM TO REVERSE/DUPLICATE A STRING

File Edit	View Go	Communicator		Help
Enter a strin	s I			
Reverse st	ring Duplic	ate string		
a		Fel Sta	10.40	

Form "Question" [htxt "Enter a string: ", textfield tref "", hr, button "Reverse string" revhandler, button "Duplicate string" duphandler]

```
where tref free
revhandler env = return $ Form "Answer"
    [h1 [htxt ("Reversed input: " ++ rev (env tref))]]
duphandler env = return $ Form "Answer"
    [h1 [htxt ("Duplicated input: " ++ env tref ++ env tref)]]
```

# ACCESSING THE WEB SERVER ENVIRONMENT

Form to show the contents of an arbitrary file stored at the server:

```
Form "Get File" [htxt "Enter local file name:",
textfield fileref "",
button "Get file!" handler]
```

where fileref free

```
handler env =
   do contents <- readFile (env fileref)
      return $ Form "Answer"
        [h1 [htxt ("Contents of file " ++ env fileref)],
            verbatim contents]</pre>
```

# HTML/CGI PROGRAMMING

The main form is executed by a wrapper function runcgi :: String -> IO HtmlForm -> IO ()

- → takes a title string and a form and transforms it into HTML text
- → replaces all CGI references by unique strings
- → decodes input values and invokes associated event handler

## Event handlers return forms rather than HTML expressions

- → sequences of interactions
- → use control abstractions (branching, recursion) of underlying language
- → state between interactions handled by CGI environments

Note: no language extension necessary (CGI library) multi-paradigm languages as scripting languages

# A Few Further Multi-Paradigm Languages

# Erlang (Ericsson)

- → developed by Ericsson for telecommunication applications
- concurrent functional language with features to support the development of robust distributed systems
- → reduced development time and maintainance

#### Escher (University of Bristol)

- → extension of Haskell by features for logic programming
- ➔ functions are evaluated by residuation
- → explicit disjunctions for logic programming
- → simplification rules for logic formulas

#### Mercury (University of Melbourne)

- → logic/functional language with highly optimized execution algorithm
- → origin: logic programming (syntax) with type/mode/determinism annotations
- → adapted concepts from functional programming, strict semantics

#### Oz (DFKI Saarbrücken)

- concurrent constraint language with features for higher-order functional, object-oriented, and distributed programming
- → operational behavior: residuation
- → search via explicit disjunctions and search operators

Toy (Univ. Complutense de Madrid)

- ➔ prototype for a functional logic language
- → based on lazy narrowing, supports non-deterministic functions
- → contraints, in particular, disequality constraints

... and, of course, there are many, many more...
## IMPLEMENTATIONS OF CURRY

Several implementations available:

- Interpreter in Prolog: TasteCurry-System
- Compiler Curry→Java [Hanus/Sadre ILPS'97/JFLP'99] (Java threads for concurrency and non-determinism)
  - → portable
  - → simplified implementation (garbage collection, threads)
  - → slow but (hopefully!) better Java implementations in the future
- [Antoy/Hanus FroCoS'00]: Efficient implementation by transformation into Sicstus-Prolog (reuse of various constraint solvers) (also Sloth-System [Mariño/Rey WFLP'98])
- ⇒ PACS (Portland Aachen Curry System)

http://www-i2.informatik.rwth-aachen.de/~hanus/pacs

• abstract Curry machine [Lux FLOPS'99]

## CONCLUSIONS

Appropriate abstractions are important for software development and maintainance

Multi-paradigm languages have the potential to express these abstractions

High-level languages support domain-specific languages

Multi-paradigm programming

- ➔ possible and advantageous
- → constraint functional logic programming: many improvements in recent years
- imperative/concurrent/distributed + declarative programming: possible but many different approaches

## More infos on Curry:

http://www-i2.informatik.rwth-aachen.de/~hanus/curry